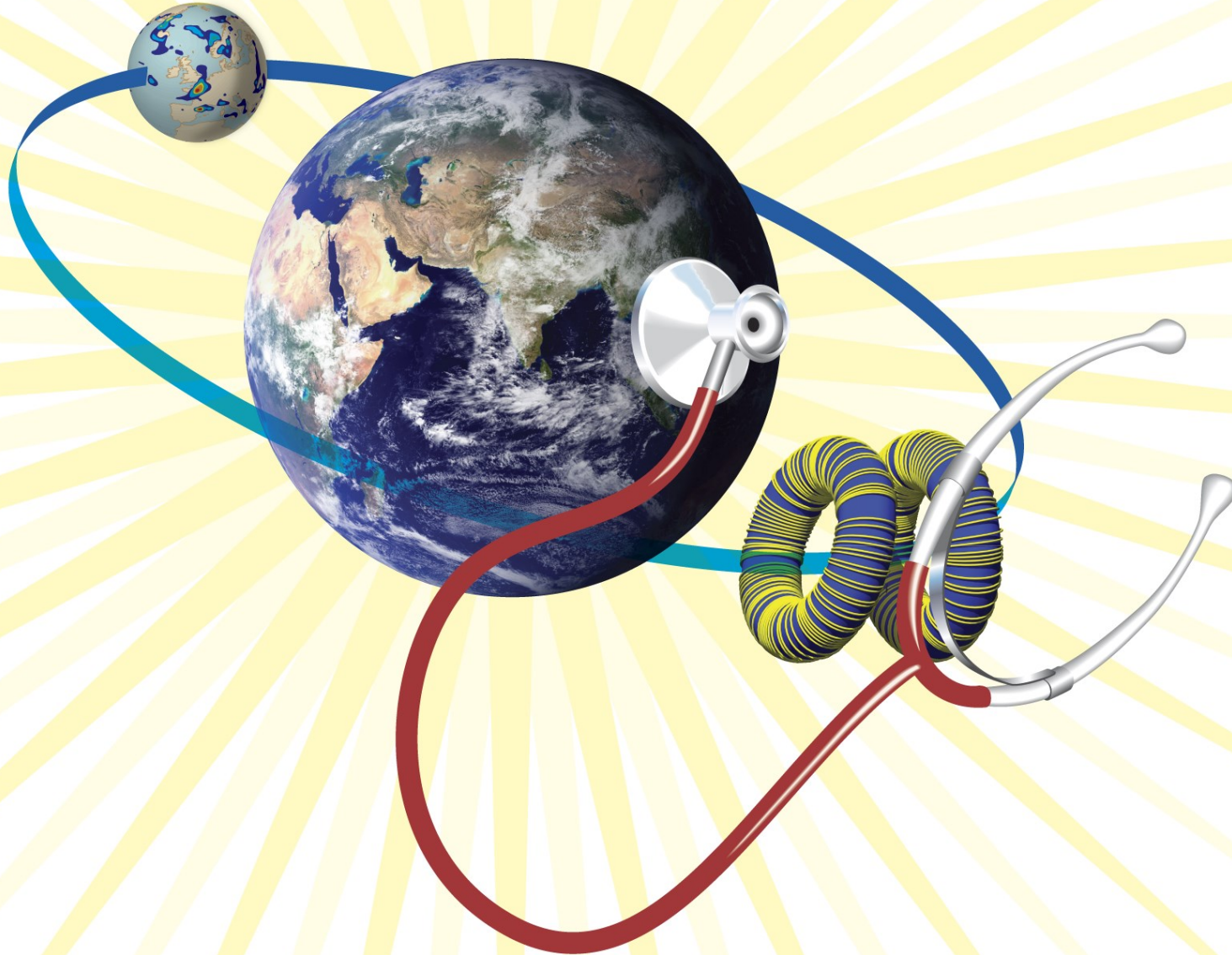


# Predictability Diagnostics 1

Mark Rodwell  
(with the support of ECMWF  
and external collaborators)



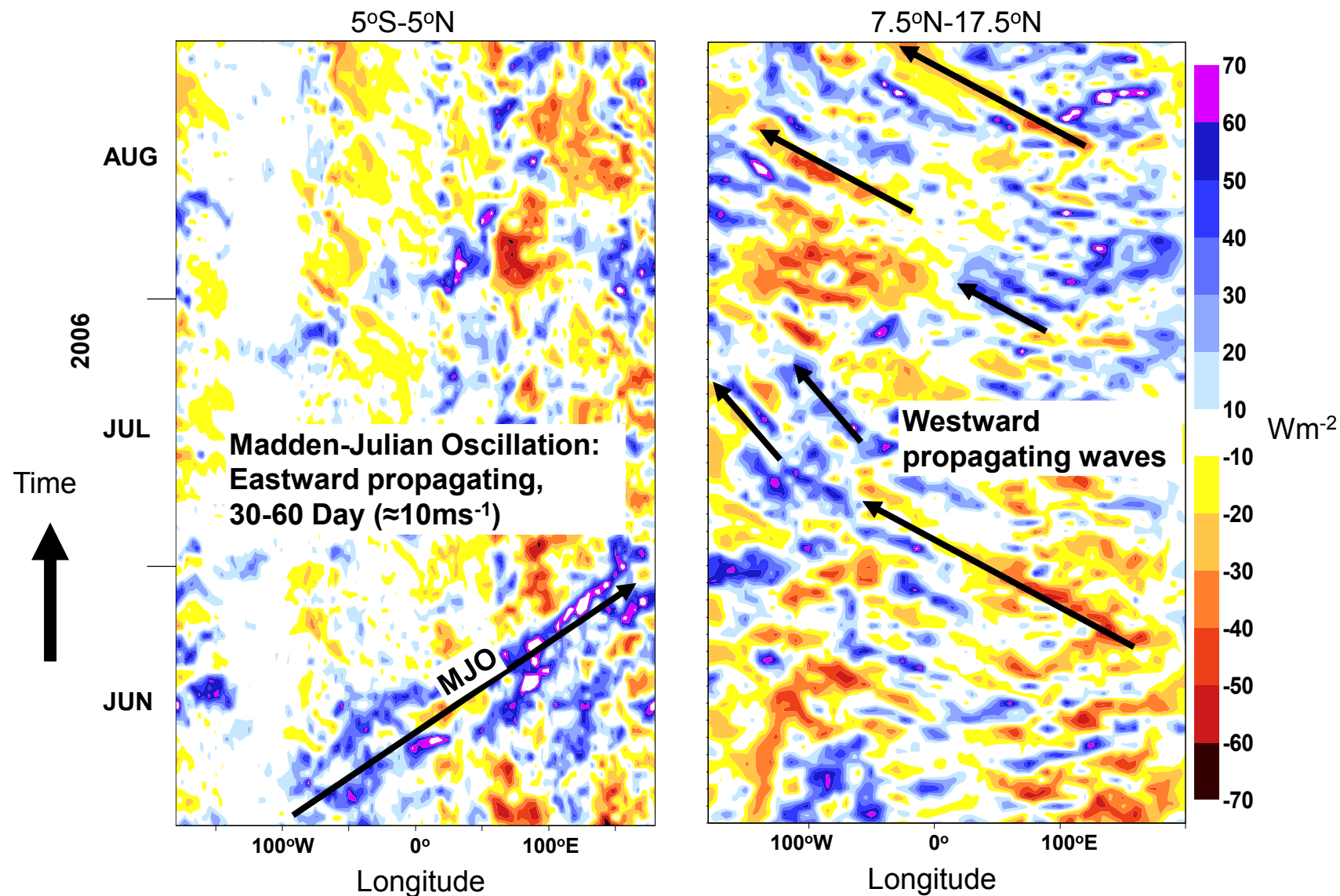
ECMWF Training Course on Predictability

10 May 2017, ECMWF Reading

# Waves propagate predictable signals and errors

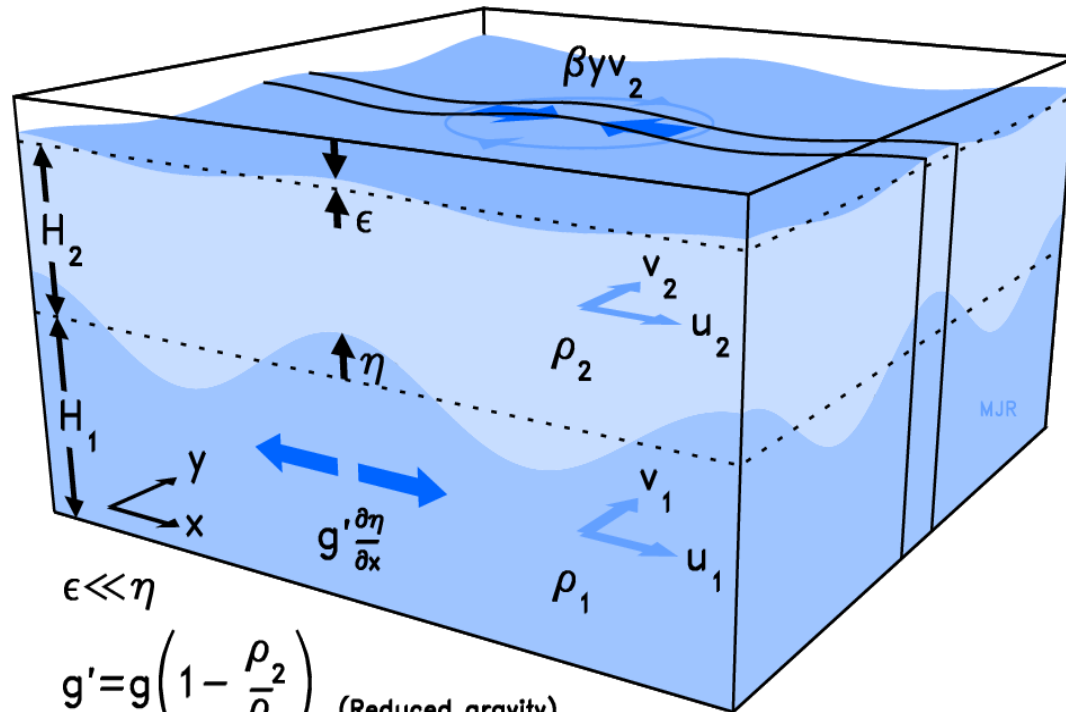
# Tropical Waves

Data from NOAA



Based on Outgoing Longwave Radiation (OLR)

