Parameterization of momentum fluxes related to sub-grid orography

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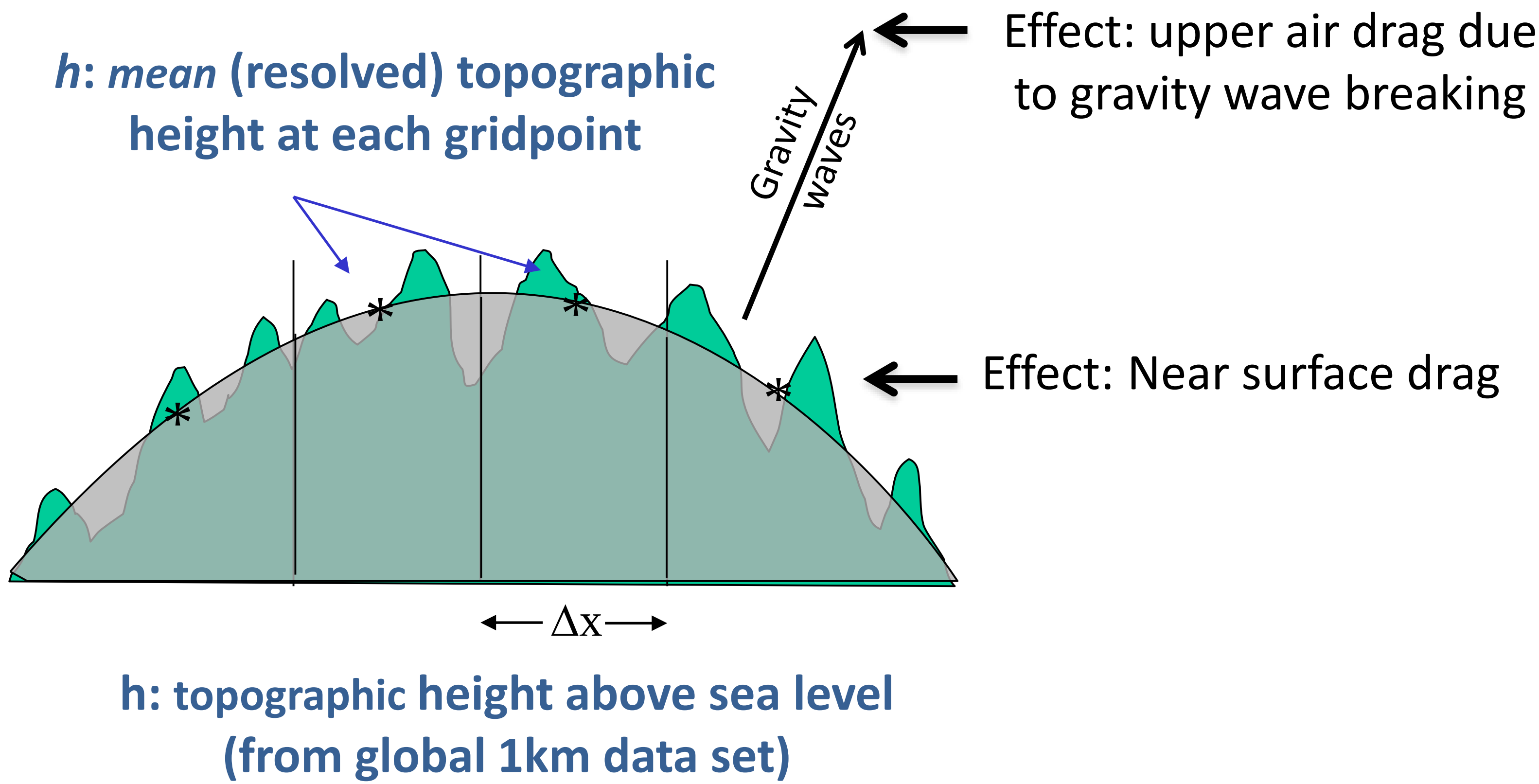
1. Gravity wave theory
2. Parameterization
3. Impact

