ECMWF Data Assimilation Training Course 2017
Coupled Data Assimilation

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ECMWF

Basics of Coupled Assimilation

Difficulties in Coupled Assimilation

Potential benefits of coupled assimilation

Coupled assimilation at ECMWF

A simple example 1

Suppose we model the temperature in the room, but we spilt the room in half and have one temperature for each half; x_1 and x_2.
Now let us measure the temperature in each half of the room; y_1 and y_2.

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Uncoupled assimilation (3DVar)

$$\boldsymbol{x} = x_1, \boldsymbol{y} = y_1, H = 1, R = \sigma_{y_1}^2, P_b = \sigma_{x_1}^2$$

$$J_1(\boldsymbol{x}) = (x_{b_1} - x_1)\sigma_{x_1}^{-2}(x_{b_1} - x_1) + (y_1 - x_1)\sigma_{y_1}^{-2}(y_1 - x_1)$$

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Uncoupled assimilation (3DVar)

$$\boldsymbol{x} = x_1, \boldsymbol{y} = y_1, H = 1, R = \sigma^2_{y_1}, P_b = \sigma^2_{x1}$$

$$J_1(\boldsymbol{x}) = (x_{b_1} - x_1)\sigma^{-2}_{x_1}(x_{b_1} - x_1) + (y_1 - x_1)\sigma^{-2}_{y_1}(y_1 - x_1)$$

$$\boldsymbol{x} = x_2, \boldsymbol{y} = y_2, H = 1, R = \sigma^2_{y_2}, P_b = \sigma^2_{x_2}$$

$$J_2(\boldsymbol{x}) = (x_{b_2} - x_2)\sigma^{-2}_{x_2}(x_{b_2} - x_2) + (y_2 - x_2)\sigma^{-2}_{y_2}(y_2 - x_2)$$

A simple example 2

Coupled assimilation (3DVar)

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, H = I, R = \begin{bmatrix} \sigma_{y_1}^2 & 0 \\ 0 & \sigma_{y_2}^2 \end{bmatrix}, P_b = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_{12}}^2 \\ \sigma_{x_{12}}^2 & \sigma_{x_2}^2 \end{bmatrix}$$

$$J(\boldsymbol{x}) = \begin{bmatrix} x_{b_1} - x_1 \\ x_{b_2} - x_2 \end{bmatrix}^T \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_{12}}^2 \\ \sigma_{x_{12}}^2 & \sigma_{x_2}^2 \end{bmatrix}^{-1} \begin{bmatrix} x_{b_1} - x_1 \\ x_{b_2} - x_2 \end{bmatrix}$$
$$+(y_1 - x_1)\sigma_{y_1}^{-2}(y_1 - x_1) + (y_2 - x_2)\sigma_{y_2}^{-2}(y_2 - x_2)$$

A simple example 3

Suppose we stop observing y_2.

- In uncoupled 3DVar, $x_2 = x_{b_2}$, i.e. nothing happens for this variable
- In coupled 3DVar x_2 is still updated if $\sigma^2_{x_{12}} \neq 0$

A simple example 3

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- In uncoupled 3DVar, $x_2 = x_{b_2}$, i.e. nothing happens for this variable
- In coupled 3DVar x_2 is still updated if $\sigma^2_{x_{12}} \neq 0$

If we never observe y_2, the *cross-covariance* $\sigma^2_{x_{12}}$ allows us to constrain x_2.

Coupled assimilation 3DVar

Recall the 3DVar cost function:

$$J(\boldsymbol{x}) = \frac{1}{2}(\boldsymbol{x_b} - \boldsymbol{x})^T P_b^{-1}(\boldsymbol{x_b} - \boldsymbol{x}) + \frac{1}{2}(\boldsymbol{y} - \mathcal{H}(\boldsymbol{x}))^T R^{-1}(\boldsymbol{y} - \mathcal{H}(\boldsymbol{x}))$$

and its gradient

$$-\nabla J(\boldsymbol{x}) = P_b^{-1}(\boldsymbol{x_b} - \boldsymbol{x}) + H^T R^{-1}(\boldsymbol{y} - \mathcal{H}(\boldsymbol{x}))$$