# Equatorial wave theory – the model



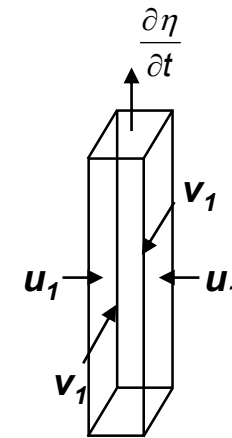
$$\epsilon \ll \eta$$

$$g' = g \left( 1 - \frac{\rho_2}{\rho_1} \right) \quad (\text{Reduced gravity})$$

$$\mathbf{u} \equiv \mathbf{u}_1 - \mathbf{u}_2 \quad \mathbf{v} \equiv \mathbf{v}_1 - \mathbf{v}_2 \quad (\text{Baroclinic mode})$$

$$c_e^2 \equiv g' \frac{H_1 H_2}{H_1 + H_2} \equiv g H_e \quad c_e \approx 20 \text{ to } 80 \text{ ms}^{-2}$$

$c_e$  is the propagation speed of a barotropic gravity wave in a single layer of depth  $H_e$



## Momentum:

$$\frac{\partial u}{\partial t} - \beta y v + g' \frac{\partial \eta}{\partial x} \approx 0$$

$$\frac{\partial v}{\partial t} + \beta y u + g' \frac{\partial \eta}{\partial y} \approx 0$$

(1)

## Continuity:

$$\frac{\partial \eta}{\partial t} + \frac{c_e^2}{g'} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \approx 0$$

(2)

## Solving for v:

$$\frac{\partial}{\partial t} \left\{ \frac{\partial^2 v}{\partial t^2} + \beta^2 y^2 v - c_e^2 \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \right\} - c_e^2 \beta \frac{\partial v}{\partial x} = 0$$

(3)

Use of the shallow water equations on the  $\beta$ -plane ( $f = \beta y$ ) for understanding tropical atmospheric waves. Note: No coupling with convection in this model

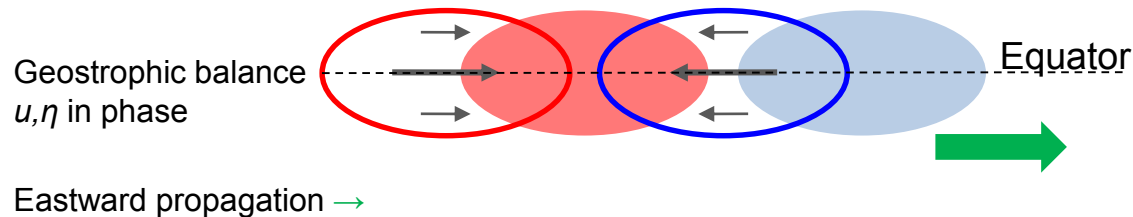
# Equatorial wave theory – limiting solutions

## Kelvin waves: $v \equiv 0$ , $u$ in geostrophic balance with meridional pressure gradient

$$(K1) \quad \frac{\partial u}{\partial t} = -g' \frac{\partial \eta}{\partial x}$$

$$(K2) \quad \beta y u = -g' \frac{\partial \eta}{\partial y}$$

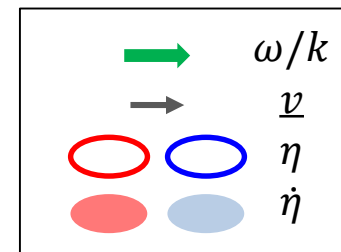
$$(K3) \quad \frac{\partial \eta}{\partial t} = -\frac{c_e^2}{g'} \frac{\partial u}{\partial x}$$



$$\frac{\partial^2 \eta}{\partial t^2} = c_e^2 \frac{\partial^2 \eta}{\partial x^2}$$

$$\eta \propto \sin(kx - \omega t) \hat{\eta}(y)$$

$$\text{phase speed} = \frac{\omega}{k} = c_e, \hat{\eta}(y) = e^{-\frac{\beta}{2c_e} y^2}$$



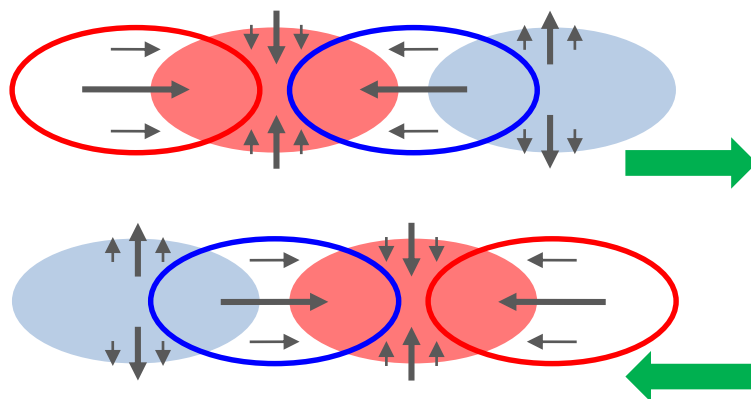
## Gravity waves: Fast, pressure gradient force dominates

$$(G1) \quad \frac{\partial u}{\partial t} = -g' \frac{\partial \eta}{\partial x}$$

$u, \eta$  in phase:  $\rightarrow$   
out of phase:  $\leftarrow$

$$(G2) \quad \frac{\partial v}{\partial t} = -g' \frac{\partial \eta}{\partial y}$$

$$(G3) \quad \frac{\partial \eta}{\partial t} = -\frac{c_e^2}{g'} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$



$$\frac{\partial^2 \eta}{\partial t^2} = c_e^2 \left( \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right)$$

$$\eta \propto \sin(kx - \omega t) \hat{\eta}(y)$$

$$\text{phase speed} = \frac{\omega}{k} = \pm c_e \left( 1 - \frac{1}{k^2 \hat{\eta}} \frac{\partial^2 \hat{\eta}}{\partial y^2} \right)^{\frac{1}{2}}$$

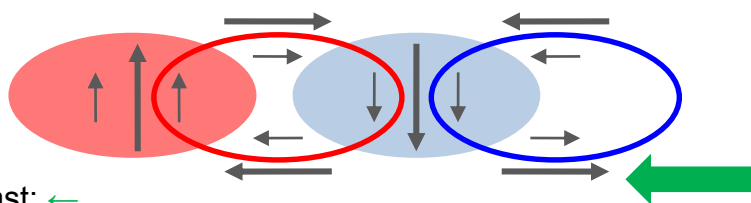
$$\rightarrow \pm c_e \text{ as } k \rightarrow \infty$$

## Rossby waves: Slow, Coriolis affect important, closer to geostrophic balance, less convergence. Take curl of (1)

$$(R1) \quad \frac{\partial \xi}{\partial t} = -\beta y \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \beta v$$

$$\approx -\beta v$$

Vorticity anomaly strengthened to west, weakened to east:  $\leftarrow$



$$\frac{\partial \nabla^2 \psi}{\partial t} = -\beta \frac{\partial \psi}{\partial x}$$

$$\psi \propto \sin(kx - \omega t) \hat{\psi}(y)$$

$$\text{phase speed} = \frac{\omega}{k} = -\frac{\beta}{k^2} \left( 1 - \frac{1}{k^2 \hat{\psi}} \frac{\partial^2 \hat{\psi}}{\partial y^2} \right)^{-1}$$

$$\rightarrow -\frac{\beta}{k^2} \text{ as } k \rightarrow \infty$$

(westward propagation, which is faster for larger waves)

$V=0$ :

$$u = u_0 e^{-y^2/2} e^{ik(x-c_e t)}$$

East propagating **Kelvin Wave**

- Non-dispersive
- In geostrophic balance

$V \neq 0$ :

$$v = \hat{v}(y) e^{i(kx - \omega t)}$$

Substitute into equation for  $v$

## Structures

(Meridional structures are solutions to Schrodinger's simple harmonic oscillator)

$$\hat{v}(y) = \begin{bmatrix} 1 \\ 2y \\ 4y^2 - 1 \\ 8y^3 - 12y \\ \vdots \\ H_n(y) \end{bmatrix} e^{-y^2/2}$$

Hermite Polynomials:  $H_n(y)$

- Each successive polynomial has one more node
- Modes alternate asymmetric / symmetric about equator

## Dispersion

(How phase speed is related to spatial scale)

$$\left( \frac{\omega^2}{c_e^2} - k^2 - \frac{\beta k}{\omega} \right) = (2n + 1) \frac{\beta}{c_e} \quad (n = 0, 1, 2, \dots)$$

