# Introduction to chaos

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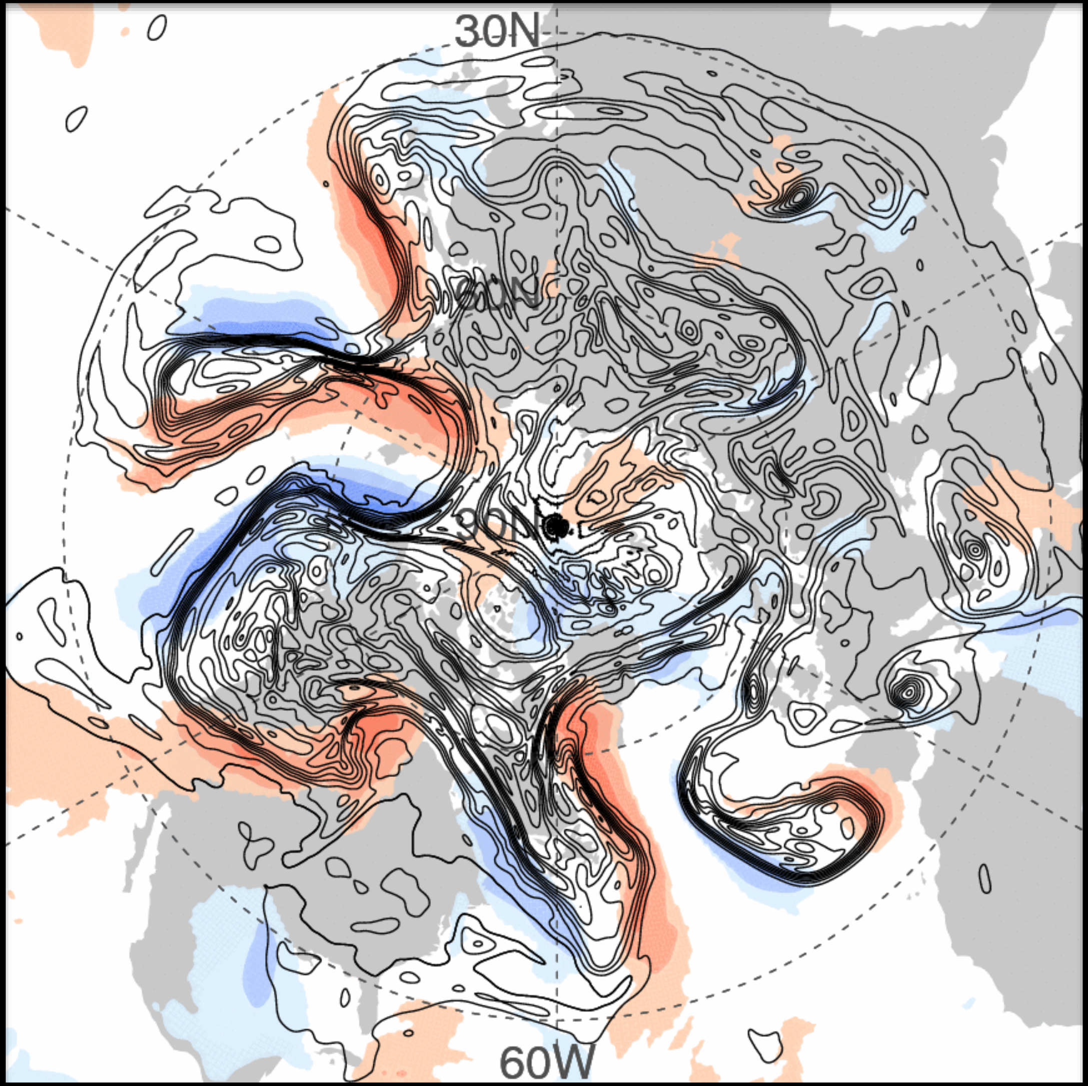
- based on some lecture material prepared in previous years by Tim Palmer and Sarah Keeley -

ECMWF

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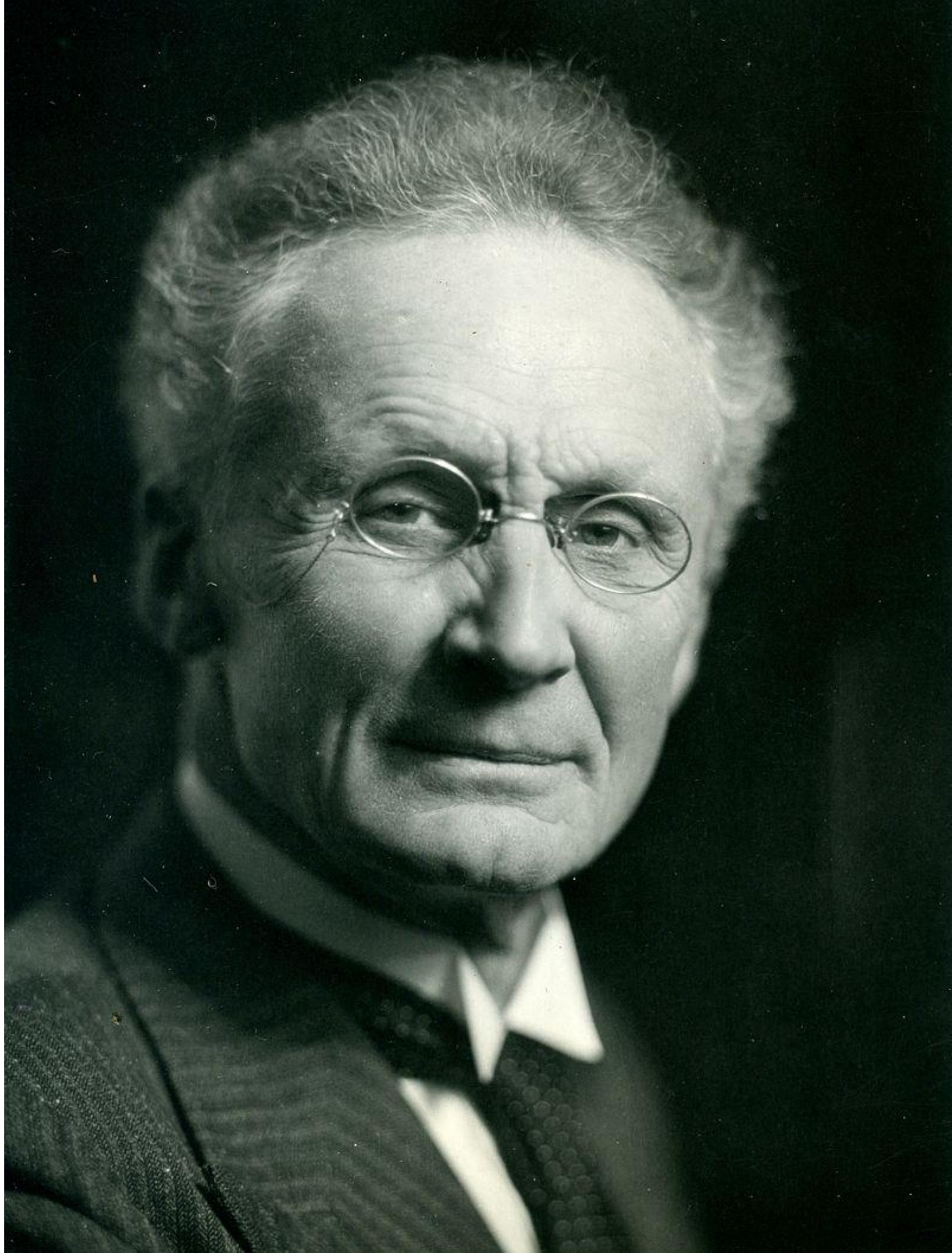
# Vilhelm Bjerknes (1862-1951)