Sub-grid orography



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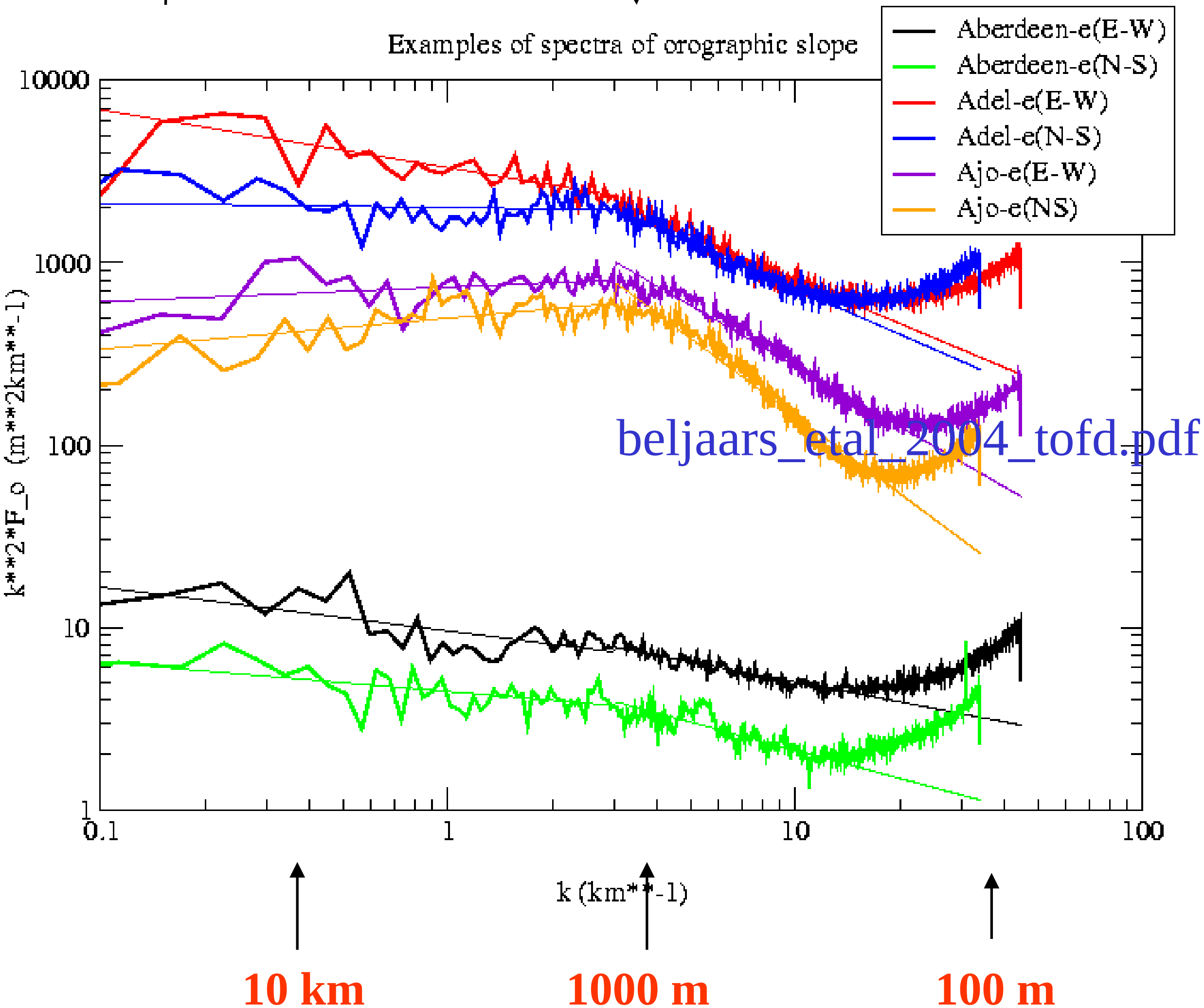
Orographic slope spectrum

$$F(k) \sim k^{-1.9} \qquad F(k) \sim k^{-2.8}$$



$$\theta^2 = \int_{k_o}^{\infty} k^2 F(k)\, dk$$

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Beljaars, Brown and Wood, 2004

Simple properties of gravity waves

In order to prepare for a description of the parameterization of gravity-wave drag, we examine some simple properties of gravity waves excited by two-dimensional stably stratified flow over orography.

We suppose that the horizontal scales of these waves is sufficiently small that the Rossby number is large (ie Coriolis forces can be neglected), and the equations of motion can be written as

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0 \qquad (1)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}+g=0 \qquad (2)$$

$$\frac{\partial u}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial z}(\rho w)=0 \qquad (3)$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+w\frac{\partial \theta}{\partial z}=0 \qquad (4)$$

(After T. Palmer 'Theory of linear gravity waves', ECMWF meteorological training course, 2004)