For  $n \neq 0$ : 3 values of  $\omega$  for each  $k$

- West propagating **Rossby Wave**
- E & W propagating **Gravity Wave**

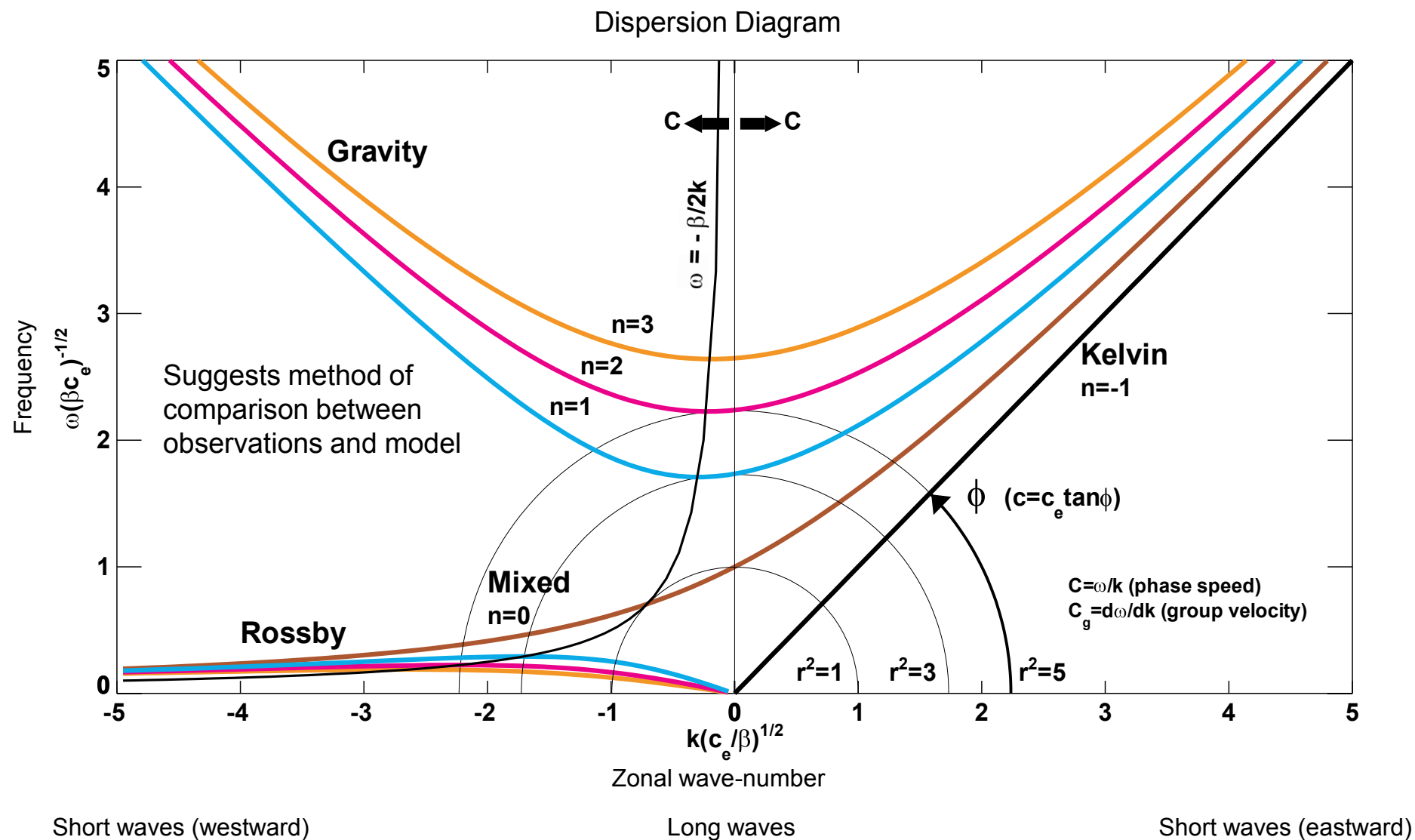
For  $n=0$ : 2 values of  $\omega$  for each  $k$

- E & W prop. **Mixed Rossby-Gravity**

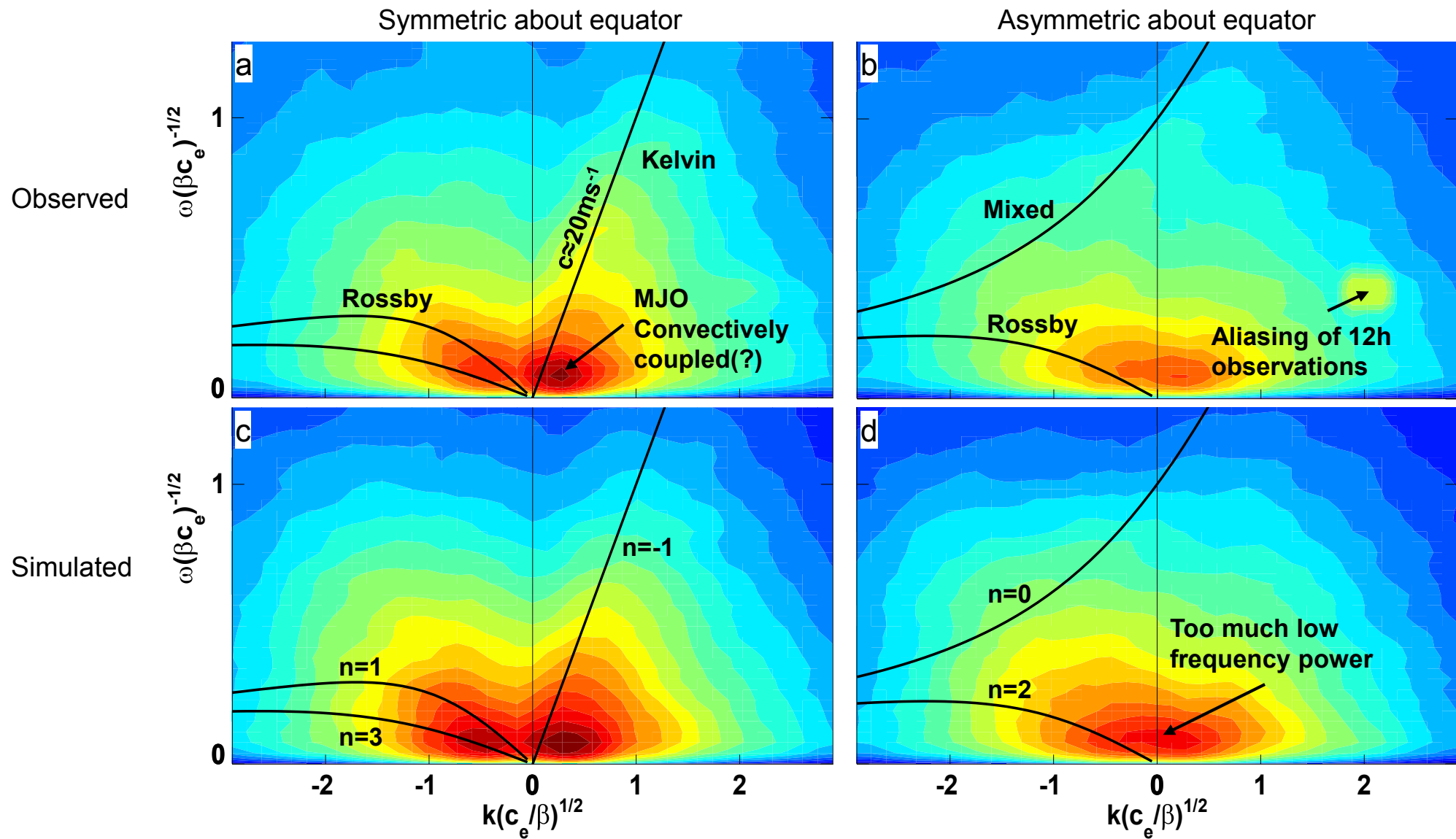
Note:  $y$  has been non-dimensionalised by the factor  $(\beta/c_e)^{1/2}$

In dispersion relation, gravity waves mainly associated with first two terms on lhs, Rossby waves with last two terms on lhs, mixed Rossby-gravity waves with all three terms