**“Founding father of modern weather forecasting”**

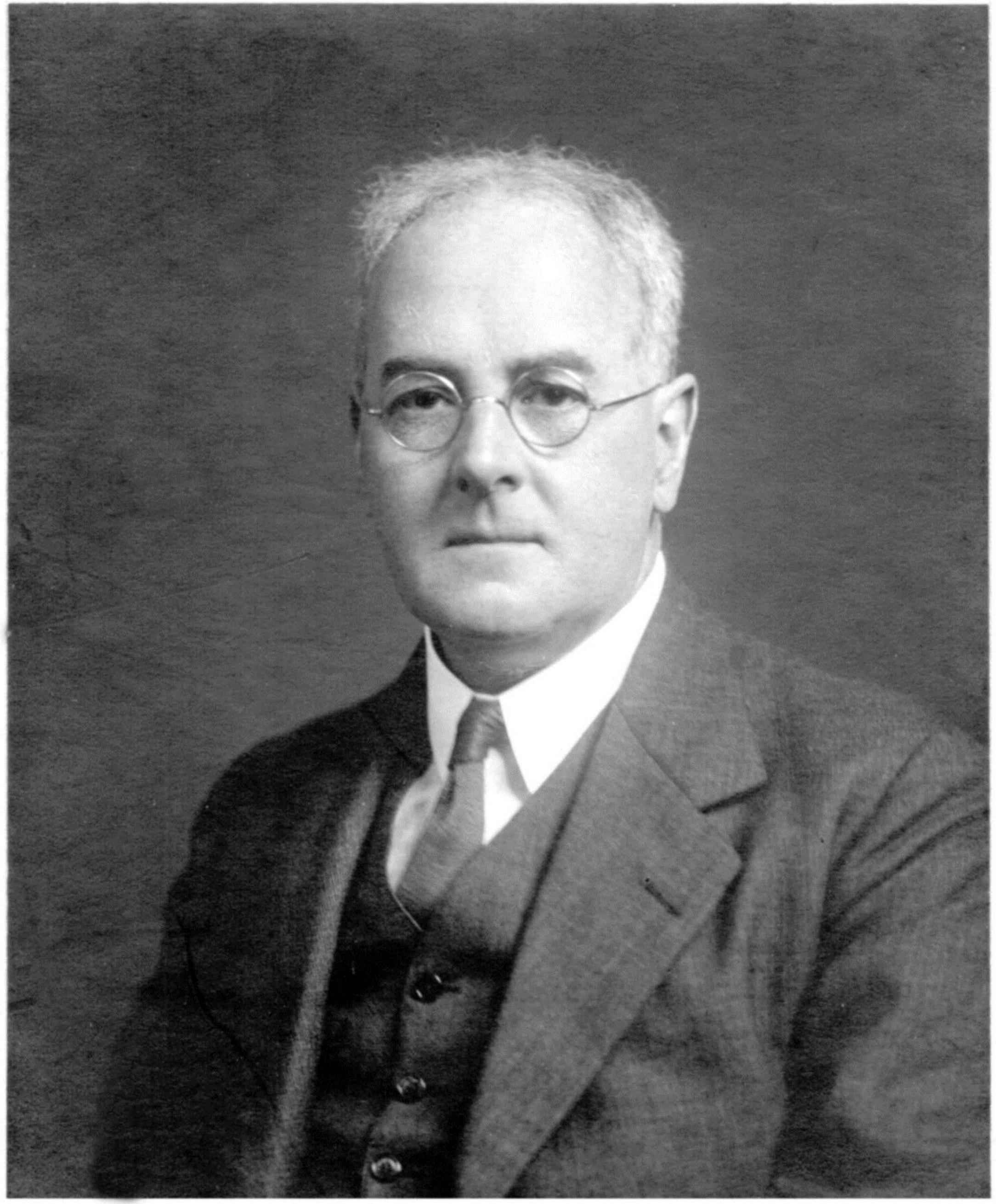
Norwegian physicist who proposed weather forecasting as a deterministic initial value problem based on the laws of physics



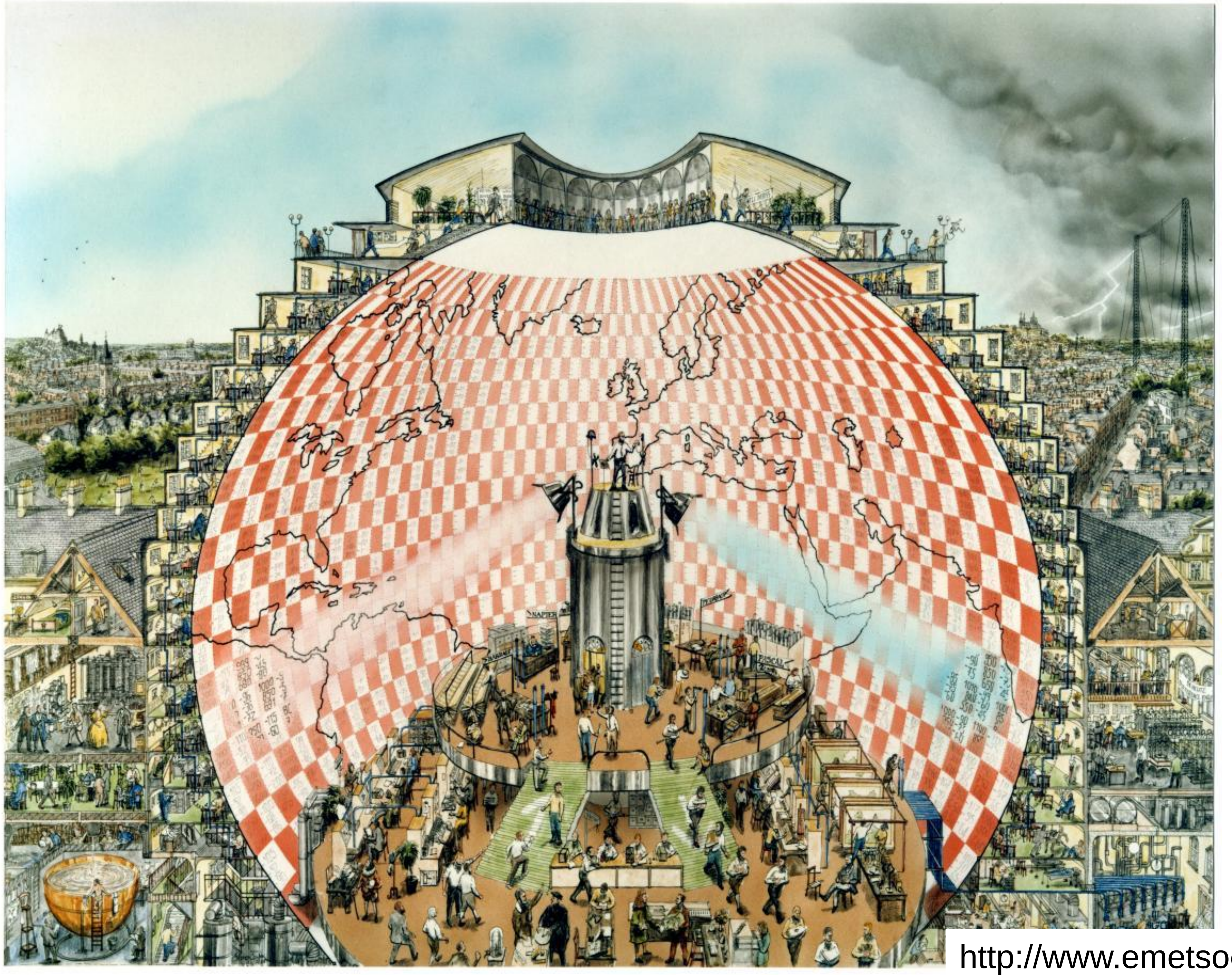
## Lewis Fry Richardson (1881-1953)