Simple properties of gravity waves

The Boussinesq approximation is used whereby density is treated as a constant except where it is coupled to gravity in the buoyancy term of the vertical momentum equation. Linearising (1)-(4) about a uniform hydrostatic flow **u_0** with constant density **ρ_0** and static stability **N**, with

$$N^2 = g\frac{d\ln\theta_0(z)}{dz}, \quad \frac{dp_0}{dz} = -\rho_0 g,$$

$$u = u_0 + u', \quad w = 0 + w', \quad \rho = \rho_0 + \rho', \quad p = p_0 + p',$$

results in the perturbation variables

$$\frac{\partial u'}{\partial t} + u_0\frac{\partial u'}{\partial x} + \frac{1}{\rho_0}\frac{\partial p'}{\partial x} = 0, \tag{5}$$

$$\frac{\partial w'}{\partial t} + u_0\frac{\partial w'}{\partial x} + \frac{1}{\rho_0}\frac{\partial p'}{\partial z} - \frac{\rho'}{\rho_0}g = 0, \tag{6}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0, \tag{7}$$

$$\frac{\partial \theta'}{\partial t} + u_0\frac{\partial \theta'}{\partial x} + w'\frac{\partial \theta_0}{\partial z} = 0. \tag{8}$$

Simple properties of gravity waves

Assuming density fluctuations to be dependent on temperature only

$$\frac{\rho'}{\rho_0} = \frac{\theta'}{\theta_0}. \qquad (9)$$

Equations (5-9) are five equations in five unknowns. These can be reduced to one linear equation

$$\left(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}\right)^2 \left(\frac{\partial^2 w'}{\partial x^2} + \frac{\partial^2 w'}{\partial z^2}\right) + N^2 \frac{\partial^2 w'}{\partial x^2} = 0. \qquad (10)$$

We now look for sinusoidal solutions of the general form

$$w' = \hat{w} \exp\left[i\left(kx + mz - \omega t\right)\right],$$

where $k = 2\pi/\lambda_x$ **is the horizontal wavenumber**

$m = 2\pi/\lambda_z$ **is the vertical wavenumber**

ω **is the wave frequency**

Substitution leads to the dispersion relation

$$\left(w - u_0 k\right)^2 \left(k^2 + m^2\right) - N^2 k^2 = 0,$$

$$\tilde{w} = w - u_0 k = \pm \frac{Nk}{\sqrt{k^2 + m^2}}.$$

$\tilde{w}$ **is the intrinsic frequency**

Derivation steps:

1. Use (9) to eliminate θ' from (8) -> resulting in equation for ρ' . Result: (8A)
2. Take total derivative of (6) and eliminate ρ' with (8A). Result: (6A)
3. Take total derivative of (5). Result (5A)
4. Take partial x-derivative of (6A) and subtract partial y-derivative of (5A). Result (6B).
5. Eliminate ∂u/∂x from (6B) using (7)

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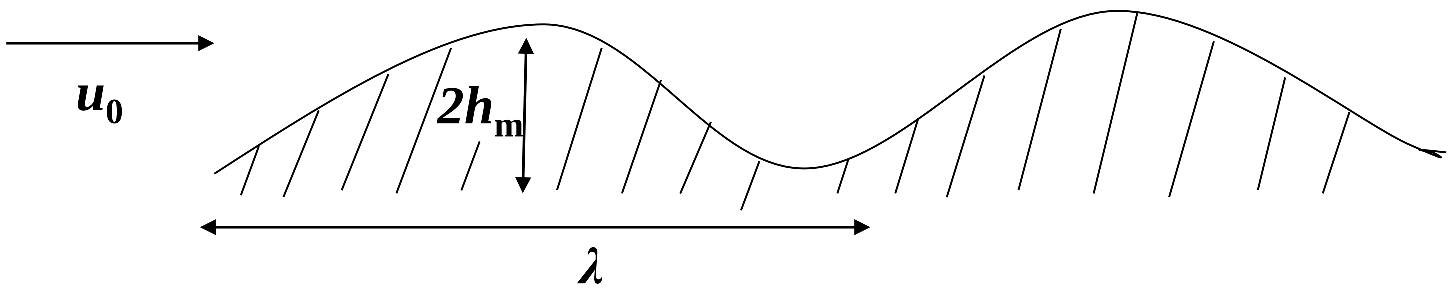
Sinusoidal hill