# Interpretation of Free Equatorial Waves



# Wave power for OLR, with dispersion relation overlaid

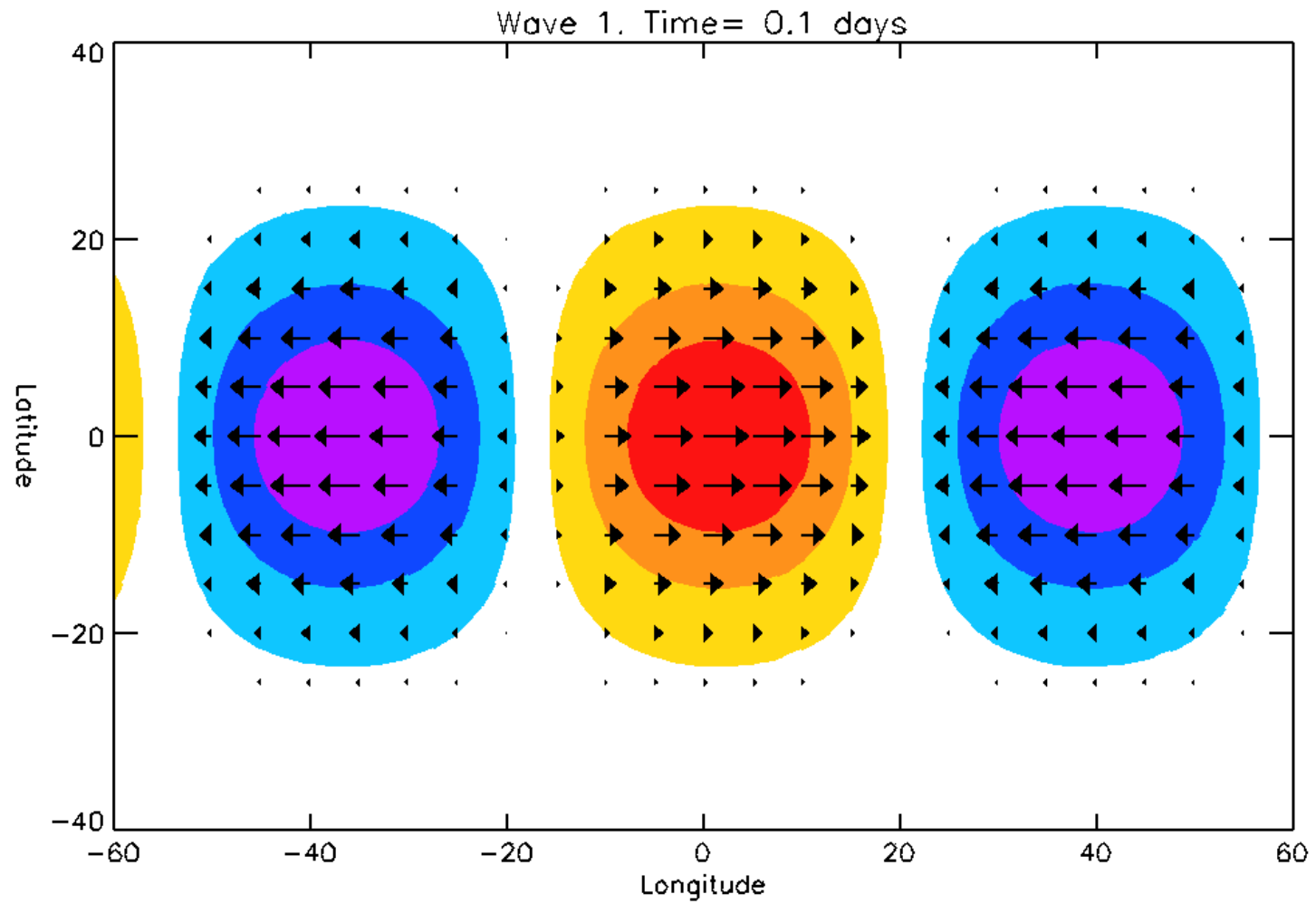


Agreement with shallow water theory if OLR is a 'slave' to the free waves, linearity, etc.

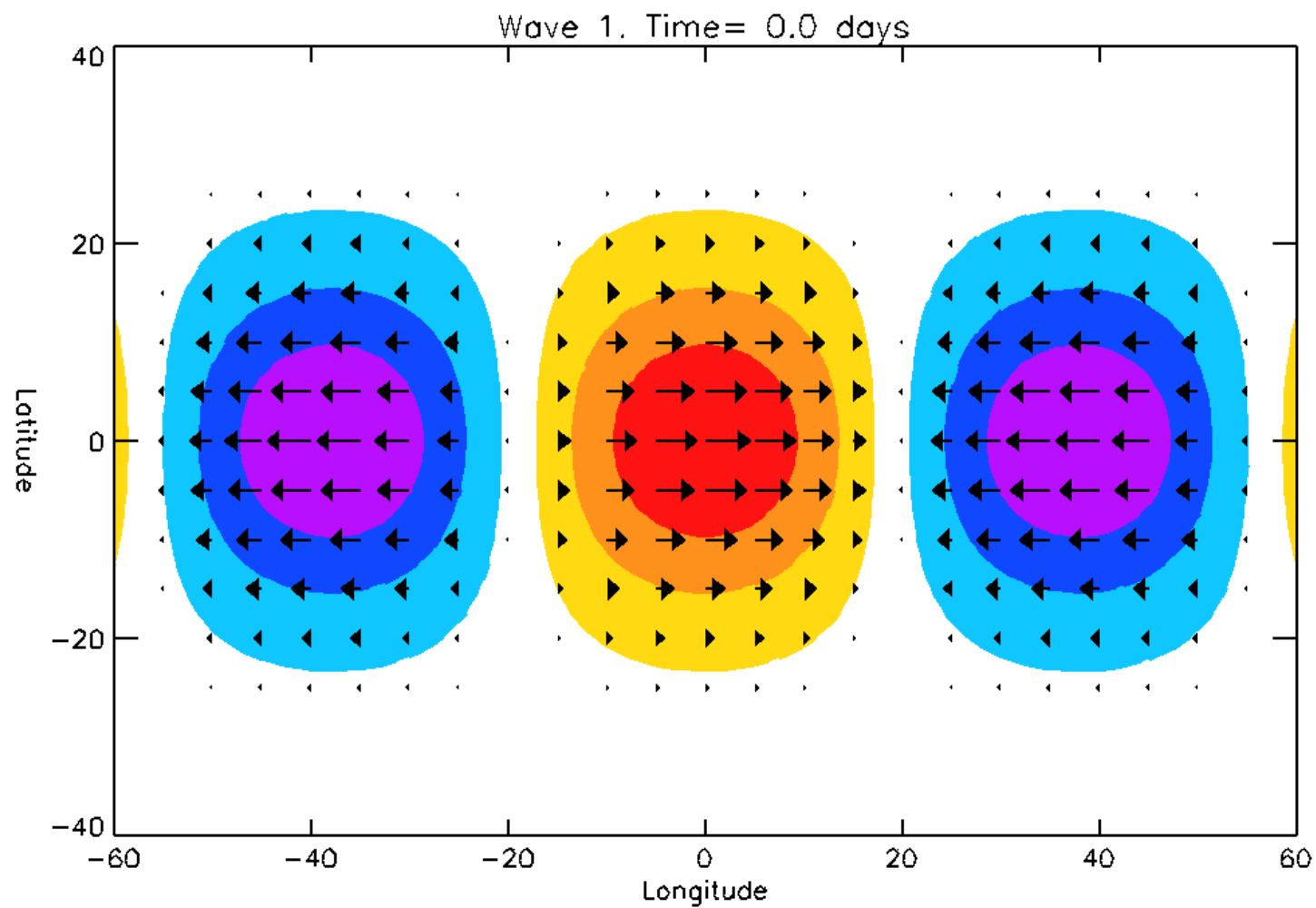
Based on data from NOAA & IFS cycle 32R3 for DJF 1990-2005



# Wave Spotting

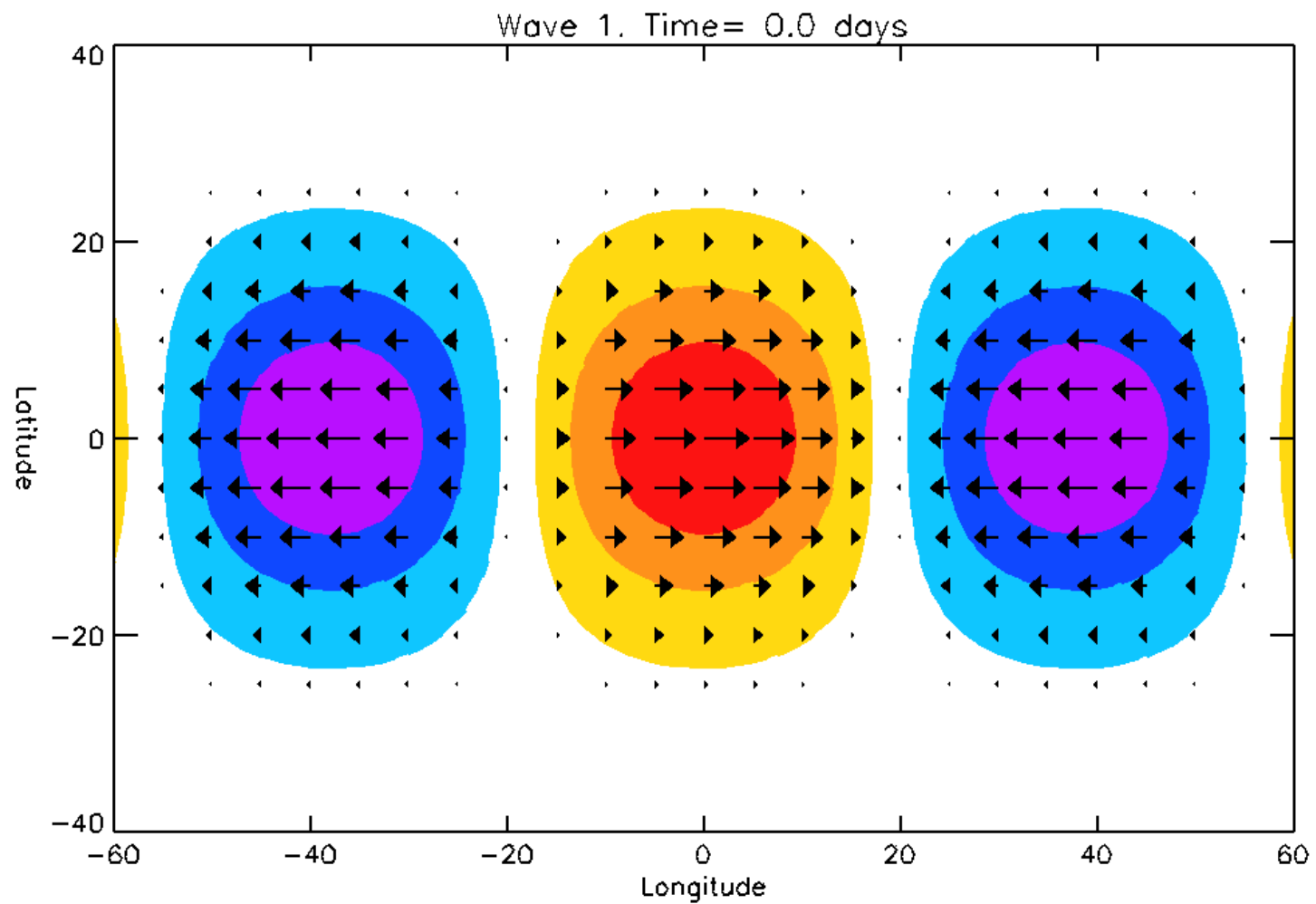


*Colours show height perturbation (red positive, blue negative), arrows show lower-level winds*



*Colours show height perturbation (red positive, blue negative), arrows show lower-level winds*

# Wave Spotting Answers



Colours show height perturbation (red positive, blue negative), arrows show lower-level winds

# Wave spotting: Your Answers

Wave	Kelvin	Mixed Rossby-Gravity	Rossby	Eastward Gravity	Westward Gravity
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