**English scientist who produced the first numerical weather forecast**

- Forecast for 20 May 1910 1pm by direct computation of the solutions to simplified flow equations using input data taken at 7am
- Forecast predicted rise in surface pressure by 145 hPa in 6 hours → dramatic failure
- A posteriori: failure to apply smoothing to data to filter out unphysical waves



L. F. Richardson, 1931



http://www.emetsoc.org/resources/rff

**“Why have meteorologists such difficulty in predicting the weather with any certainty?** Why is it that showers and even storms seem to come by chance ... a tenth of a degree (C) more or less at any given point, and the cyclone will burst here and not there, and extend its ravages over districts that it would otherwise have spared. **If (the meteorologists) had been aware of this tenth of a degree, they could have known (about the cyclone) beforehand, but the observations were neither sufficiently comprehensive nor sufficiently precise,** and that is the reason why it all seems due to the intervention of chance”

Poincaré, 1909

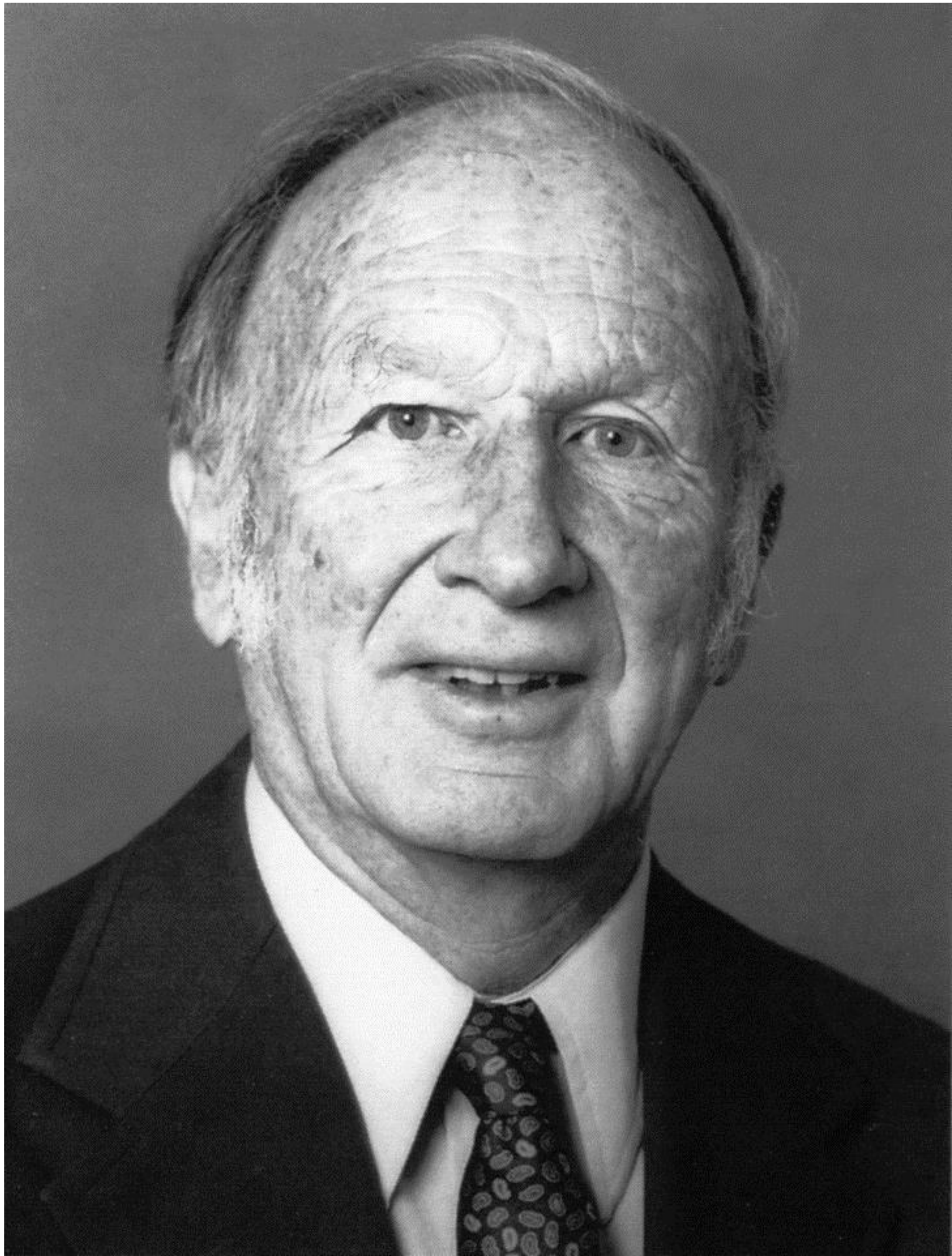
## Edward Lorenz (1917 –2008)

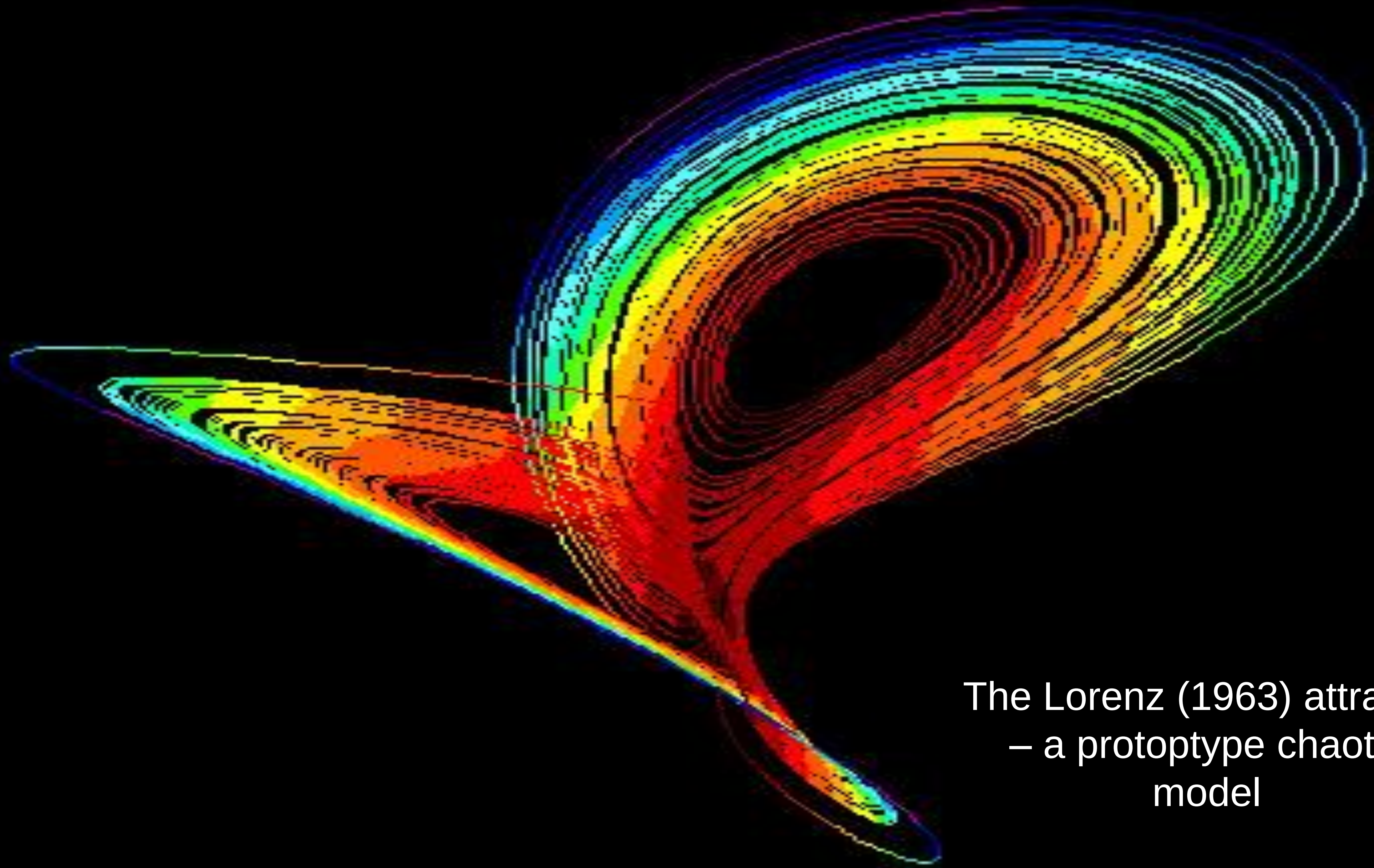
"… one flap of a sea-gull's wing may forever change the future course of the weather" (Lorenz, 1963)