There are two ways x_2 can influence x_1:

- H is a function of both x_1 and x_2
- $P_{b12} \neq 0$

Coupled assimilation 4DVar

Recall the 4DVar cost function:

$$J(\boldsymbol{x}) = \frac{1}{2}(\boldsymbol{x_b} - \boldsymbol{x})^T P_b^{-1}(\boldsymbol{x_b} - \boldsymbol{x}) + \frac{1}{2}\sum_k (\boldsymbol{y}_k - \mathcal{G}_k(\boldsymbol{x}))^T R_k^{-1}(\boldsymbol{y}_k - \mathcal{G}_k(\boldsymbol{x}))$$

and its gradient

$$-\nabla J(\boldsymbol{x}) = P_b^{-1}(\boldsymbol{x_b} - \boldsymbol{x}) + \sum_k M_k^T H_k^T R_k^{-1}(\boldsymbol{y}_k - \mathcal{G}_k(\boldsymbol{x}))$$

There is a third way that x_2 can influence x_1:

- $\mathcal{G}_k$ is a function of both x_1 and x_2, i.e. the coupled model has mixed the information over time

Thus in 4DVar, the implied cross-covariance between elements of the system also allow for information to be transferred, on top of those supplied in P_b.

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Long assimilation windows

- The longer the assimilation window, the more observations we get to put into our systems
- The longer the assimilation window, the more flow dependence we obtain in our solution - i.e. we become less reliant on the background error covariance that we specify at $t = 0$.

Timescales in the Earth System

▸ Microscale turbulence	minutes
▸ Mesoscale storms (tornadoes/thunderstorms)	hours
▸ Synoptic scale cyclones	days
▸ Planetary waves/blocking structures	weeks
▸ Intraseasonal features	months
▸ Seasonal cycles/ENSO	years

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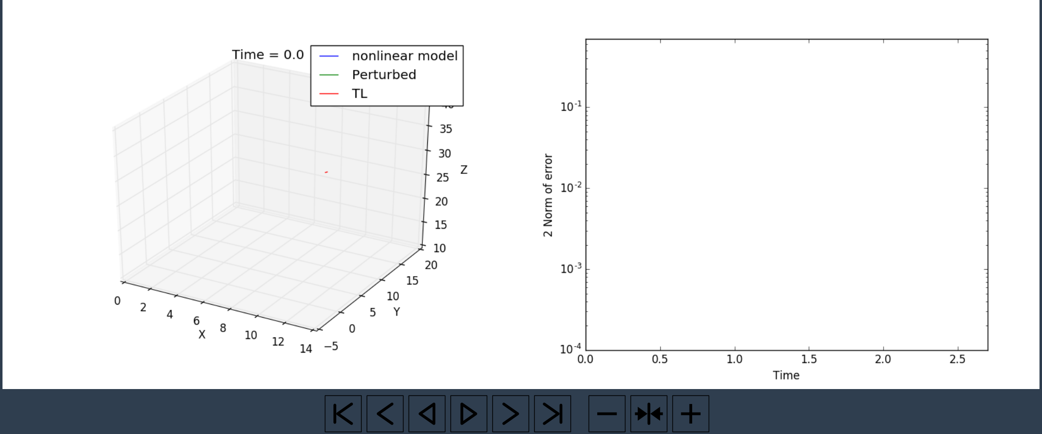
▸ Internal waves	hours
▸ Tides	days
▸ Mesoscale eddies	weeks/months
▸ ENSO	years
▸ Thermohaline circulation	centuries

Tangent linear model and approximation



Strongly and weakly coupled assimilation

Solving the system as described previously is known as strongly coupled 4DVar.

- P_b may or may not have off-diagonal blocks, i.e. cross-covariances
- Requires M^T of the fully coupled system.
- ! Problem: what if the coupled system is implemented in entirely different computer codes?

Strongly and weakly coupled assimilation

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- P_b may or may not have off-diagonal blocks, i.e. cross-covariances
- Requires M^T of the fully coupled system.
- ! Problem: what if the coupled system is implemented in entirely different computer codes?