Consider stationary waves forced by sinusoidal orography with elevation h(x)

$$h = h_m \sin kx \quad k = 2\pi/\lambda$$



The lower boundary condition (the vertical component of the wind at the surface must vanish) is

$$w(z=0) = u_0 \frac{\partial h}{\partial x} = u_0 k h_m \cos kx$$

For steady state situations with *ω*=0, *m* can be derived from the dispersion relation:

$$u_0^2 \left(k^2 + m^2\right) - N^2 = 0 \quad \rightarrow \quad m^2 = \frac{N^2}{u_0^2} - k^2$$

$$m^2 = N^2/u_0^2 - k^2$$

Solutions periodic in *x* that satisfy the surface boundary condition:

$$w' = \mathrm{Re}\left\{ \hat{w} e^{ikx+imz} \right\} = \mathrm{Re}\left\{ u_0 k h_m e^{ikx+imz} \right\}$$

$$k > N/u_0$$

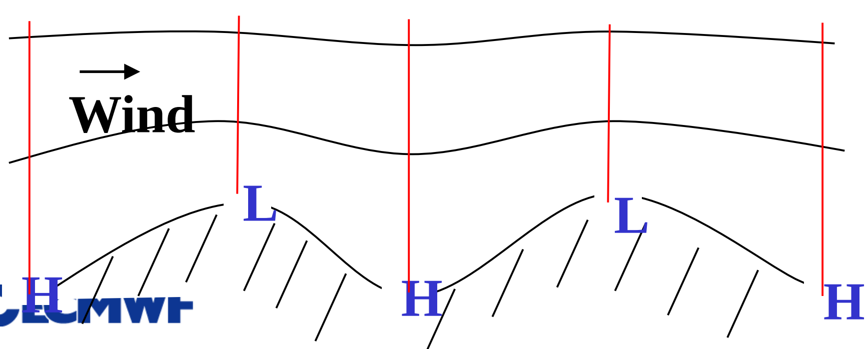
m is imaginary: Evanescent solution

$$w' = u_0 k h_m e^{-|m|z} \cos kx$$

From the continuity equation,

$$u' = u_0 h_m |m| e^{-|m|z} \sin kx$$

$$\overline{\rho_0 u' w'} = 0$$



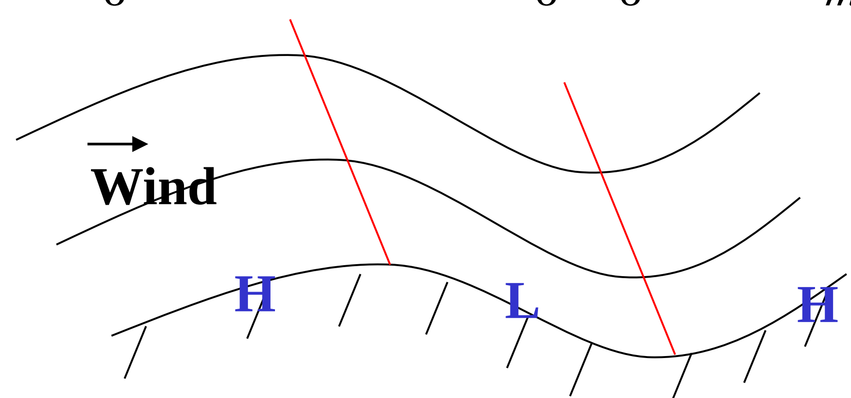
$$k < N/u_0$$

m is real: Propagating solution

$$w' = u_0 k h_m \cos(kx + mz)$$

$$u' = -u_0 m h_m \cos(kx + mz)$$

$$\overline{\rho_0 u' w'} = -0.5 \rho_0 u_0^2 k m h_m^2$$



Summary: two regimes

k>N/U (i.e. narrow-ridge case)
(or equivalently Uπ/L>N, i.e. high frequency)
Evanescent solution (i.e. fading away)
Non-dimensional length *NL/U*<π

- waves decay exponentially with height
- vertical phase lines
- linear theory -> no drag. Steep small scales leading to form drag -> TOFD scheme