$$\dot{X} = -\sigma X + \sigma Y$$

$$\dot{Y} = -XZ + rX - Y$$

$$\dot{Z} = XY - bZ$$





The Lorenz (1963) attractor – a protoptype chaotic model

# What is deterministic chaos?

A physical system that

- follows deterministic rules (absence of randomness)
- *but appears* to behave randomly; it *looks* random
- needs to be nonlinear, dissipative and at least 3-dimensional

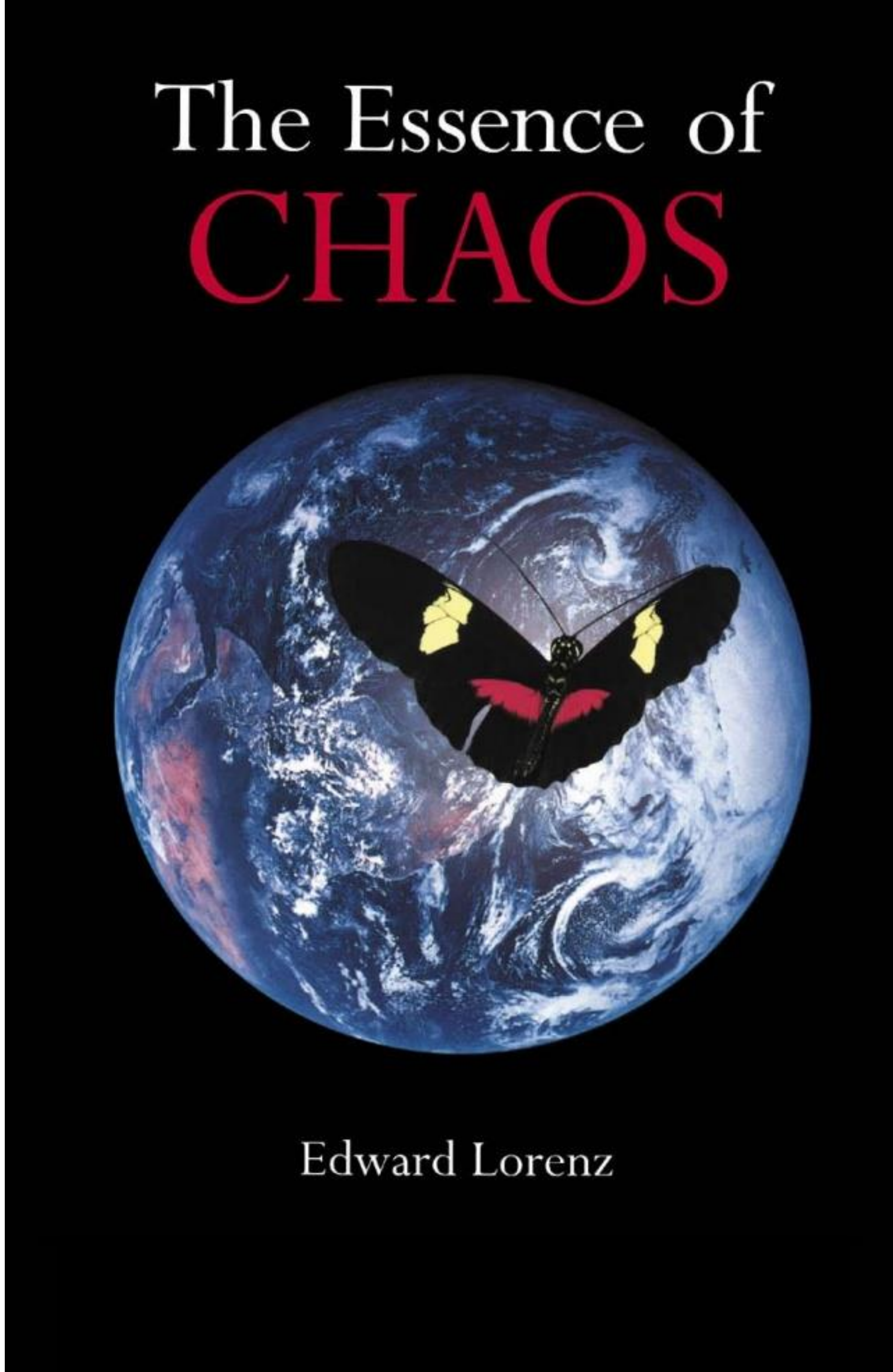
$$\frac{dX}{dt} = F[X] \quad \text{is a nonlinear system}$$

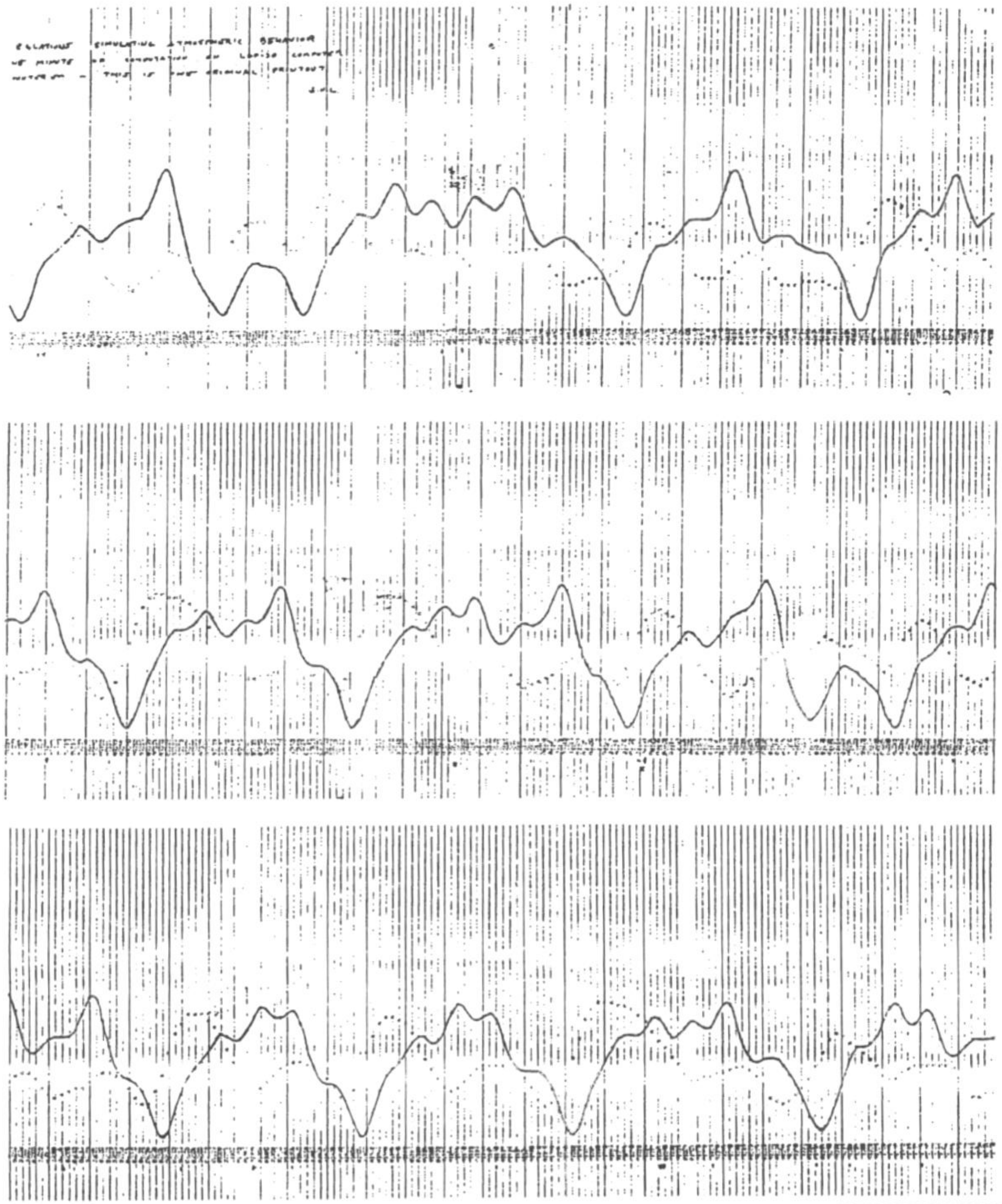
$$\Rightarrow \frac{d\delta X}{dt} = \frac{dF}{dX} \delta X \equiv J \quad \delta X$$