One approach is known as weakly coupled 4DVar in which

$$M = \begin{bmatrix} M_1 & M_{12} \\ M_{21} & M_2 \end{bmatrix} := \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \quad \text{i.e.} \quad M\boldsymbol{x} = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} M_1 x_1 \\ M_2 x_2 \end{bmatrix}$$

$$M^T = \begin{bmatrix} M_1^T & M_{21}^T \\ M_{12}^T & M_2^T \end{bmatrix} := \begin{bmatrix} M_1^T & 0 \\ 0 & M_2^T \end{bmatrix} \quad \text{i.e.} \quad M^T\boldsymbol{x} = \begin{bmatrix} M_1^T & 0 \\ 0 & M_2^T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} M_1^T x_1 \\ M_2^T x_2 \end{bmatrix}$$

Weakly coupled assimilation in incremental 4DVar (1)

$$J(\boldsymbol{x}) = \tfrac{1}{2}(\boldsymbol{x}_b - \boldsymbol{x})^T P_b^{-1}(\boldsymbol{x}_b - \boldsymbol{x}) + \tfrac{1}{2}\sum_k (\boldsymbol{y}_k - \mathcal{G}_k(\boldsymbol{x}))^T R_k^{-1}(\boldsymbol{y}_k - \mathcal{G}_k(\boldsymbol{x}))$$

$$-\nabla J(\boldsymbol{x}) = P_b^{-1}(\boldsymbol{x}_b - \boldsymbol{x}) + \sum_k G_k^T R_k^{-1}(\boldsymbol{y}_k - \mathcal{G}_k(\boldsymbol{x}))$$

where

$$\mathcal{G}_k = \mathcal{H}_k \mathcal{M}_{t_0 \to t_k} \quad \text{and} \quad G_k^T = M_{t_0 \to t_k}^T H_k^T$$

Weakly coupled assimilation in incremental 4DVar (2)

Recall the linearisation state $\boldsymbol{x}^{(m)}$ such that

$$\boldsymbol{x} = \boldsymbol{x}^{(m)} + \delta\boldsymbol{x}^{(m)}$$

Then the cost function becomes

$$\begin{aligned} J(\delta\boldsymbol{x}^{(m)}) =& \tfrac{1}{2}(\boldsymbol{x}_b - \boldsymbol{x}^{(m)} - \delta\boldsymbol{x}^{(m)})^T P_b^{-1}(\boldsymbol{x}_b - \boldsymbol{x}^{(m)} - \delta\boldsymbol{x}^{(m)}) \\ &+ \tfrac{1}{2}(\boldsymbol{y} - \mathcal{G}(\boldsymbol{x}_b - \boldsymbol{x}^{(m)} - \delta\boldsymbol{x}^{(m)}))^T R^{-1}(\boldsymbol{y} - \mathcal{G}(\boldsymbol{x}_b - \boldsymbol{x}^{(m)} - \delta\boldsymbol{x}^{(m)})) \end{aligned}$$

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=& \tfrac{1}{2}(\boldsymbol{x}_b - \boldsymbol{x}^{(m)} - \delta\boldsymbol{x}^{(m)})^T P_b^{-1}(\boldsymbol{x}_b - \boldsymbol{x}^{(m)} - \delta\boldsymbol{x}^{(m)}) \\
&+ \tfrac{1}{2}(\boldsymbol{d}^{(m)} - G\delta\boldsymbol{x}^{(m)})^T R^{-1}(\boldsymbol{d}^{(m)} - G\delta\boldsymbol{x}^{(m)})
\end{aligned}$$

where $\boldsymbol{d}^{(m)} = \boldsymbol{y} - \mathcal{G}(\boldsymbol{x}^{(m)})$

Weakly coupled assimilation in incremental 4DVar (3)