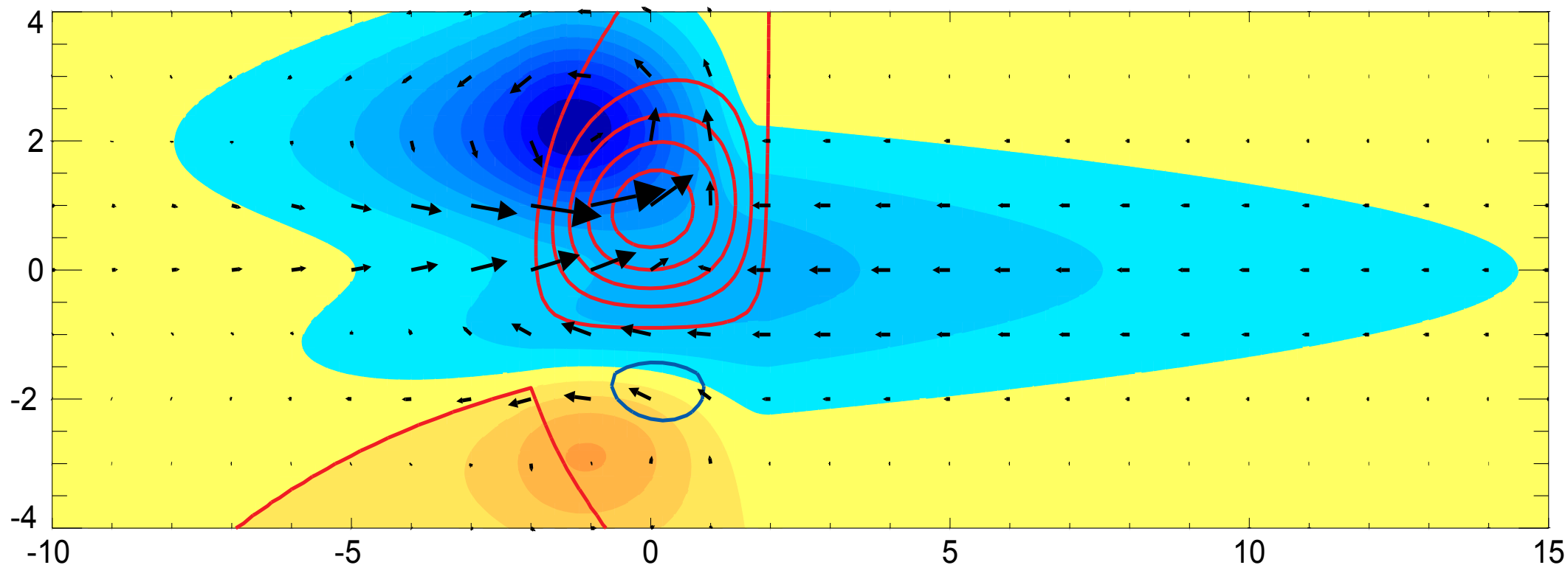
# Gill's steady solution to monsoon heating

Following Gill (1980). See also Matsuno (1966)

Damping/heating terms take the place of the time derivatives. Explicitly solve for the x-dependence

Good agreement with the aerosol change results (opposite sign):

- North Atlantic subtropical anticyclone
- Convective coupling in Kelvin wave regime

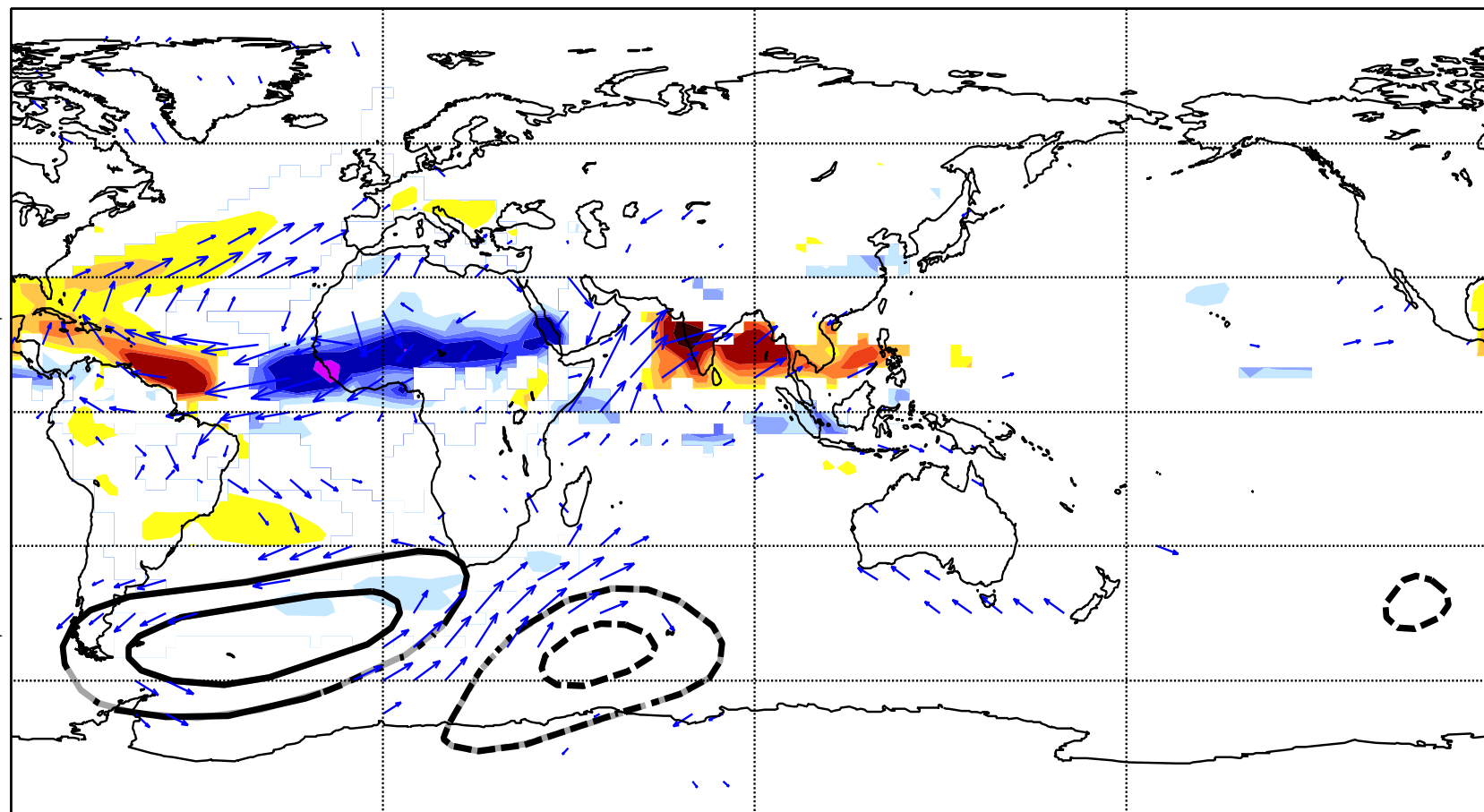


Colours show perturbation pressure, vectors show velocity field for lower level, contours show vertical motion (blue = -0.1, red = 0.0, 0.3, 0.6, ...)

# Model climate response to a change in aerosol climatology

Change in JJA precipitation (shaded),  $v_{925}$  (vectors) and Z500 (contours)

mm day<sup>-1</sup>. 10% Sig.



Gill's steady solution helps explain the model's climate response in the tropics and subtropics, but could we have identified aerosol as a problem *a priori*?