Since $F$ is a nonlinear function of $X$

$$\Rightarrow J = J(X)$$

**→ sensitive dependence on initial conditions**





A fifteen-month section of the original print-out of symbols representing two variables of the twelve-variable model. A solid curve had been drawn through the symbols for one variable, while the symbols for the other are faintly visible. The section has been broken into three five-month segments, shown on consecutive rows.

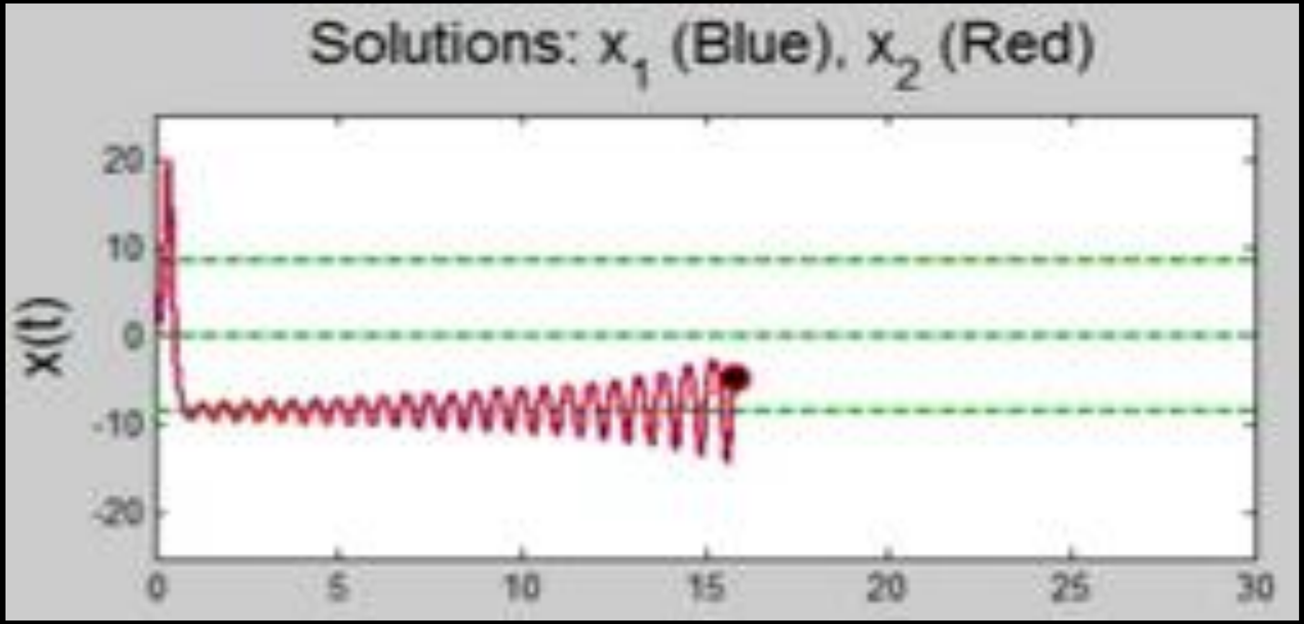
(*Ed Lorenz: The Essence of Chaos*)

# Ed Lorenz (1963): Deterministic Nonperiodic Flow

**Dynamical system that is highly sensitive to perturbations of the initial conditions**

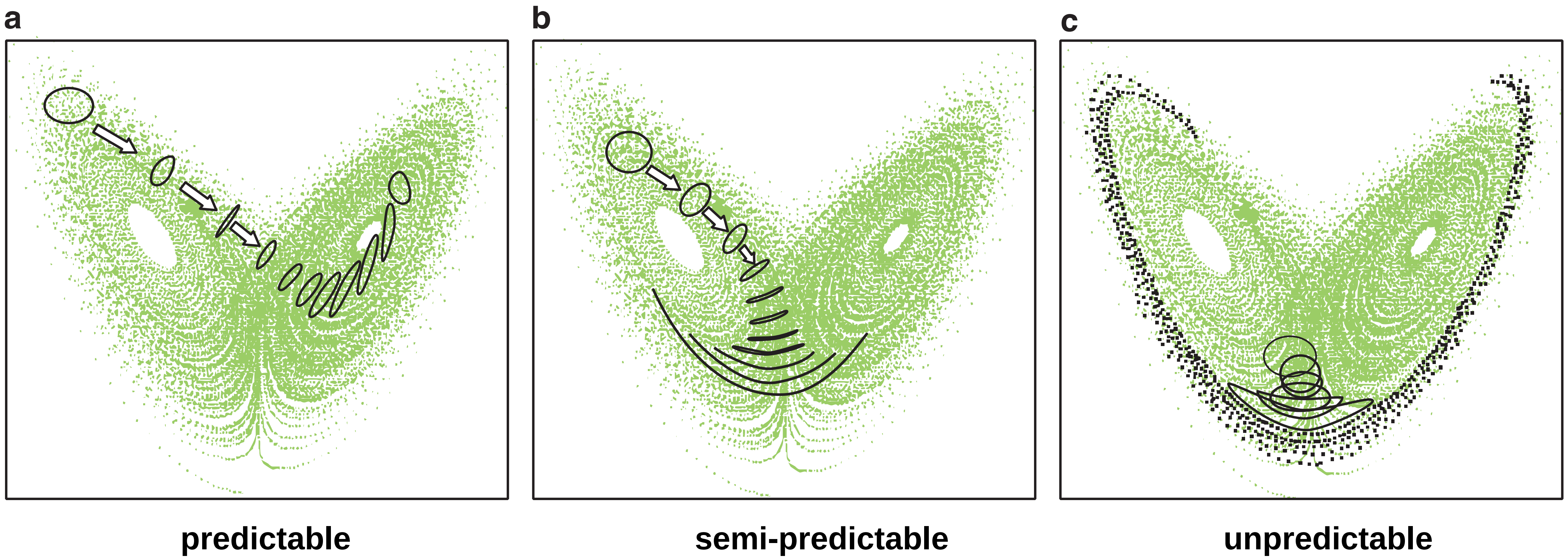
(deterministic chaos)

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -\sigma(x-y) \\ (r-z)x-y \\ xy-bz \end{pmatrix}$$