- $\boldsymbol{d}^{(m)} = \boldsymbol{y} - \mathcal{G}(\boldsymbol{x}^{(m)})$
  - $\mathcal{G}(\boldsymbol{x}^{(m)}) = \mathcal{G}\begin{pmatrix} \boldsymbol{x}_1^{(m)} \\ \boldsymbol{x}_2^{(m)} \end{pmatrix}$ is computed with the coupled nonlinear model
- $G\delta\boldsymbol{x}^{(m)} = \begin{pmatrix} H_1 M_1 \delta\boldsymbol{x}_1^{(m)} \\ H_2 M_2 \delta\boldsymbol{x}_2^{(m)} \end{pmatrix}$
  - Computed using the uncoupled linearised model and observation operator (suitably interpolated)
- Thus in weakly coupled 4DVar the interaction between components happens though $\mathcal{G}$ each outer loop of the minimisation

Coupled assimilation with sequential DA techniques

- Many sequential techniques do not require the adjoint of the coupled model (i.e. Kalman filter, EnKF, particle filters, 3DVar, 4DEnVar)
- Thus the issue of multiple timescales are avoided
- Similarly, the issue of an incomplete gradient is avoided

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- Thus the issue of multiple timescales are avoided
- Similarly, the issue of an incomplete gradient is avoided
- Explicit cross-covariances may need to be specified (3DVar, particle filters)
- Localisation methods across the different components need to be specified (EnKF, 4DEnVar)

More general weakly coupled assimilation

We have seen that in weakly coupled 4DVar, the only interaction between components comes through the evolution of the nonlinear coupled models.

- This concept can be extended to assimilation methods other than 4DVar.

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We have seen that in weakly coupled 4DVar, the only interaction between components comes through the evolution of the nonlinear coupled models.

- This concept can be extended to assimilation methods other than 4DVar.
- The weak coupling refers to the coupling in the nonlinear trajectory.
- Each component does not need to use 4DVar as its assimilation method.
- For example, one could use a (simplified extended) Kalman filter for a soil moisture model.

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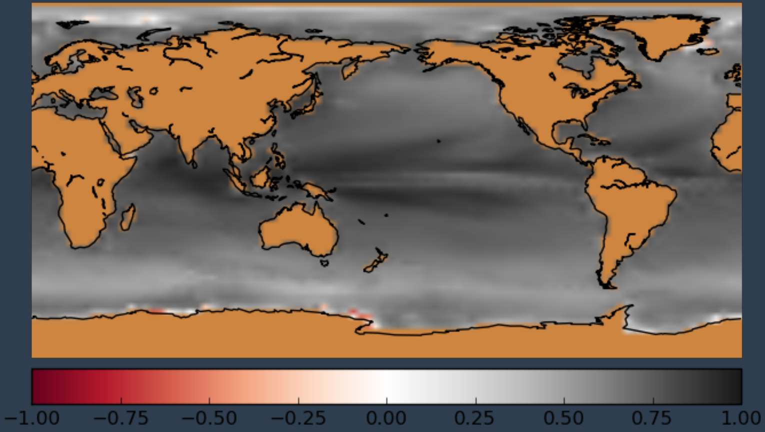
Better information flow

Observations of one component of the system can updated another, e.g.

- Observing surface winds could directly impact on the land surface model, e.g. through updating both surface temperatures and evaporation rates
- Observations of passive (chemical) tracers can be used to update atmospheric winds

Coupled balances

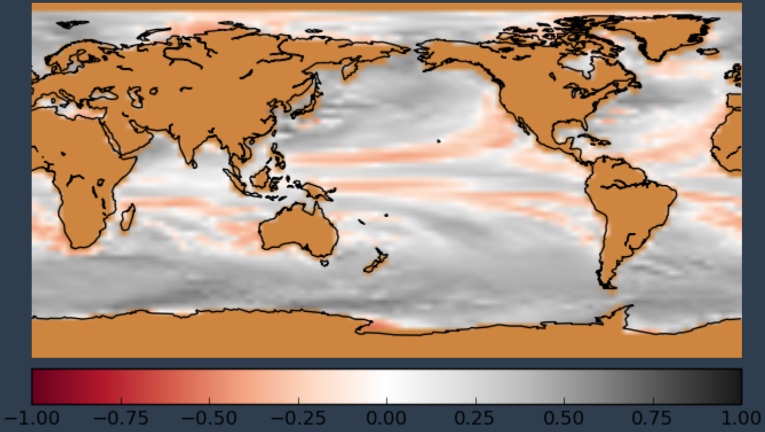
Temperature errors are correlated
Atmosphere Temperature - Ocean Temperature



