

# Initial ensemble perturbations - basic concepts

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Acknowledgements: Erland Källén, Martin Leutbecher, ..., ...

# Introduction

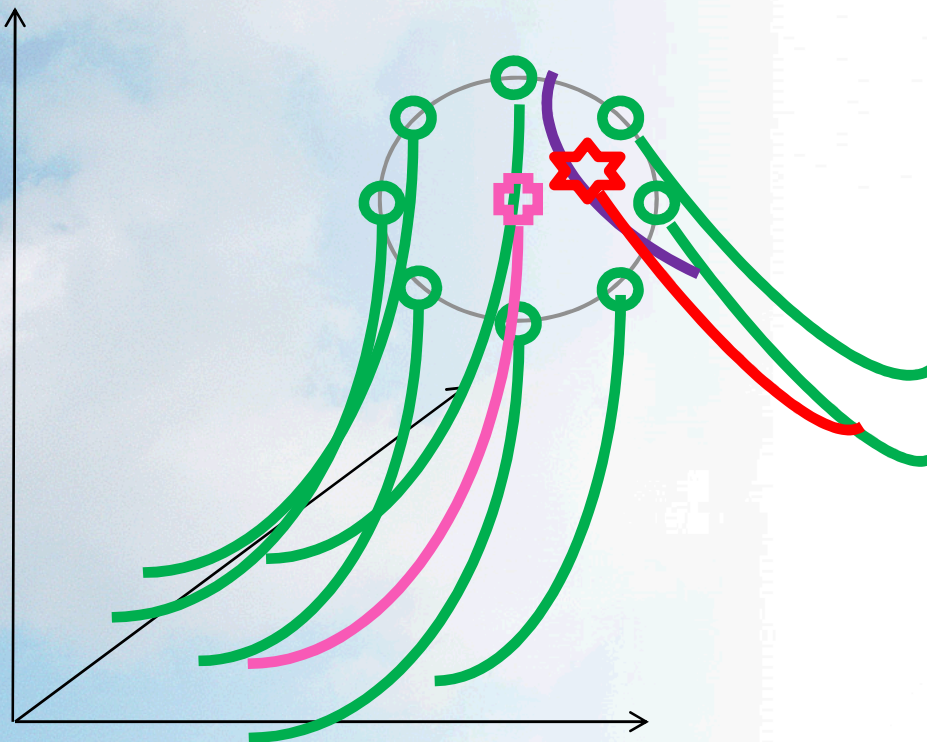
$$\text{Perturbed forecast} = \text{Optimal analysis} + \text{Perturbation}$$

Perturbations added to a subset or all model variables

***“How to construct the initial perturbations?”***

***“Why does not pure random numbers work?”***

# Introduction



Starring list

✚ Analysis – best estimate

○ Uncertainty estimate

— Bifurcation line

○ Perturbed members

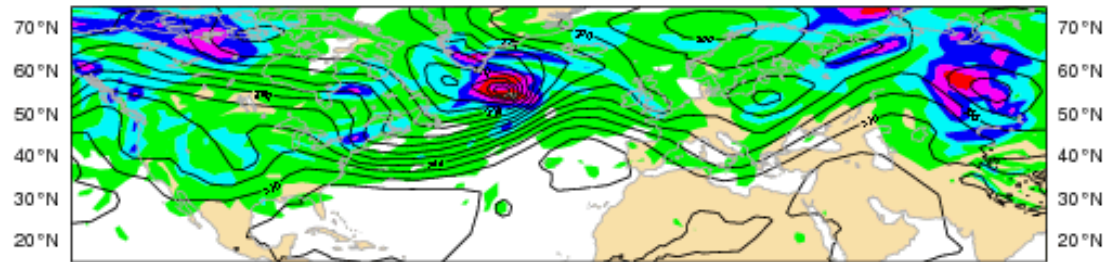
★ Truth (unknown)

***“If we knew the truth, we have started the forecast for it.”***



# Desirable properties of initial perturbations

- Sampling analysis uncertainty
  - Mean amplitude
  - Geographical distribution
  - Spatial scale
  - "Error of the day"
- Sustainable growth
- High quality of probabilistic forecast



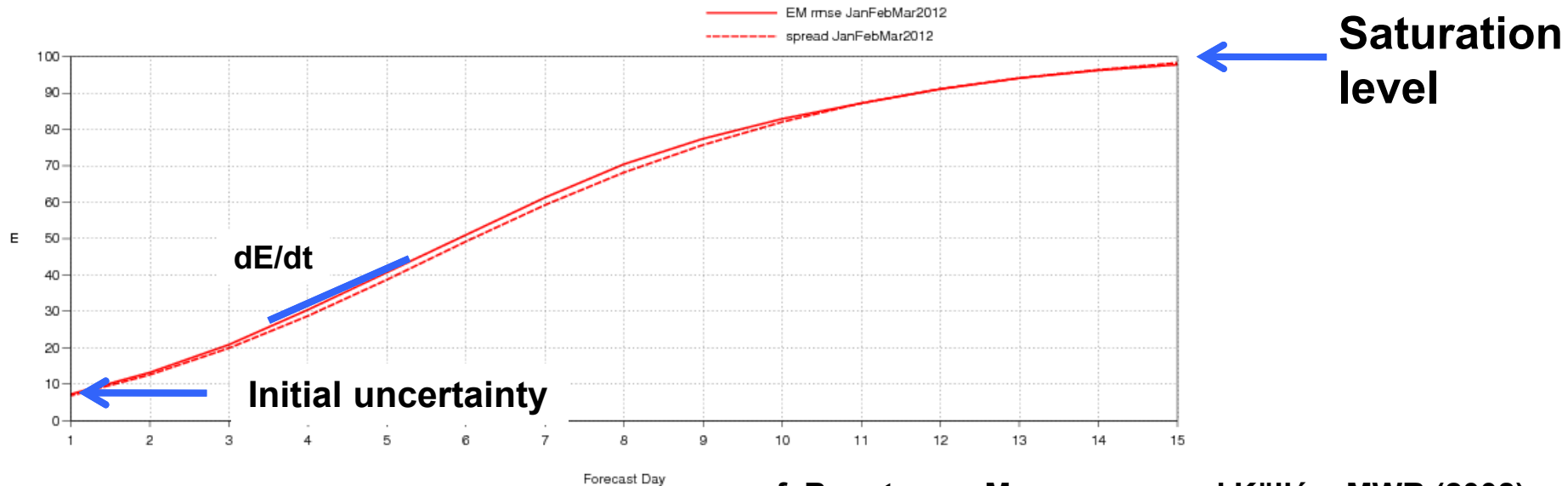
# Ensemble mean RMSE and ensemble spread

## Ensemble Mean RMSE and Ensemble Spread

500hPa geopotential

NHem Extratropics (lat 20.0 to 90.0, lon -180.0 to 180.0)

JanFebMar



cf. Bengtsson, Magnusson and Källén, MWR (2008)