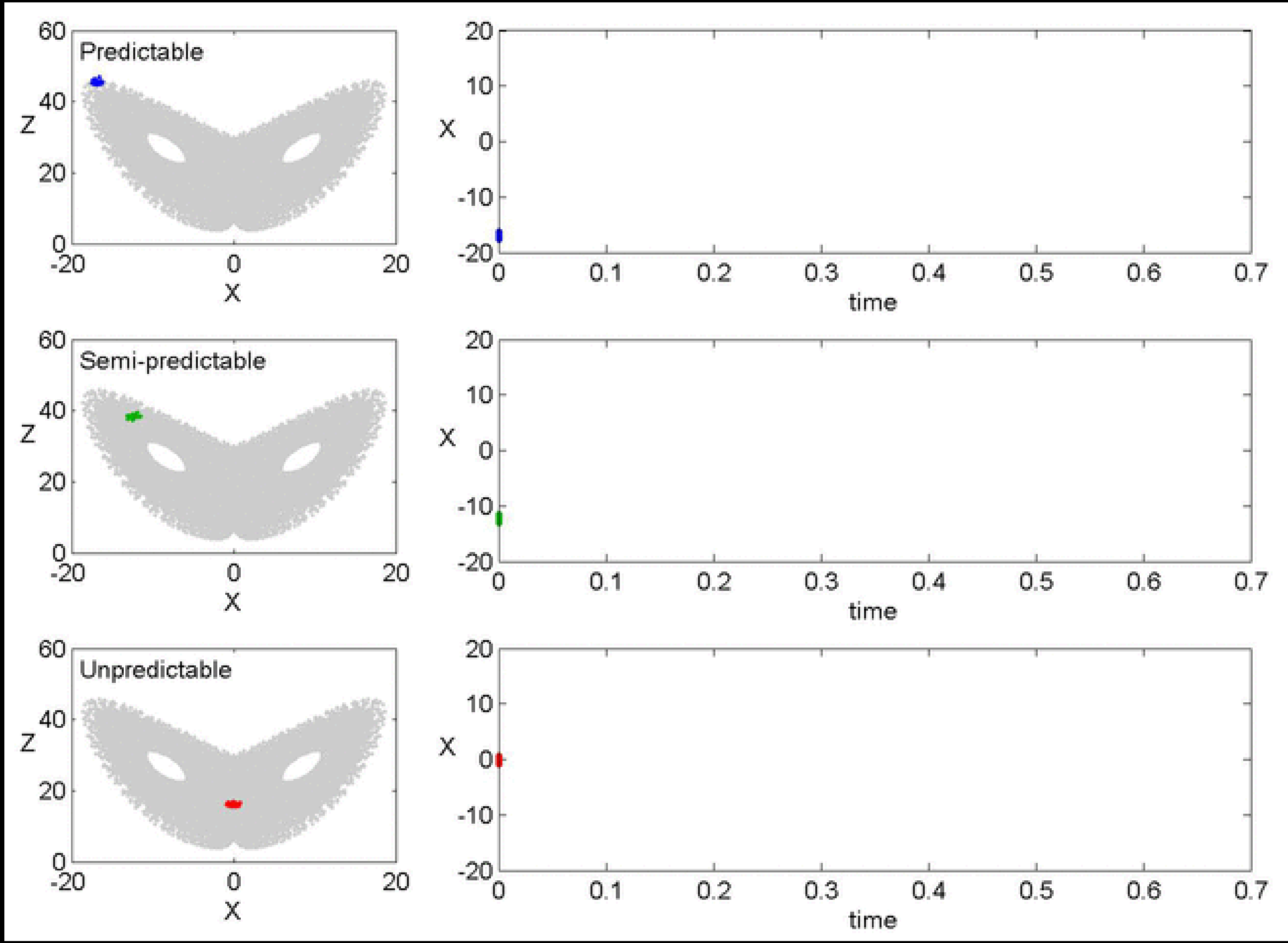

*Courtesy Hannah Christensen*

In a nonlinear system the growth of initial uncertainty is flow dependent.




The set of initial conditions (black circle) is located in different regions of the attractor in a), b) and c) and leads to different error growth and predictability in each case.

# Lorenz (1963)




*Courtesy Hannah Christensen*

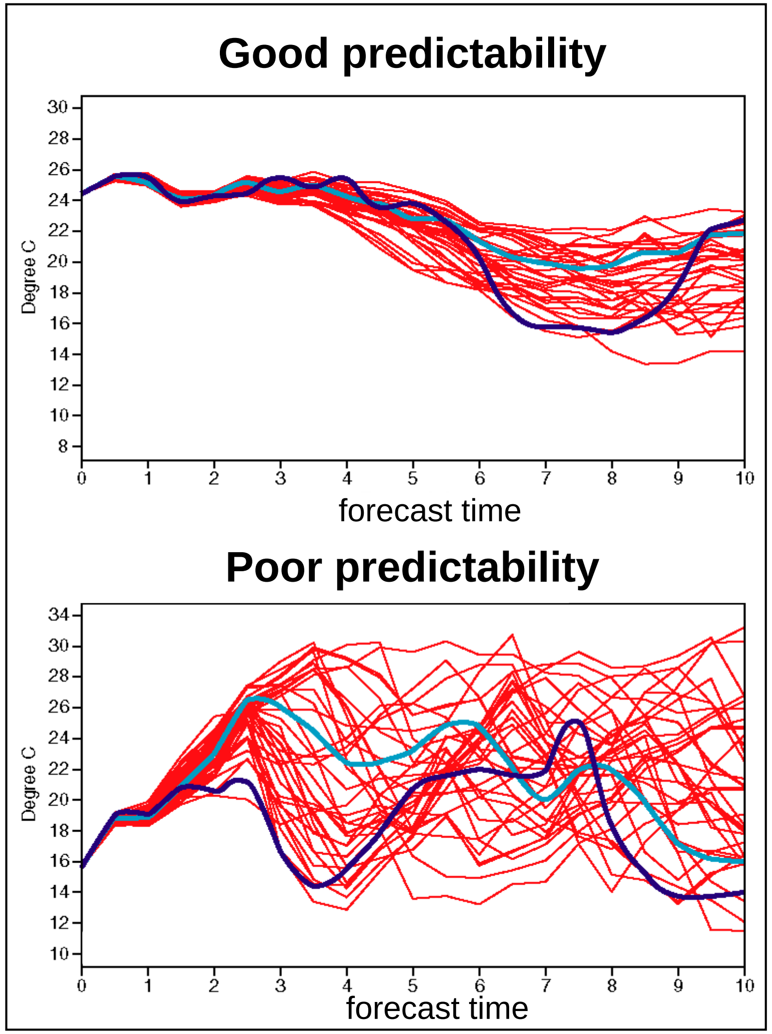
# Chaos and ensemble forecasting

The climate is a **chaotic** system where the future state of the system can be very sensitive to small differences in the current (initial) state of the system.

In practice, the initial state of the system is always uncertain.

Our forecast models are not perfect in all aspects (e.g. small-scale features such as clouds).

**Ensemble forecasting** takes into account these inherent uncertainties by running a large number of similar but not identical versions of the model in parallel. The resulting forecasts are expressed in **probabilities**.