P.A. Browne and P.J. van Leeuwen. Twin experiments with the equivalent weights particle filter and HadCM3. *Quarterly Journal of the Royal Meteorological Society*, 141(693 October 2015 Part B):3399--3414, 2015

Coupled balances

Wind speeds affect ocean cooling rates
Atmosphere Zonal Wind - Ocean Temperature



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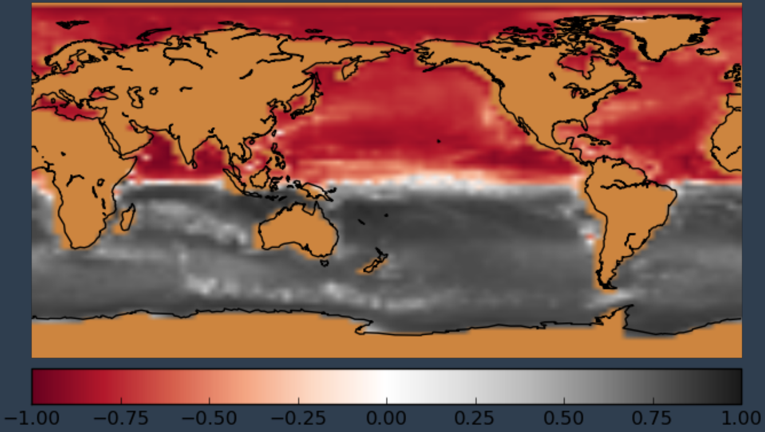
Coupled balances

Ekman circulation
Atmosphere Zonal Wind - Ocean Meridional Current



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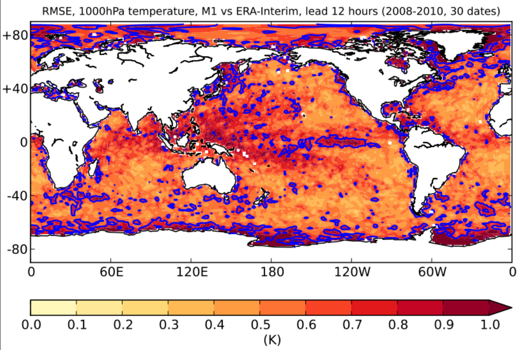
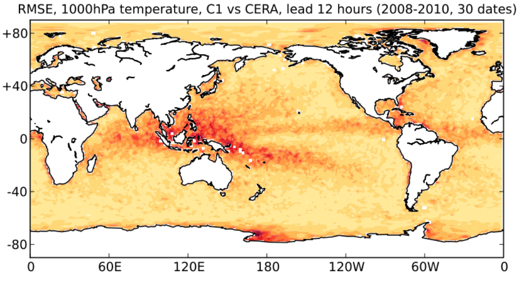
Reduced initialisation shock

Uncoupled assimilation

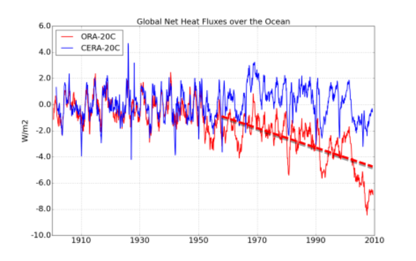


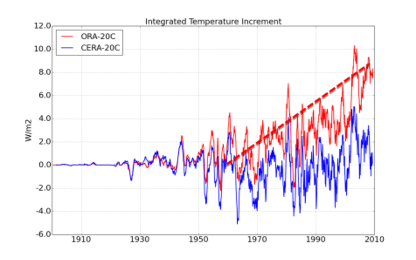
Coupled assimilation



David P. Mulholland, Patrick Laloyaux, Keith Haines, and Magdalena Alonso Balmaseda. Origin and Impact of Initialization Shocks in Coupled Atmosphere–Ocean Forecasts. *Monthly Weather Review*, 143:4631--4644, 2015

Balanced ocean-atmosphere analysis





Global net air-sea fluxes toward the ocean in CERA-20C and ORA-20C.