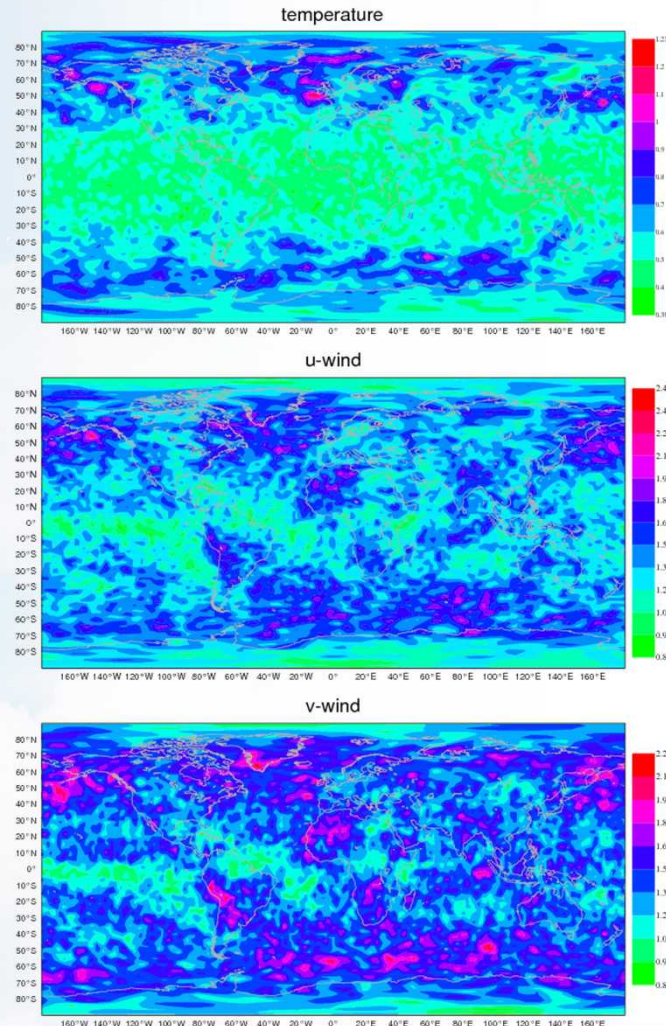
**Optimally: Ensemble mean RMSE = ensemble spread**