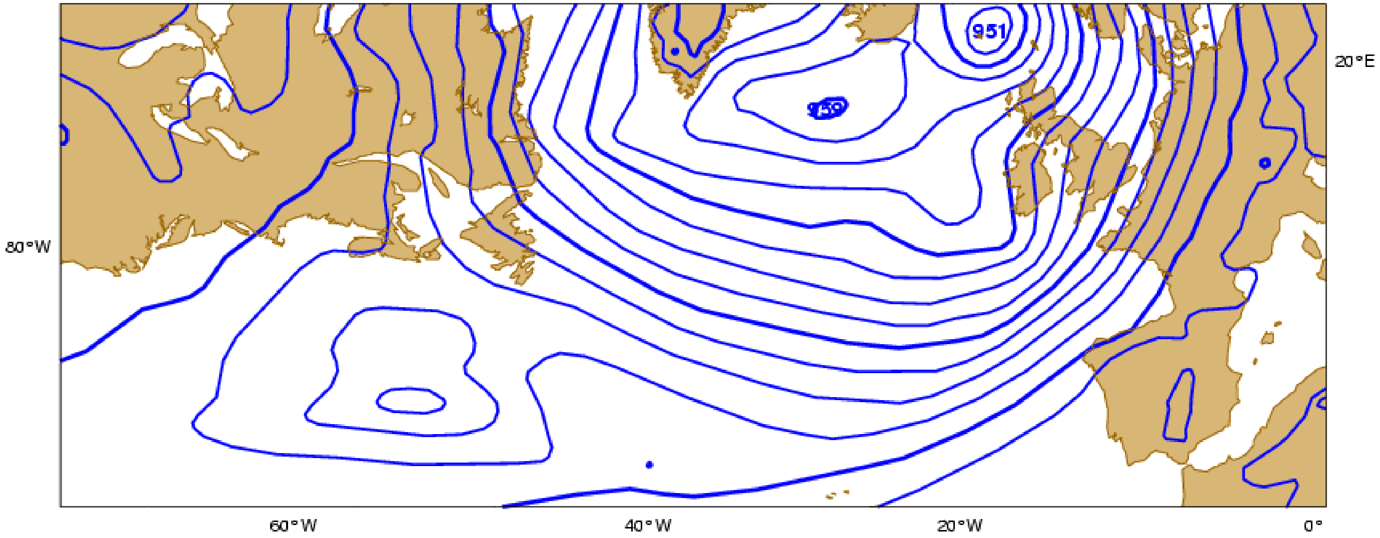


## Lothar: 08Z, 26 Dec. 1999

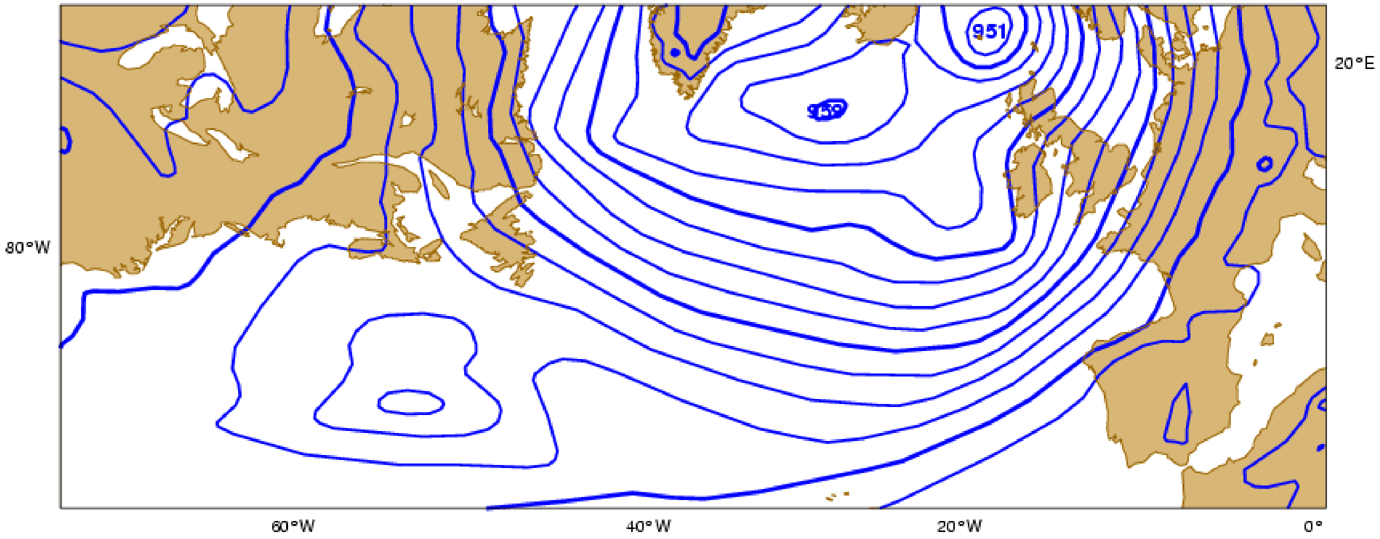
### Storms Lothar +Martin

- 100 fatalities
- 400 million trees blown down
- 3.5 million electricity users affected for 20 days
- 3 million people without water

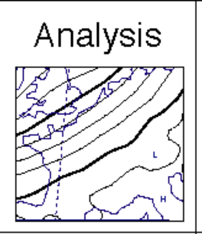
forecast 24 December 1999, 12UTC +0 h - mem no. 13 of 17