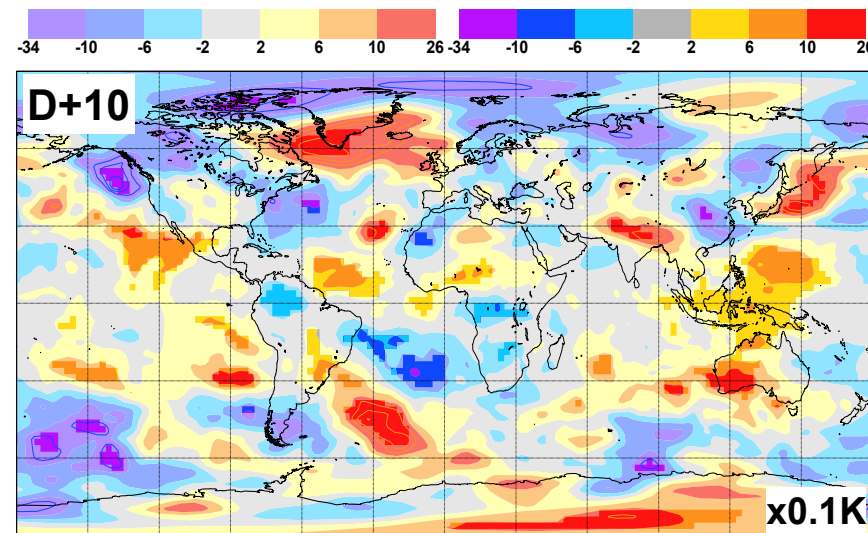
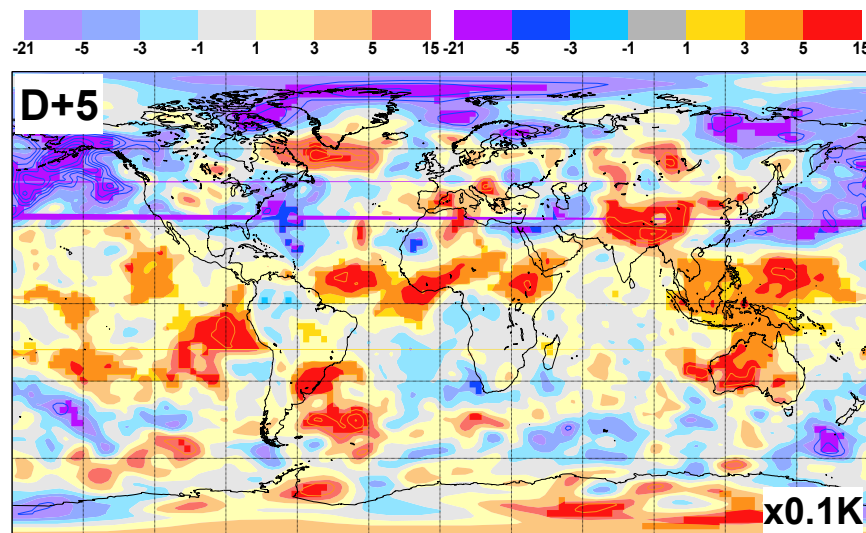
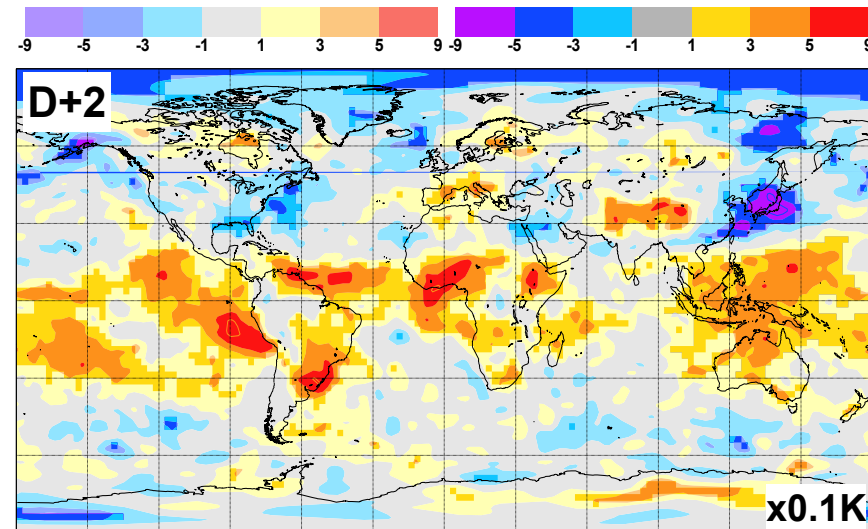
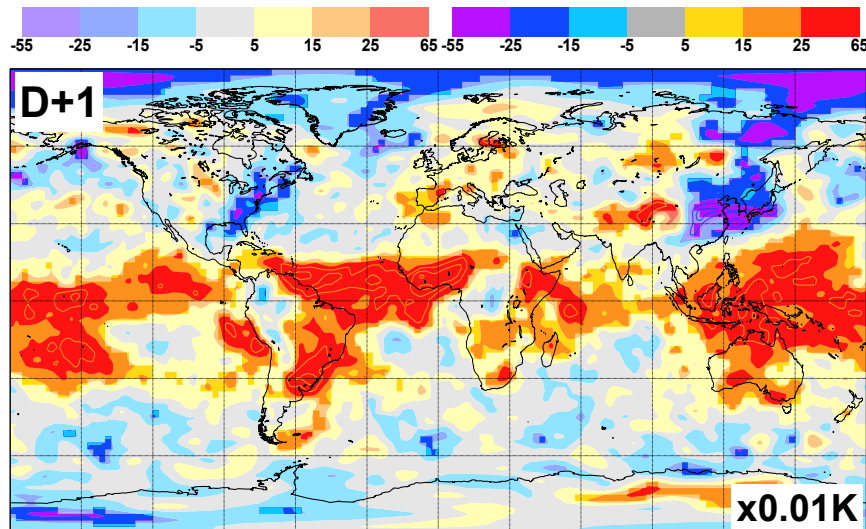
The extratropical response will be discussed later

“A stitch in time saves nine”

# T500 forecast error as function of lead-time

At short lead-times errors are more coherent, statistically significant and linked to model deficiencies

At longer lead-times, errors are more associated with lack of predictability



Based on DJF 2007/8 operational analyses and forecasts. Significant values (5% level) in deep colours.

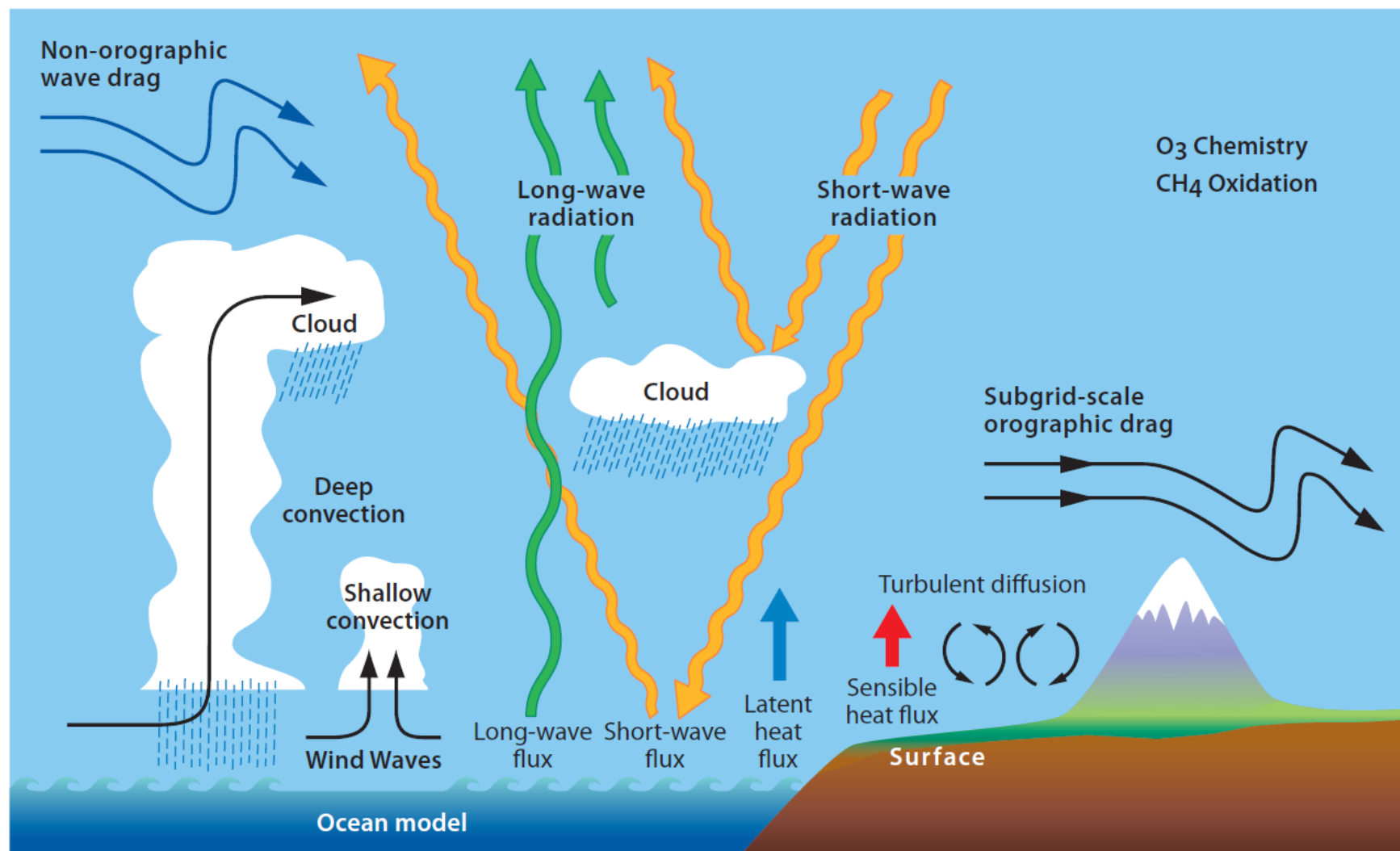


# The complexity of present-day model physics

Figure from Peter Bechtold

Ideally, we wish to identify deficiencies at the process level. Again, this should be easier at short timescales since interactions between physical processes and the resolved flow (including teleconnections) are minimised.

Single column and LES models can help, but these do not take into account the evolution of the resolved flow.



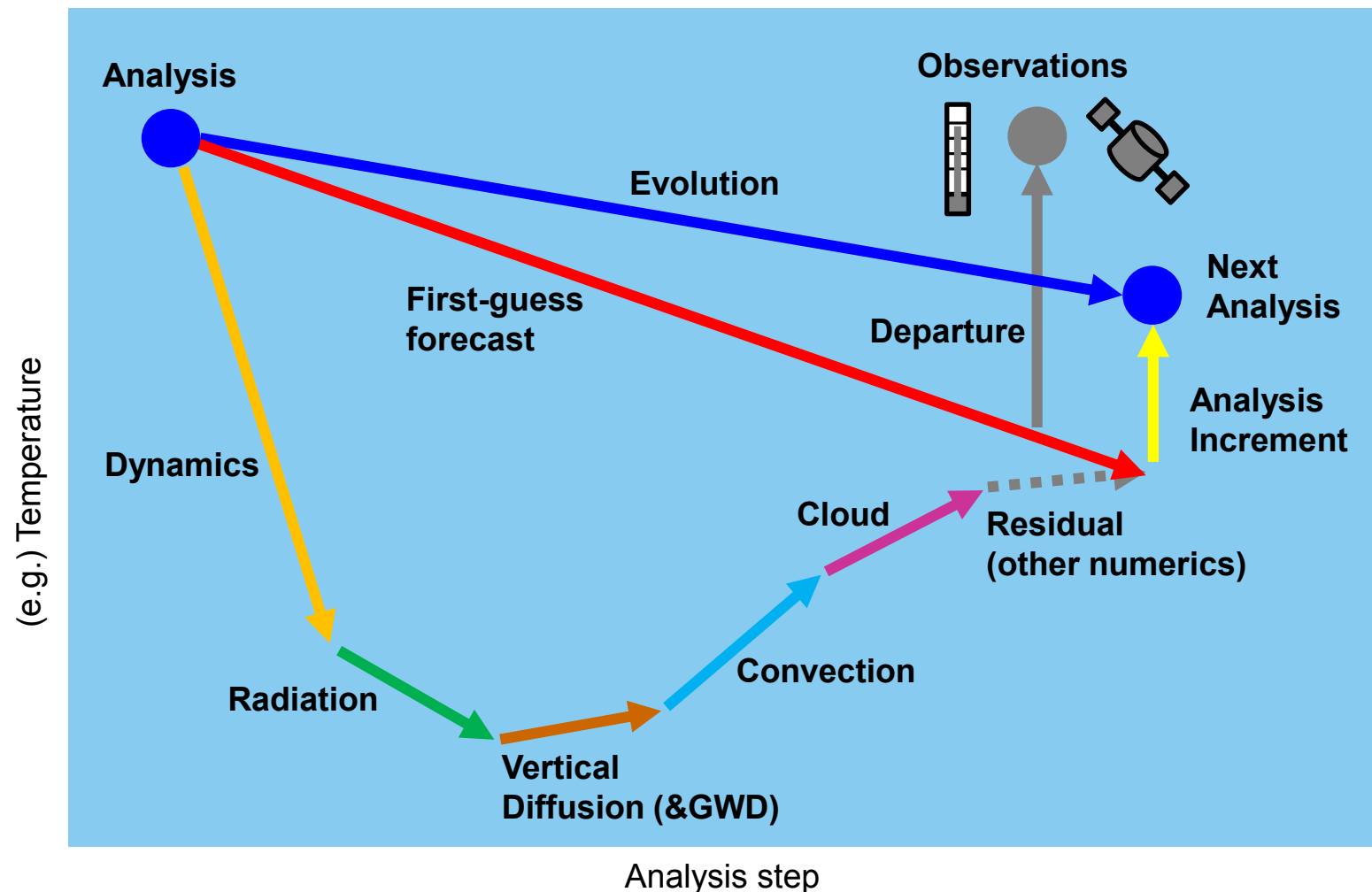
# The Initial Tendency approach to diagnosing model error