# Random Perturbations (why does it not work?)

Random grid-point  
number (-1 to 1) x

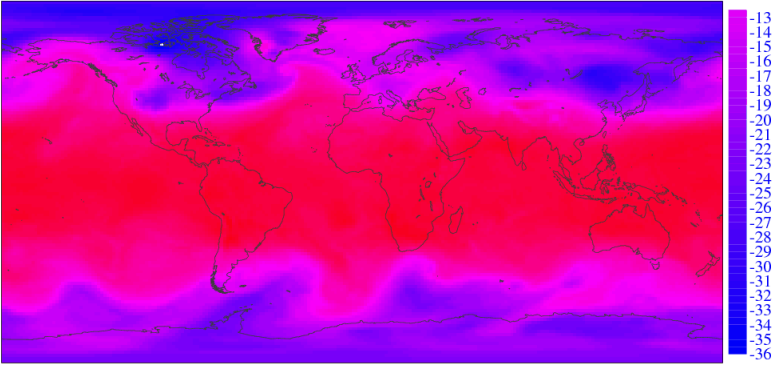
## Analysis error estimate



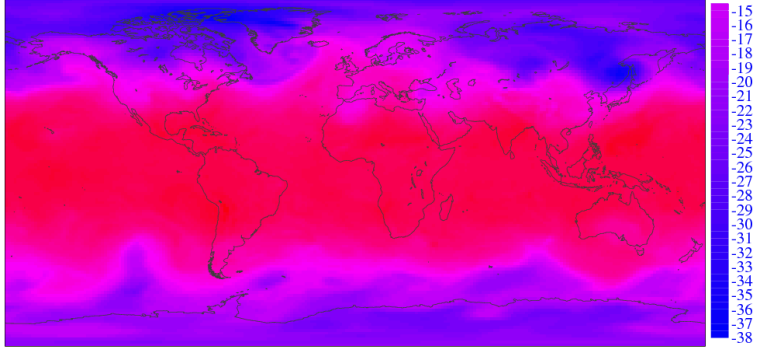
x global tuning factor

# Random Field (RF) perturbation (for benchmarking)

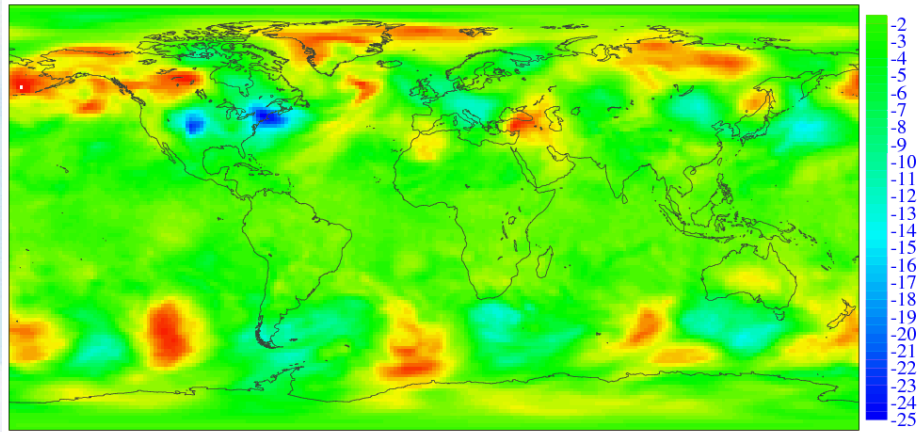
Analysis Random date 1



Analysis Random date 2



Random Field Perturbation



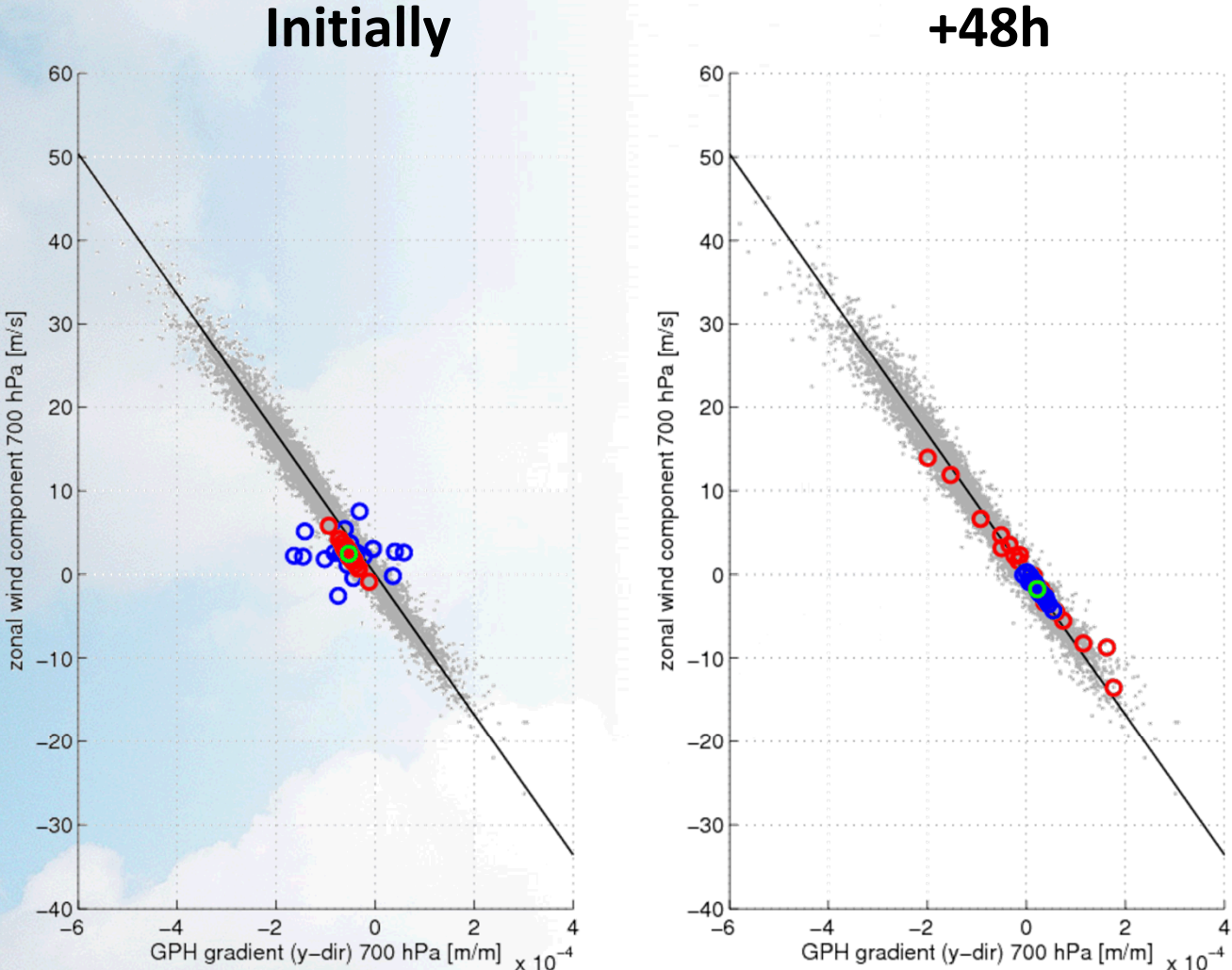
x normalizing factor =

Not flow-dependent, but linear balances maintained

cf. Magnusson, Nycander and Källén, Tellus A (2009)



# Geostrophic balance and perturbations



Blue – random pert., Red – random field, climatology - grey



# Singular vectors

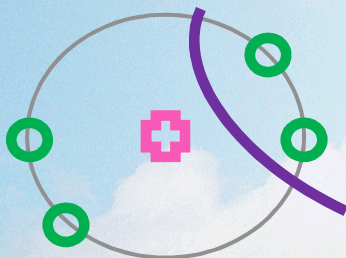
$$\alpha = \frac{\langle \Delta \vec{x}(t), \Delta \vec{x}(t) \rangle}{\langle \Delta \vec{x}(0), \Delta \vec{x}(0) \rangle}$$

Optimize perturbation growth for a time interval

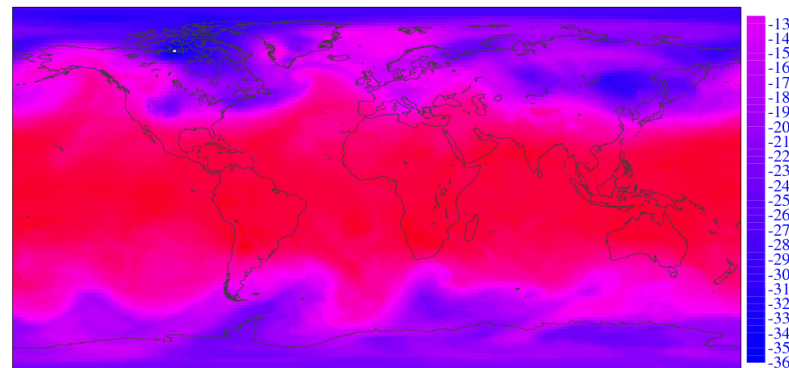
Norm dependent!

M tangent linear operator.  $\Delta \mathbf{x}(t) = \mathbf{M}(t, \mathbf{x}_0) \Delta \mathbf{x}(0)$

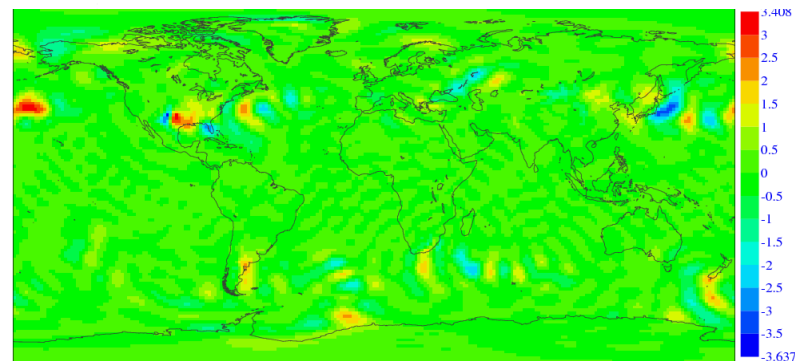
(Will be further explained by Simon Lang)