- Spurious trend in ORA-20C probably due to shift in wind forcing in ERA-20C (heat lost)

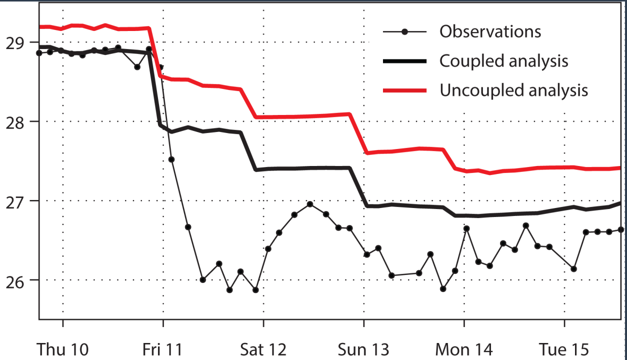
Ocean temperature increment in CERA-20C and ORA-20C.

- Increment in ORA-20C is trying to compensate for the heat lost
- CERA-20C fluctuates around zero suggesting a more balanced air-sea interface

Courtesy of E. de Boisséson

Cyclone tracking

Ocean temperature at 40 metres observed by an Argo float located on the track of the cyclone Phailin, 11 October 2013 in the Bay of Bengal.



Patrick Laloyaux, Jean-Noël Thépaut, and Dick Dee. Impact of Scatterometer Surface Wind Data in the ECMWF Coupled Assimilation System. *Monthly Weather Review*, 144(3):1203--1217, 2016

The temperature drop is due to the cold wake induced by the cyclone. The difference between the red and the black thick lines shows the impact of using a coupled assimilation system. Improvement through the better use of surface wind satellite measurements.

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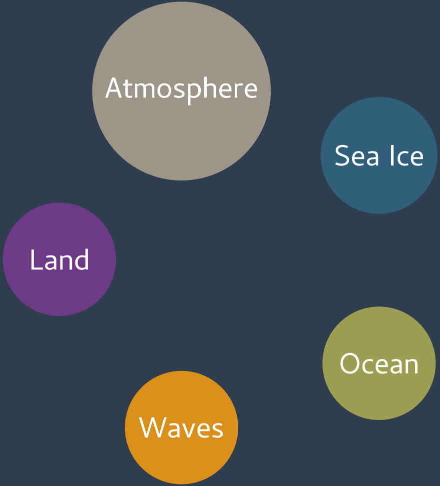
ECMWF
THE STRENGTH OF A COMMON GOAL
A ROADMAP TO 2025

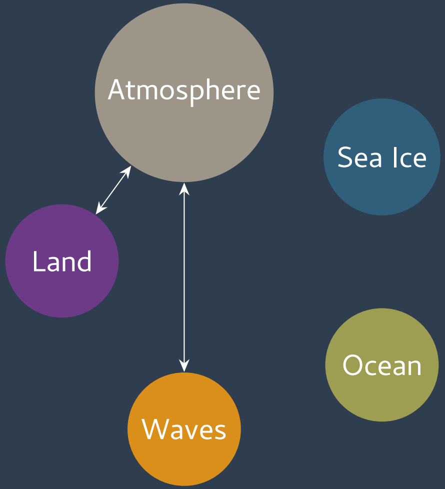
"ECMWF has started to explore a new coupled assimilation system to initialise the numerical weather forecast in a more comprehensive and balanced manner. Such an approach has the potential to better use satellite measurements and to improve the quality of our forecasts. It will generate a reduction of initialisation shocks in coupled forecasts by fully accounting for interactions between the components. It will also lead to the generation of a consistent Earth-system state for the initialisation of forecasts across all timescales",
ECMWF Roadmap to 2025