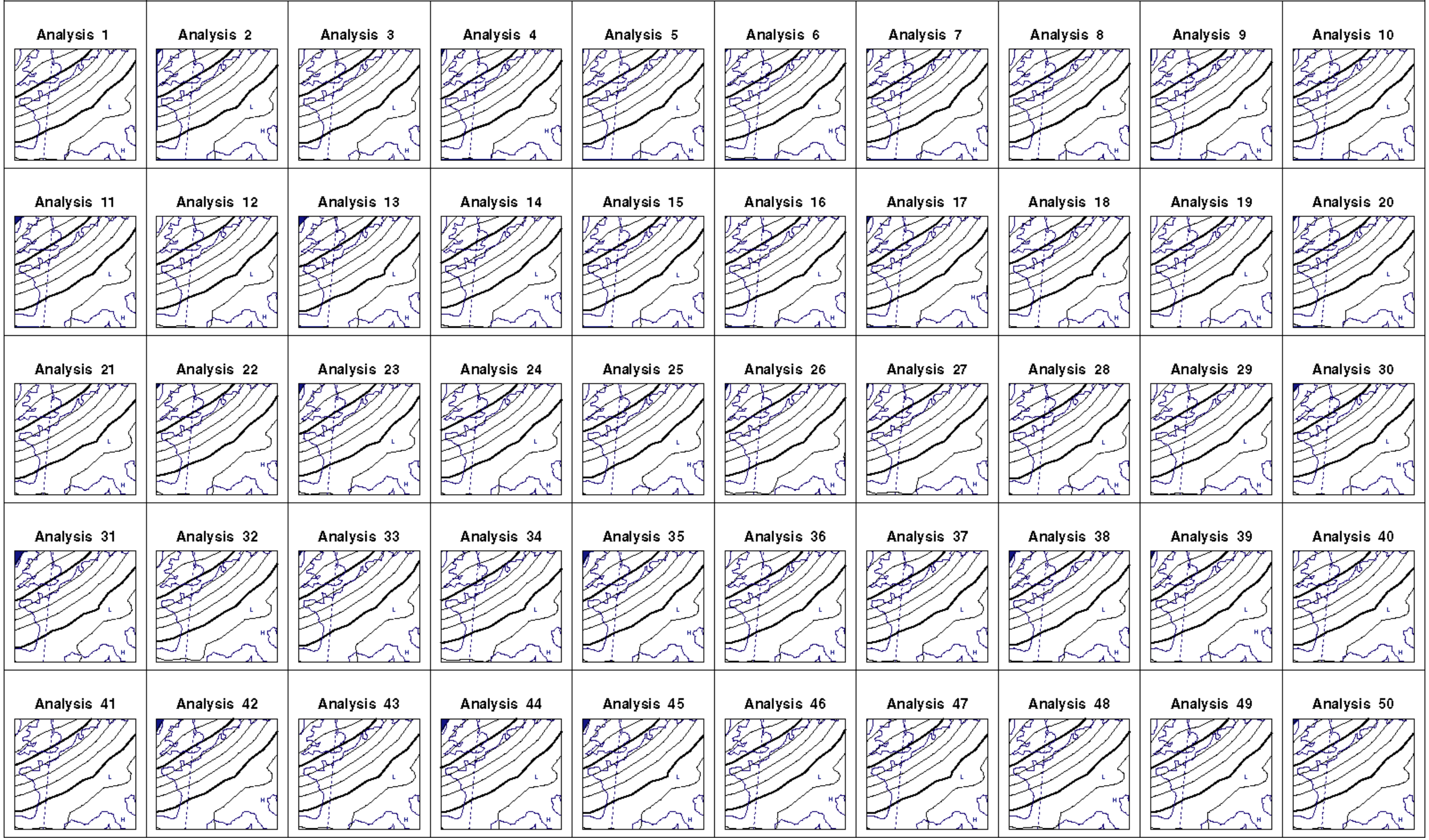



forecast 24 December 1999, 12UTC +0 h - mem no. 14 of 17

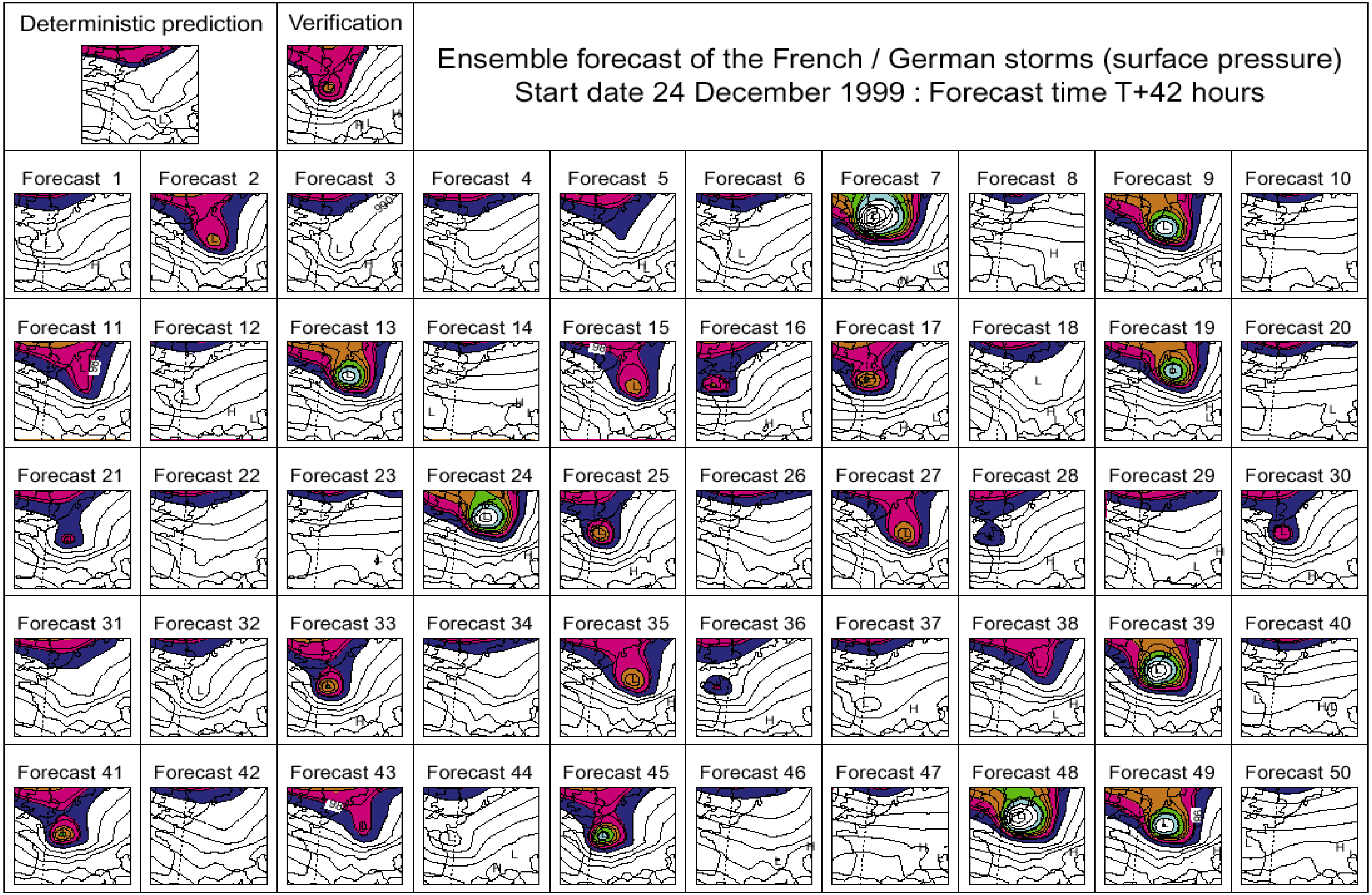


# Ensemble Initial Conditions 24 December 1999

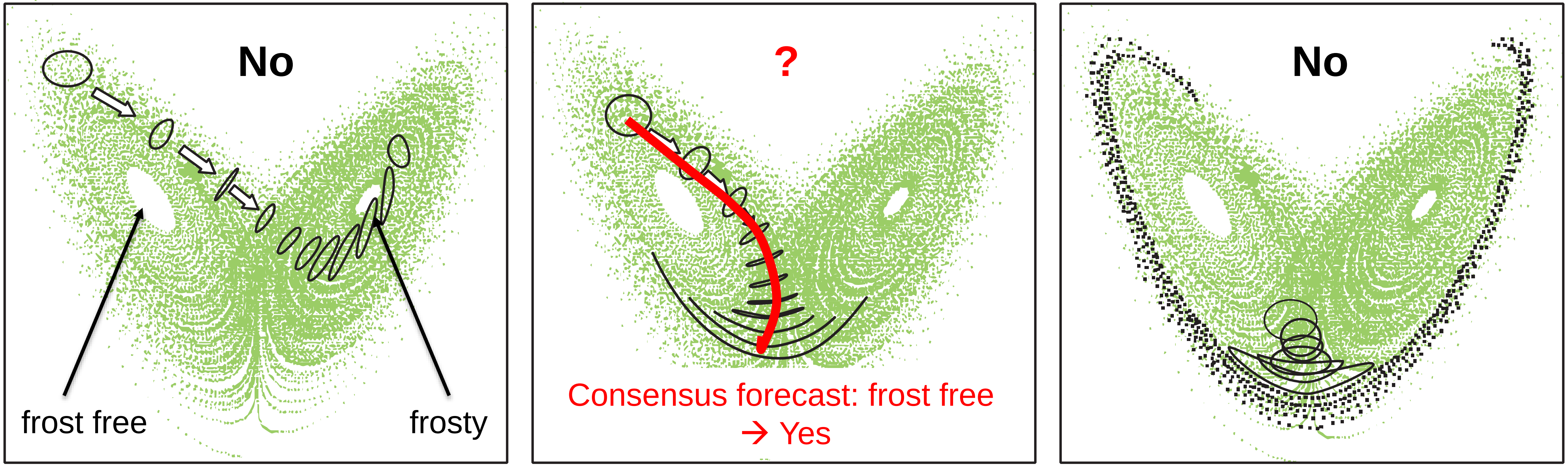


# Lothar (T+42 hours)



# Probabilistic forecasting and the cost-loss concept




Charlie is planning to lay concrete tomorrow. Should he?

Let ***p*** denote the probability of frost. Charlie loses **L** if concrete freezes. But Charlie also has fixed (e.g. staff) costs. There may be a penalty for late completion of this job - by delaying completion of this job, he will miss out on other jobs. These costs are **C.**

Is **L*$p$ > C** ?

**If $p$ > C/L don't lay concrete!**