## Atmospheric state

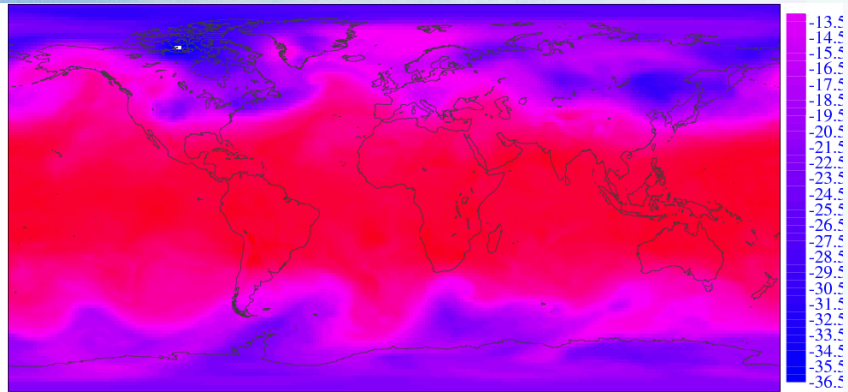


## Singular vector perturbation

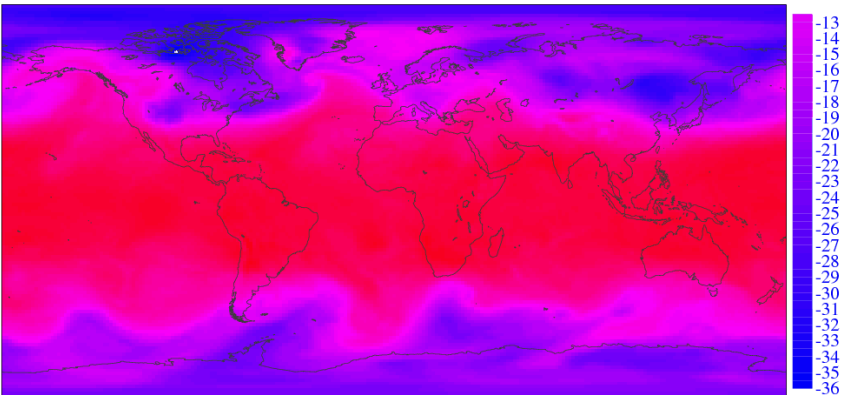


# Breeding perturbations

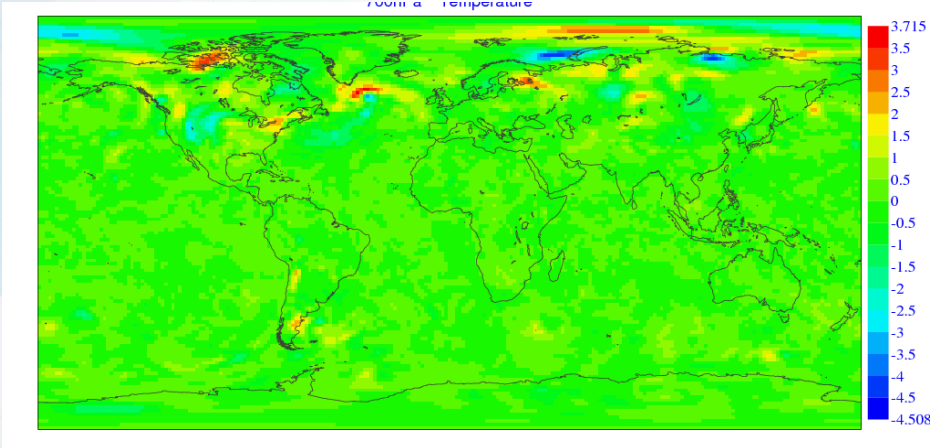
## Perturbed Forecast +06h



## Unperturbed Forecast +06h



## Breeding vector



x normalizing factor =

# Ensemble transform perturbations (further development of BV)

(Wei et al., Tellus A, 2008)

$$X^f = [x_1^f, x_2^f, \dots, x_m^f, \dots, x_k^f] \quad (\text{x=pf-em}), \text{ compare with BV}$$

$$X^a = [x_1^a, x_2^a, \dots, x_m^a, \dots, x_k^a]$$

$$X^a = X^f C \Gamma^{-1/2}$$

Make the perturbations orthogonal

C and  $\Gamma$  from eigenvalue problem:

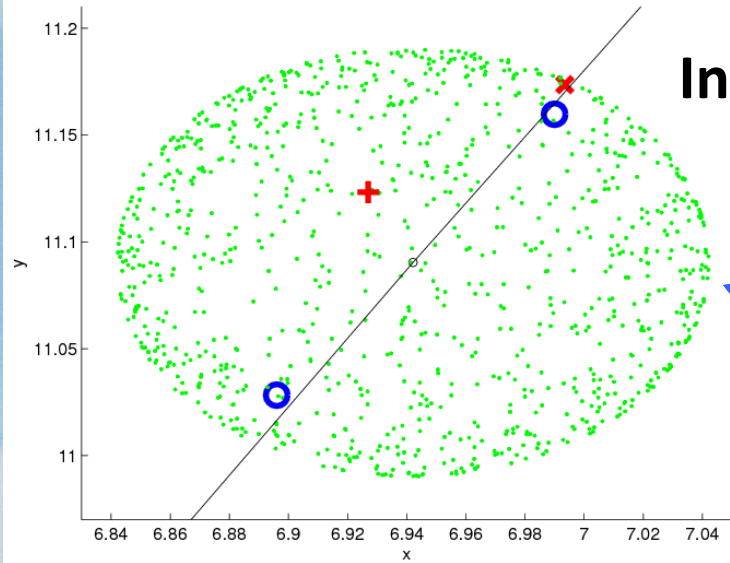
$$\frac{X^{fT} P_{op}^{-1} X^f}{n}$$

Error norm

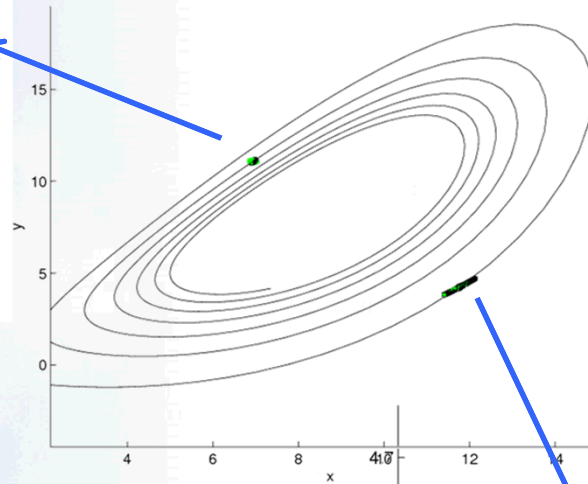
- Simplex transformation
- Regional re-scaling
- ETKF perturbations similar idea (Wang and Bishop, JAS, 2003)



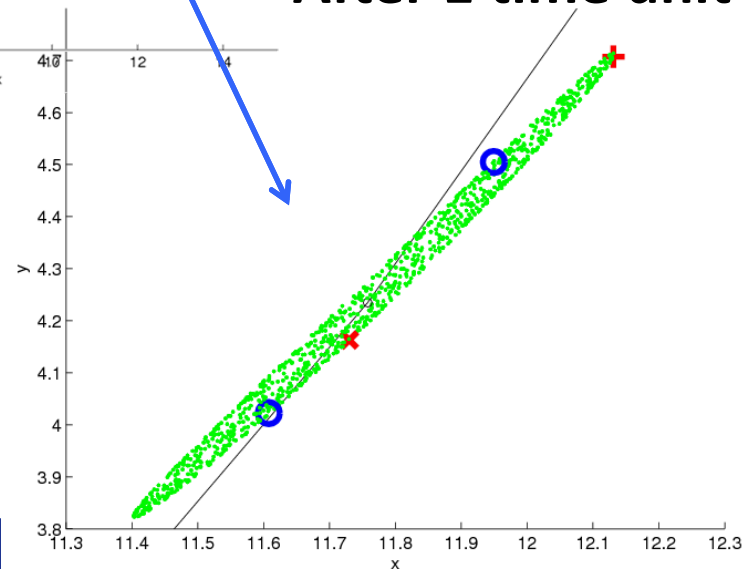
# Perturbation methods Lorenz-63



Initial point



After 1 time unit



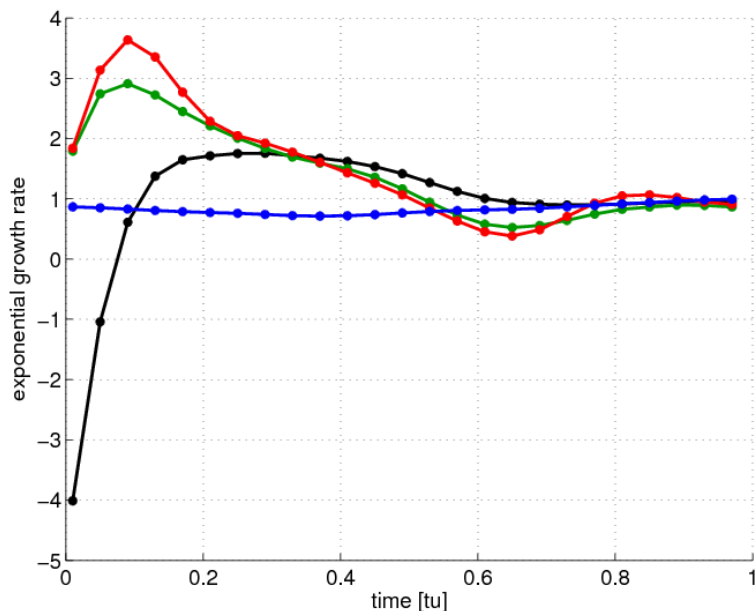
• Random pert.