Components of the Earth System



Components of the Earth System

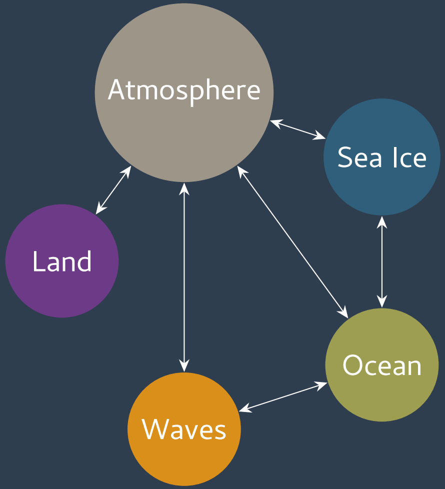
Coupled Models



- HRES NWP and ERA5

Components of the Earth System

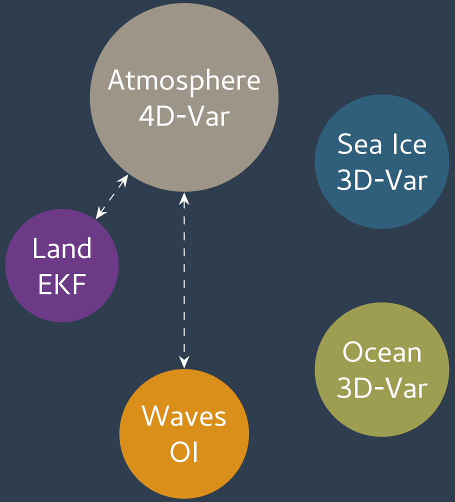
Coupled Models



- ENS/monthly, seasonal, and CERA

Components of the Earth System

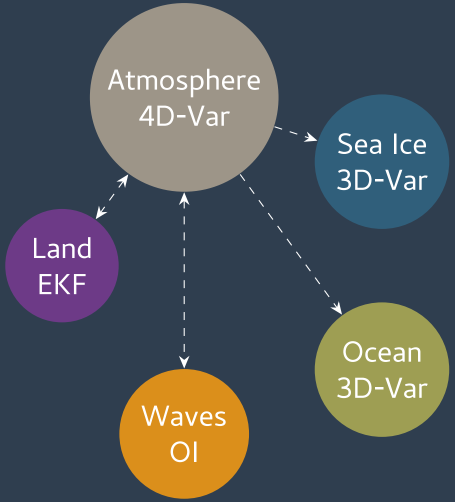
Coupled Assimilation



- HRES NWP and ERA5: land and waves weakly coupled

Components of the Earth System

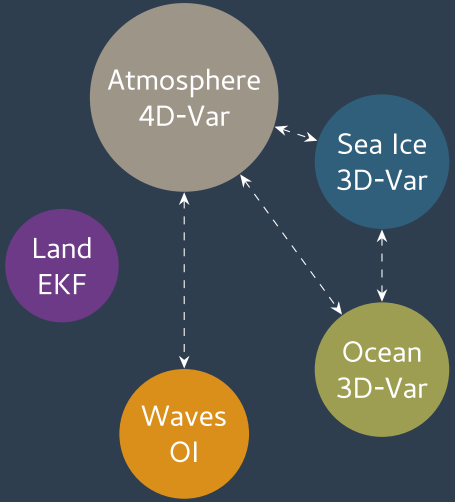
Coupled Assimilation



- ENS,monthly and seasonal, Ocean5/ORAS5: uncoupled ocean and sea ice assimilation

Components of the Earth System

Coupled Assimilation



- CERA-20C: outer-loop coupling for atm-ocean, sea ice

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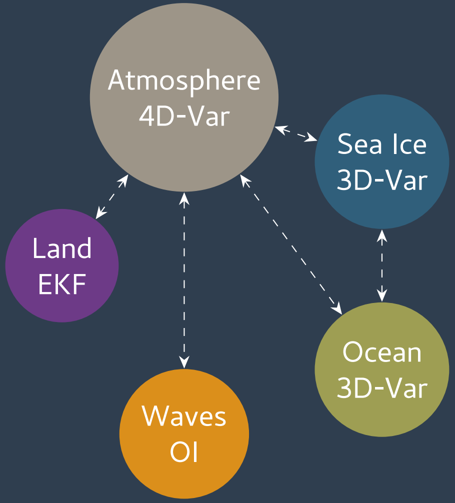
Coupled Assimilation