Schematic of the data assimilation process – a diagnostic perspective

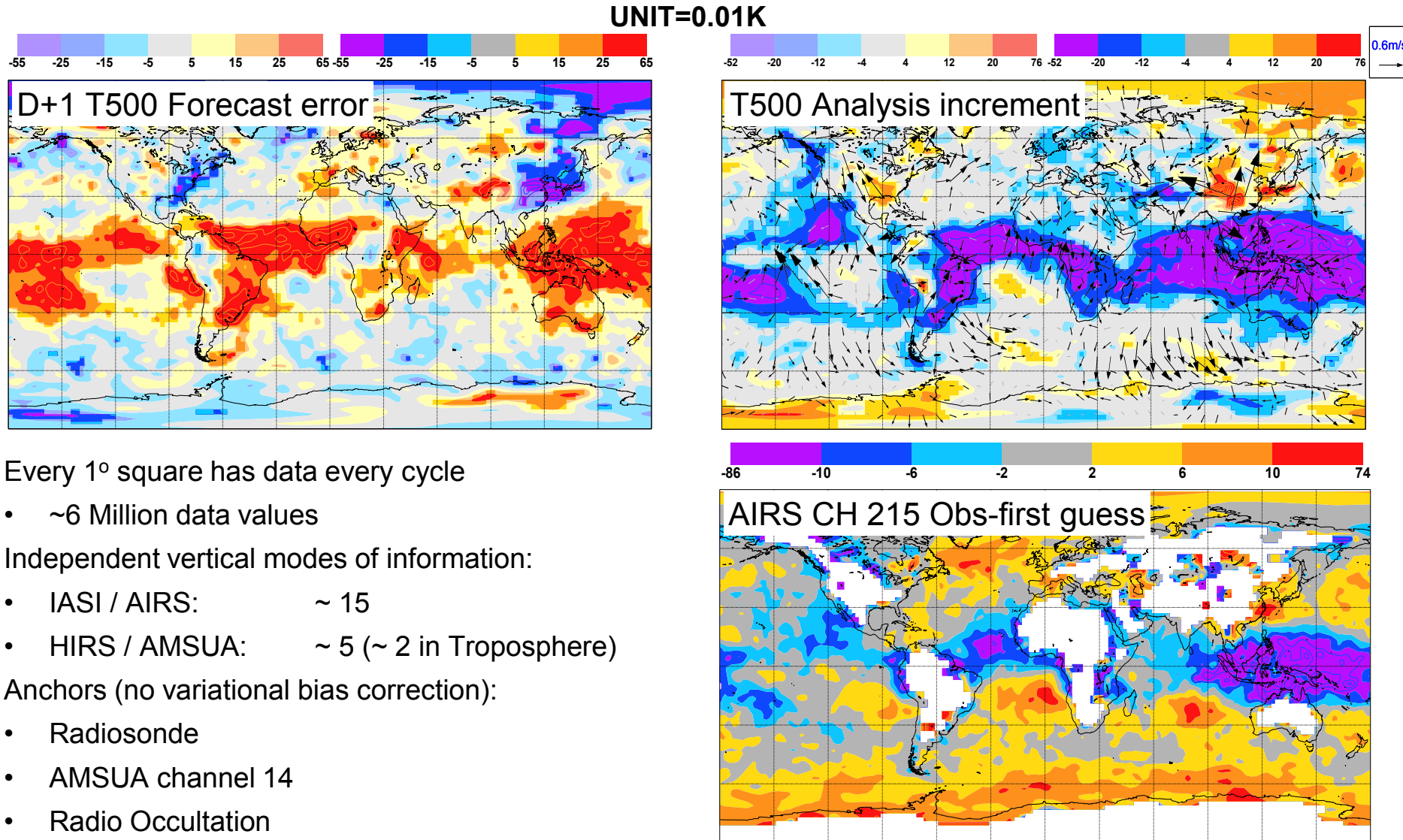
Analysis increment corrects first-guess error, and draws next analysis closer to observations.

First-guess = sum of all processes

Relationship between increment and individual process tendencies can help identify key errors.



“Initial Tendency” approach discussed by Klinker & Sardeshmukh (1992). Refined by Rodwell & Palmer (2007)



Every 1° square has data every cycle

- ~6 Million data values

Independent vertical modes of information:

- IASI / AIRS: ~ 15
- HIRS / AMSUA: ~ 5 (~ 2 in Troposphere)

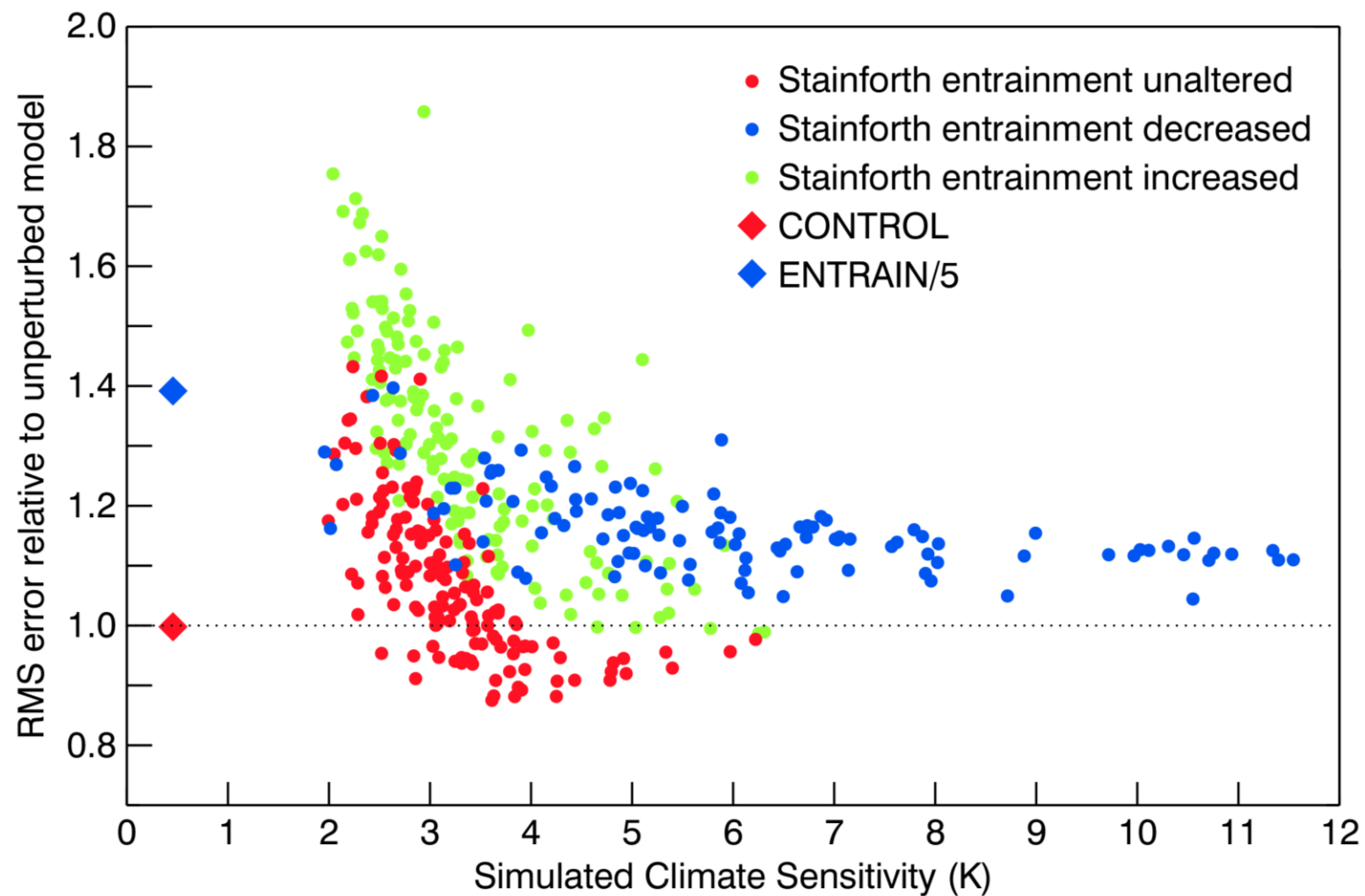
Anchors (no variational bias correction):