- + 1st SV
- × 2nd SV
- BV

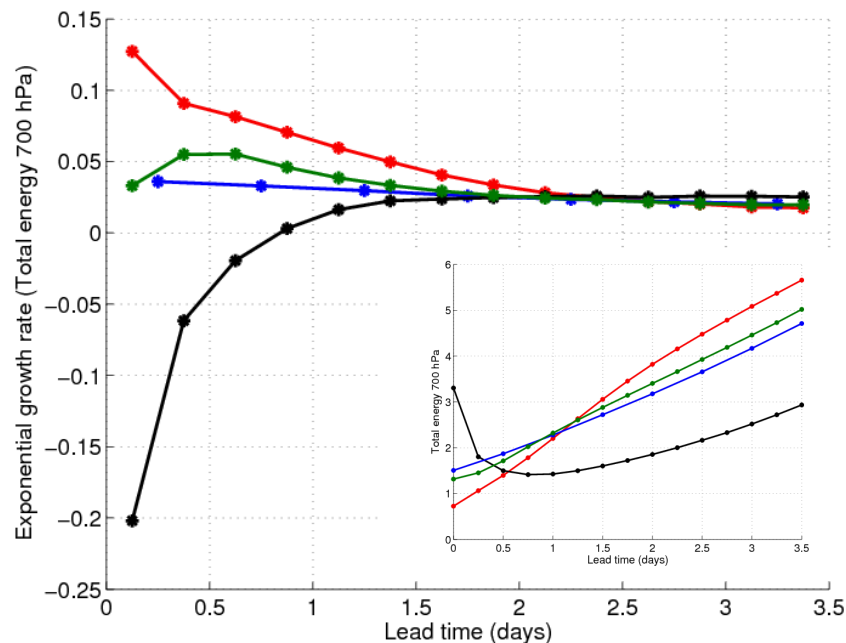
# Exponential perturbation growth

$$\lambda = \frac{1}{\Delta t} \ln \left( \frac{\|\Delta x(t + \Delta t)\|}{\|\Delta x(t)\|} \right)$$

## Lorenz-63



## NWP-model (ECMWF)

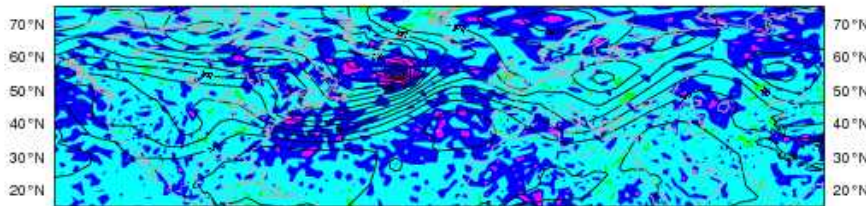


SV – red, BV – blue, Random Field Pert. – Green, Random Pert. - black

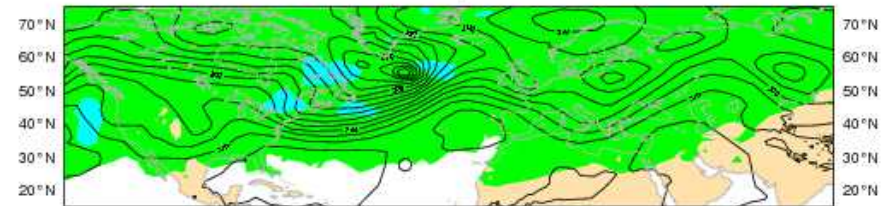
# Evolution of ensemble spread (one case, total pert. energy 700 hPa)

Initially

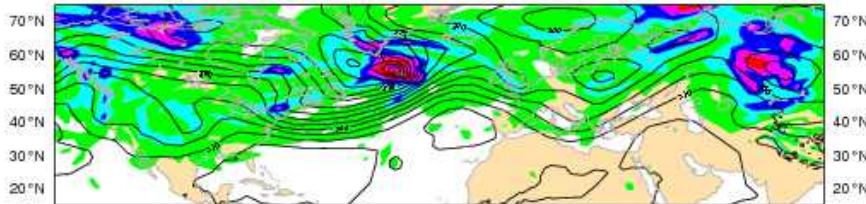
Random perturbations



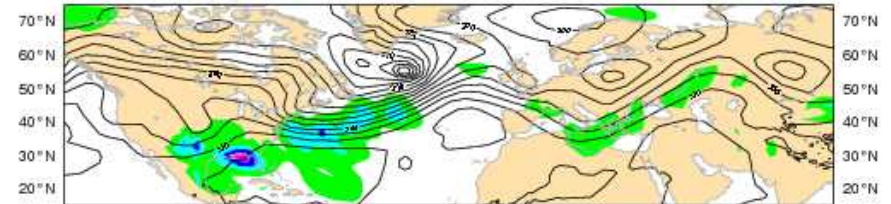
Random Field perturbations



Breeding perturbations



Singular Vector perturbations



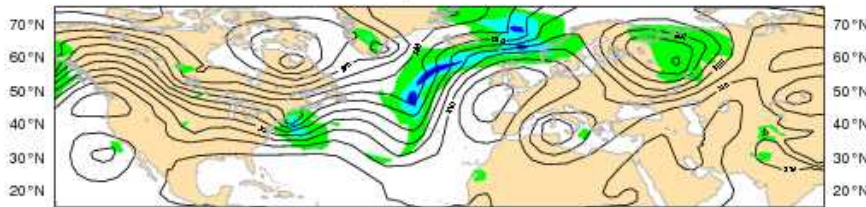
Maximum – red



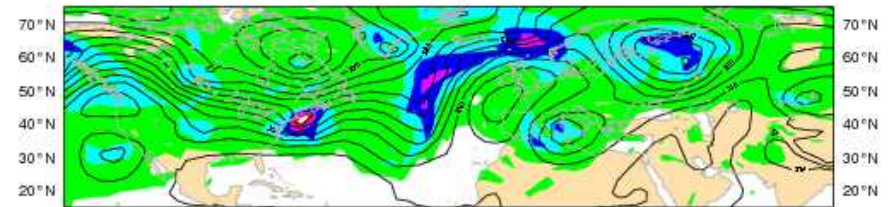
# Evolution of ensemble spread (one case, total pert. energy 700 hPa)