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- CERA-SAT with also land-atm weak coupling and full observing system

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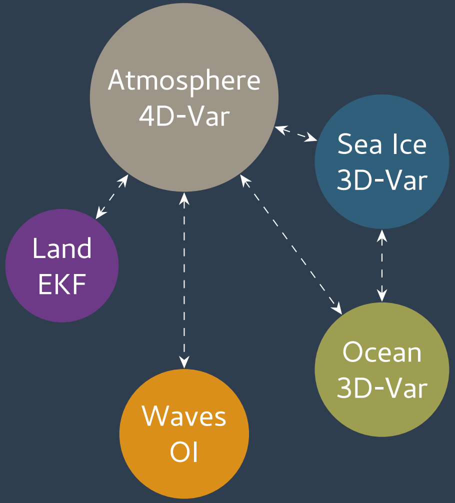
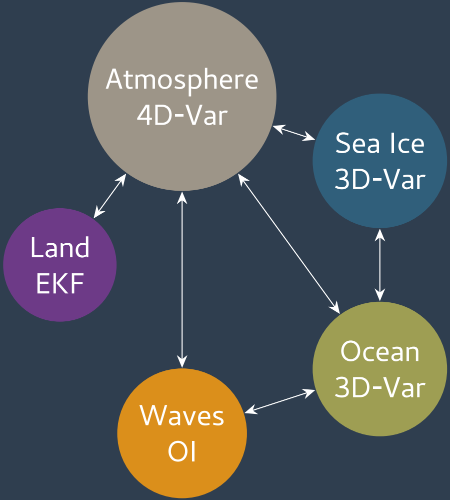


- CERA-20C: outer-loop coupling for atm-ocean, sea ice
- CERA-SAT with also land-atm weak coupling and full observing system
- Hence different coupling strategies are used for the different configurations

Towards an Earth System Approach

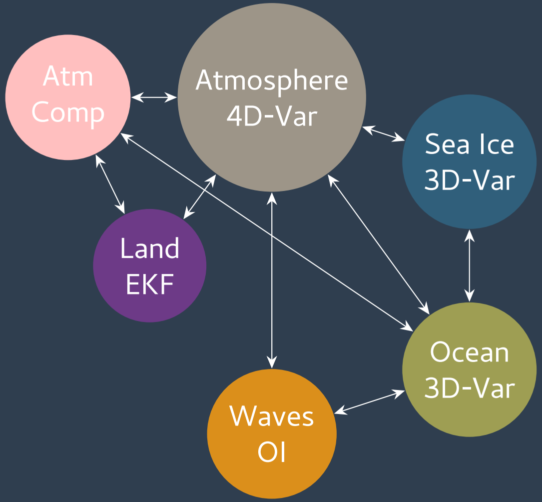
Coupled Assimilation - CERAv3/CERA100



- Consistency of the coupling approaches across the different components of the Earth system
- Comprehensive Earth system approach; atmosphere, land, ocean, sea ice, waves

Towards an Earth System Approach

Coupled Assimilation - CERAv3/CERA100



- Consistency of the coupling approaches across the different components of the Earth system
- Comprehensive Earth system approach; atmosphere, land, ocean, sea ice, waves, atmospheric composition

Summary

- Coupled data assimilation is, in theory, the same as multivariate DA
- Coupled data assimilation can improve balance in analyses and can increase the use of, and information gained from, observations
- Issues arise from:
  - Varying timescales in the different components - leads to poor TL approximation for "long" windows
  - Various components of the Earth system running separate models/executables - the full adjoint is not always available
- Weakly coupled assimilation is coupling at the outer loop level, where the full nonlinear model is coupled
- ECMWF is regularly doing coupled assimilation:
  - Atmosphere - land - wave in high resolution NWP and ERA5 reanalysis
  - Atmosphere - land - wave - sea ice - ocean in CERA-20C and CERA-SAT reanalyses
- Specifying cross-covariances is a big future challenge

Assimilation, Star Trek style