- Radiosonde
- AMSUA channel 14
- Radio Occultation

*Based on DJF 2007/8 operational analyses and forecasts. Significant values (5% level) in deep colours. AIRS Channel 215 Brightness temperature (~T500)*

# Climate sensitivity of perturbed climate models

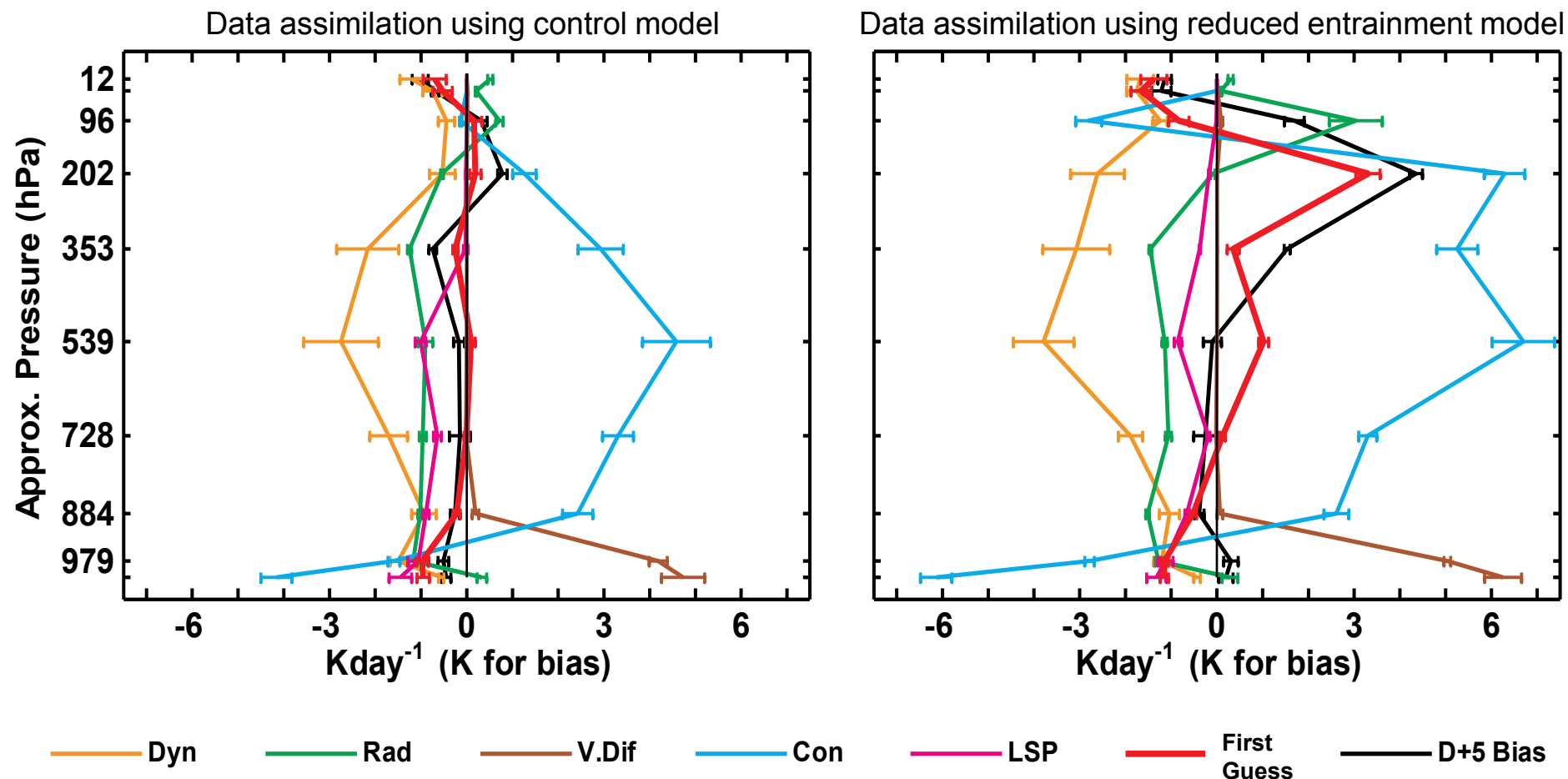
Rodwell and Palmer (2007) with data from Stainforth et al (2005)



# Using Initial Tendencies to investigate 12K warming possibility in climate ensemble

Temperature tendency profiles over the Amazon (300-320°E, 20°S-0°N)

Rodwell and Palmer (2007)

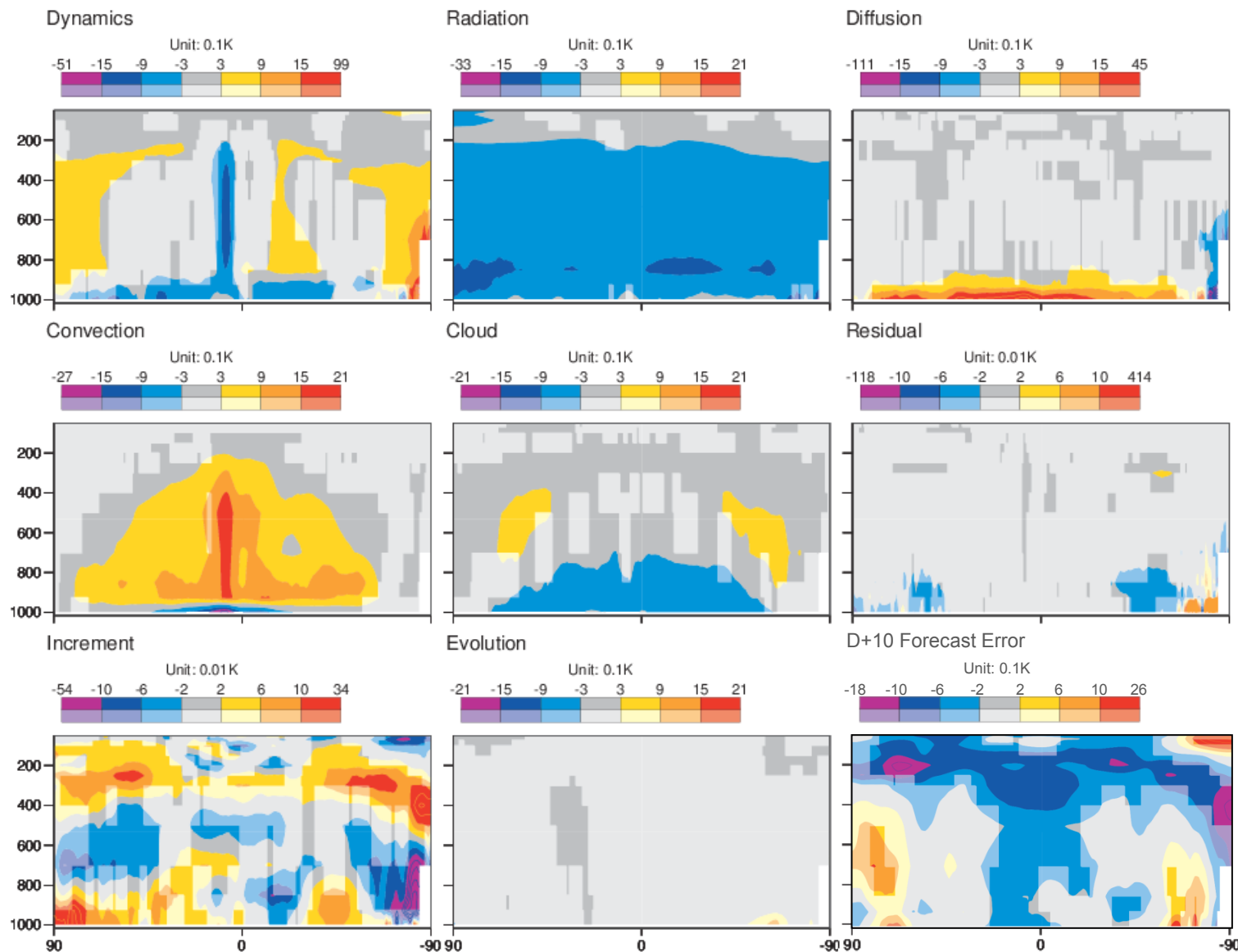


Mean first guess tendency, red, (the sum of all processes) is 'quite small': A reference value for the realism of the model's physics

Greatly increased time-mean first-guess tendency: Perturbation leads to poorer physics. Reject this perturbation from climate ensemble?

6hr tendencies. 31 days (January 2005) X 4 forecasts per day. 70% conf.int. T159, L60, 1800s.

# Using Initial Tendencies to investigate upper-tropospheric cooling error



Strong upper-tropospheric increments (where radiation is not balanced by dynamics)

Error grows x10 by D+10 (due to poorly constrained humidities?)

Zonal-mean temperature tendencies, analysis increments and day 10 forecast error for SON 2013. Note that increment and residual plotted with smaller contour interval

- The aim of this (and my next) lecture is to illustrate ways in which we might evaluate our forecast system and diagnose deficiencies.
- I have demonstrated that waves are important in the atmosphere – they propagate predictable signals. However, they also propagate errors and uncertainty, and interact with physical processes.
- Evaluation of spatio-temporal variability (waves *etc.*) is an increasingly important test of the forecast system. Do analysis increments, ensemble spread, *etc.* project onto such waves?
- With increasing model complexity and accuracy, how can the diagnosis of model error remain effective and affordable? A key approach (Initial Tendencies) is to look at the very short leadtimes (associated with data assimilation) which can localise error (before, *e.g.*, waves have had a chance to propagate these errors) and minimise the effects of chaos.