+48h

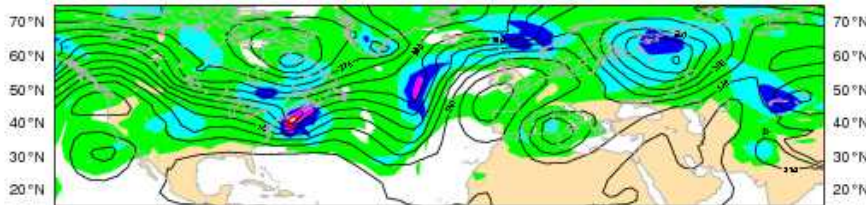
Random perturbations



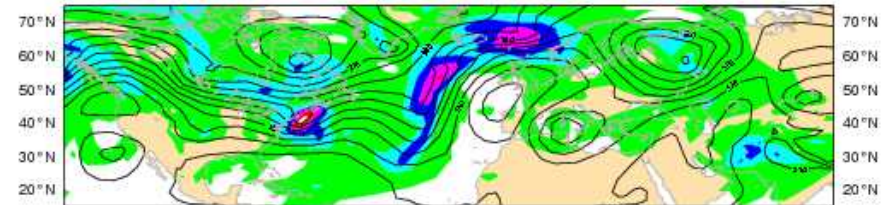
Random Field perturbations



Breeding perturbations



Singular Vector perturbations

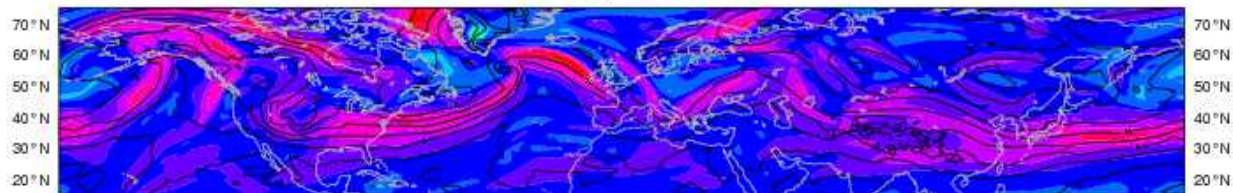


Maximum – red, Scale 48: twice the scale for +00h

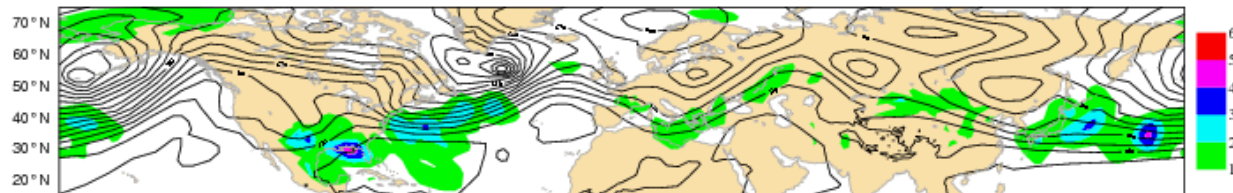
# Connections between perturbations and baroclinic zones

$$E = 0.3125 \frac{f}{N} \frac{dV}{dz} \quad \text{Fastest growth rate of normal modes}$$

Eady index

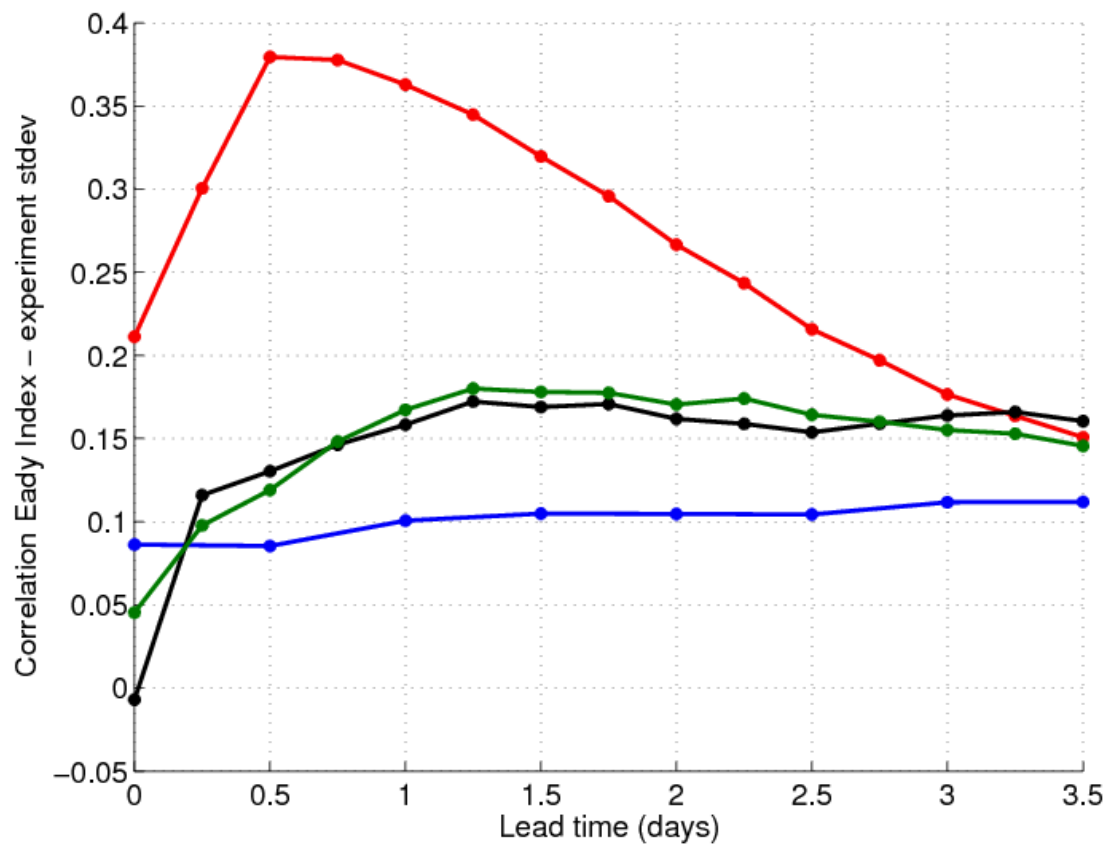


(d) Singular Vector perturbations





# Correlation Eady index – Ens. Stdev z500



SV - Red  
ET - Blue  
RF - Green  
RP - Black

(20N-70N)



# Mean initial perturbation distribution

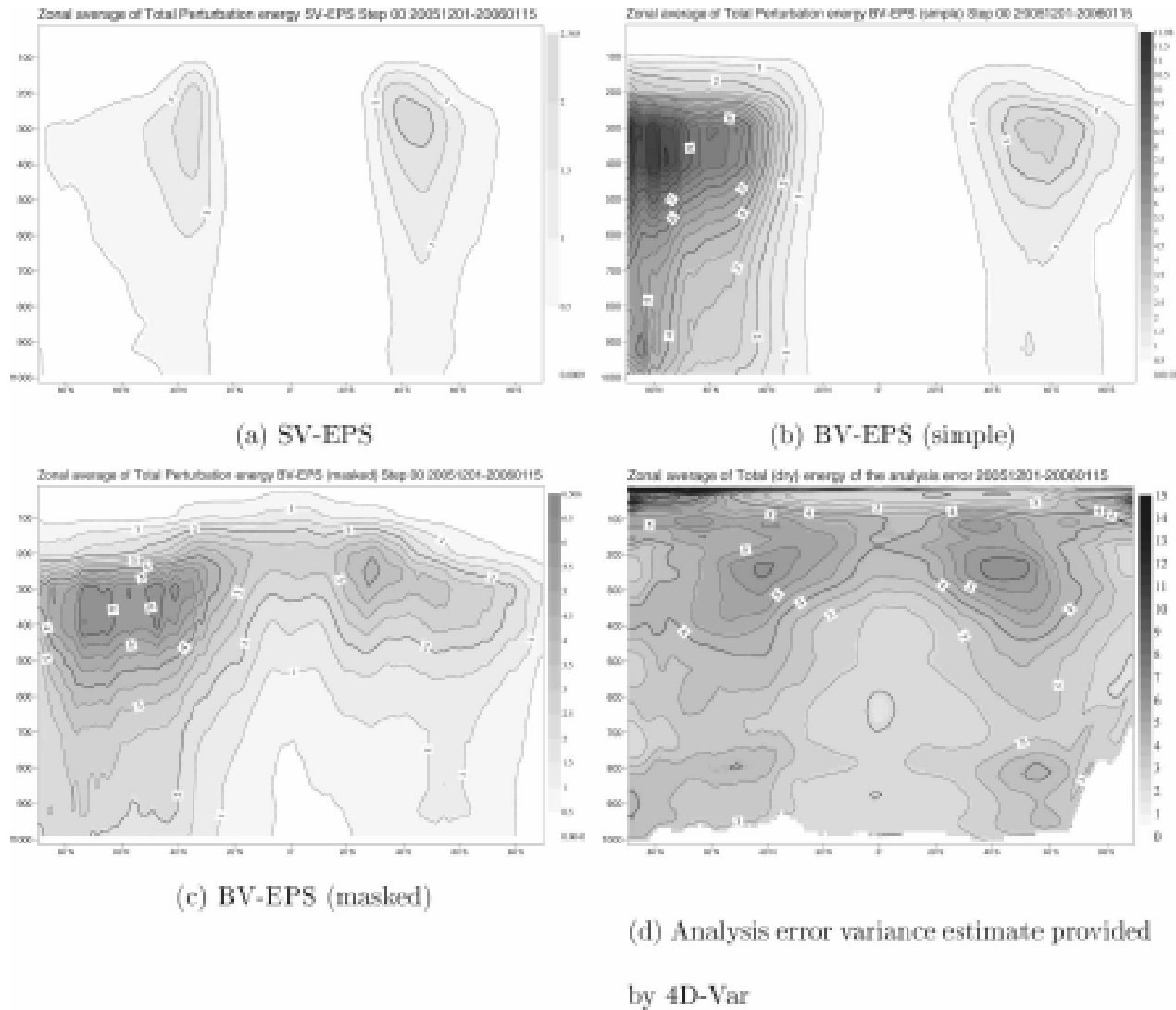


Figure 2. Magnusson, Leutbecher and Källén. 2008 (MWR)

# Mean perturbation distribution after 48 hours

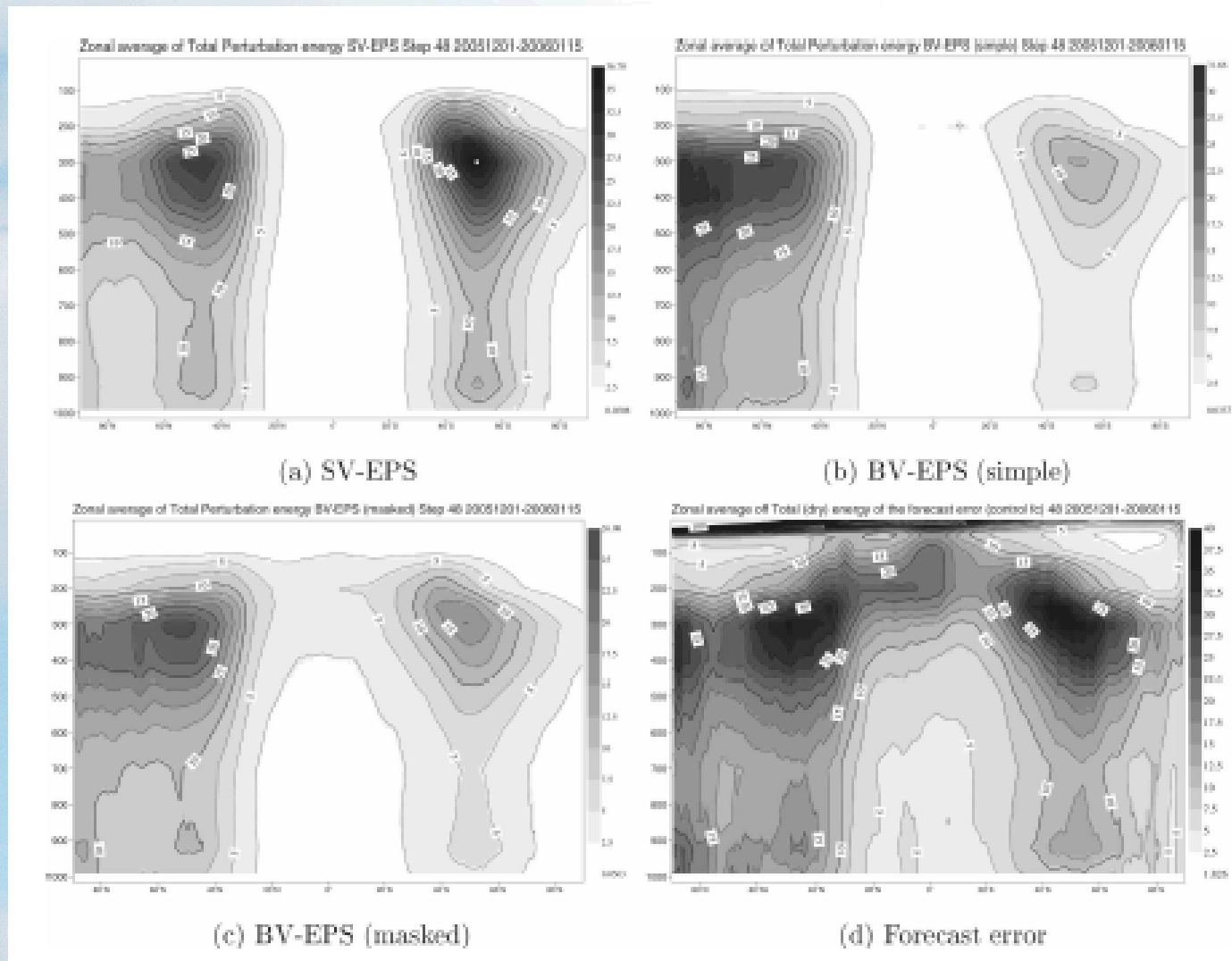
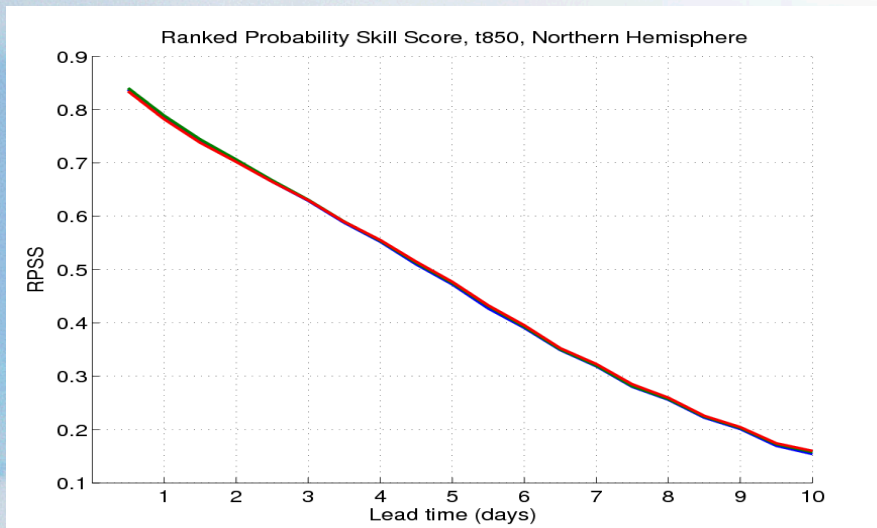


Figure 3. Magnusson, Leutbecher and Källén. 2008 (MWR)

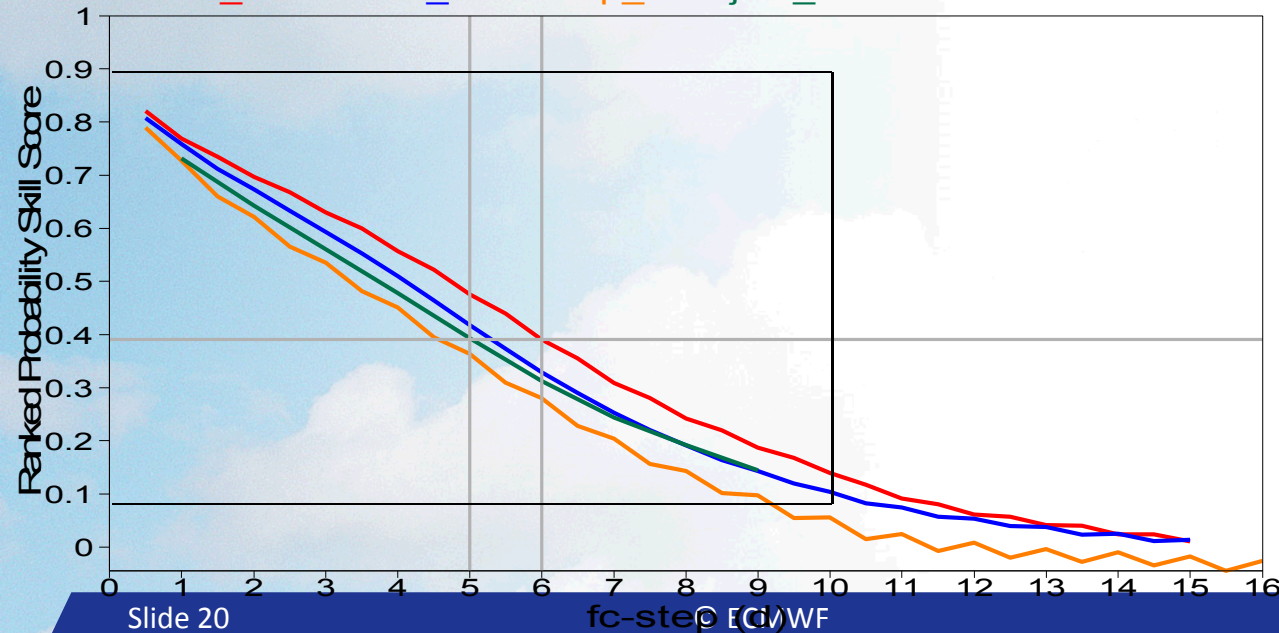
# Ranked Probability skill score - t850



Different initial perturbations  
(N.Hem)

SV - Red  
ET - Blue  
RF - Green

t at 850hPa (cf. as\_ref)  
10 categories, cases 20070328-20070528\_N62, area n.hem  
ecmwf\_vt12 ukmo\_vt12 ncep\_vt12 jma\_vt12



Different Centres  
(from Park et al.(2008),  
Courtesy R. Buizza )



## Other things to consider - Perturbation symmetry

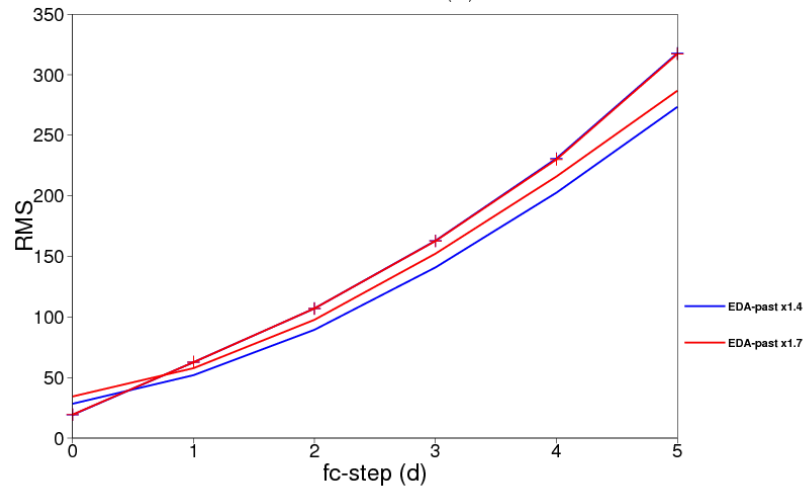
- +/- symmetry -> rank  $N/2$
- Simplex transformation -> rank  $N-1$
  
- No clear advantage for simplex transformation in our metrics

# Other things to consider - Importance of initial amplitude scaling

## Two models with different tuning of the initial amplitude

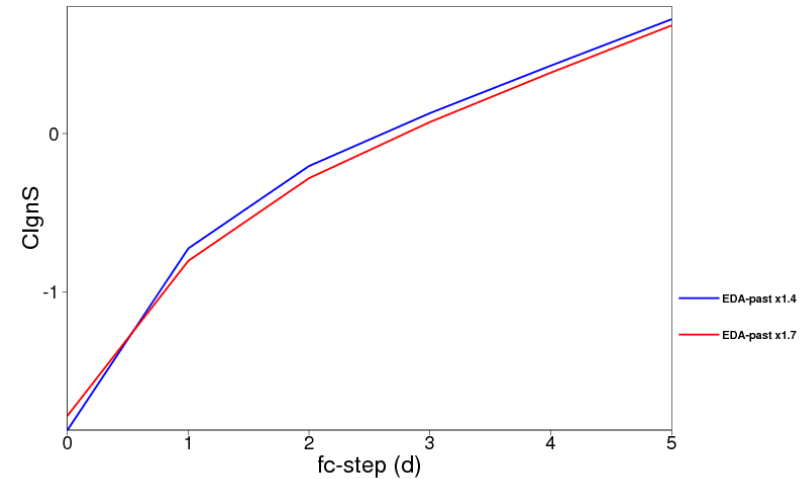
z500hPa, Northern Extra-tropics

spread\_em, rmse\_em  
2011052200-2011061100 (11)



z500hPa, Northern Extra-tropics

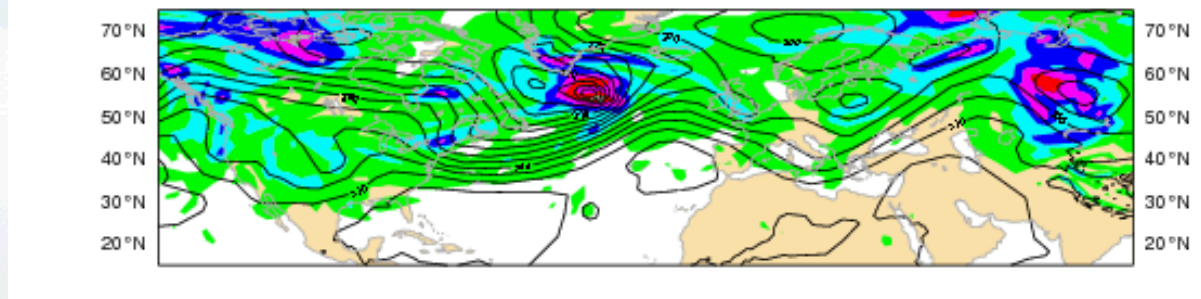
ContinuousIgnoranceScoreGaussian  
2011052200-2011061100 (11)



**Perturbation growth is highly model dependent!**

# Desirable properties for initial perturbations

	SV	BV and ET	RF	Random
<u>Sampling analysis uncertainty</u>				
Mean amplitude		v	v	
Geographical			v	V
Spatial scale	v	V	v	
Growth	v (too fast?)	V	v	
An Error of the day	V	v		
<u>Fc Error of the day</u>	V	V	V	V
<u>Fc Quality</u>	V	V	V	





# Ensemble assimilation and prediction