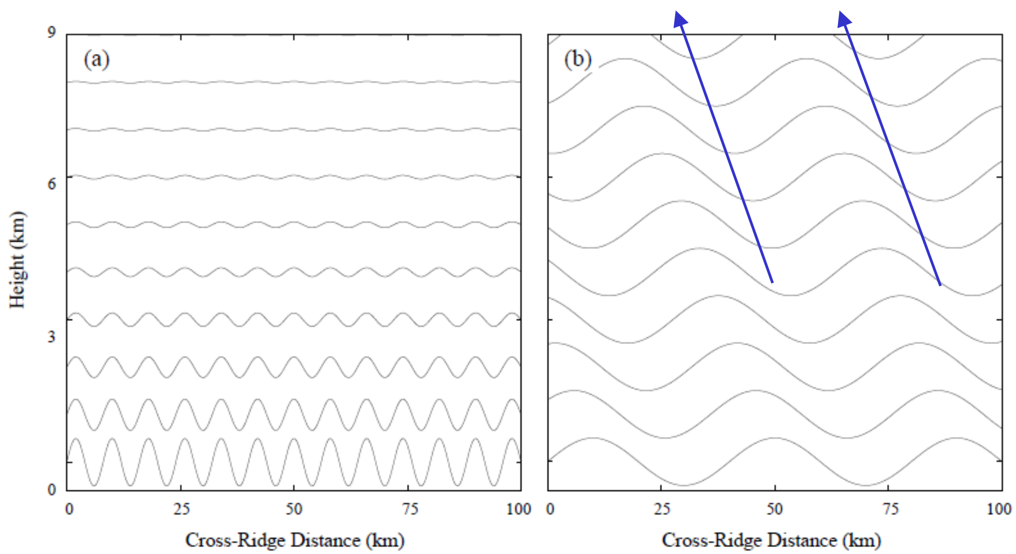
$$w = Ae^{-|m|z}\cos kx$$

k<N/U (i.e. wider mountains)
(or equivalently Uπ/L<N, i.e. low frequency)
Wave solution
Non-dimensional length *NL/U*>π

- energy/momentum transported upwards
- waves propagate without loss of amplitude
- phase lines tilt upstream as z increases

$$w = A\cos(kx + mz)$$

For typical atmospheric wind and stability (U=10 m/s and N=0.01 s-1):
L ≈ 3 km



Durran, 2003

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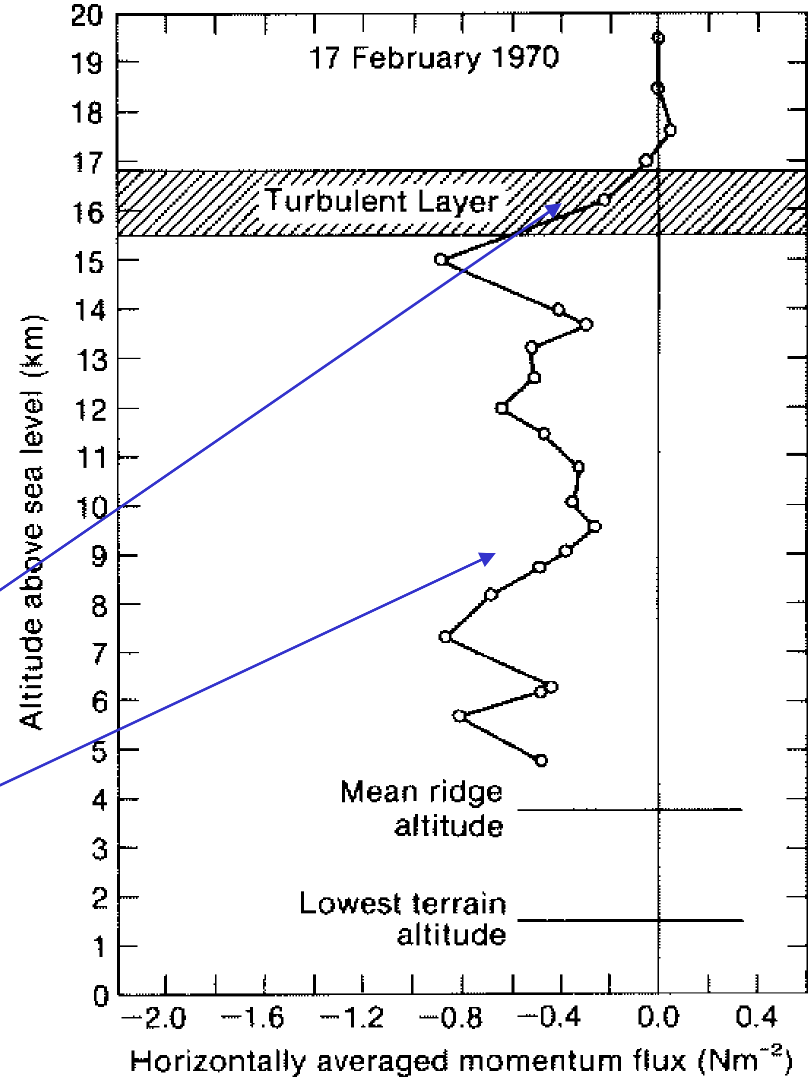
What happens to the moment flux associated with gravity waves

Momentum flux: $-\rho\left(\overline{u'w'}\right)$
is constant and density decreases with height, so the amplitude of gravity wave increases until they break

$$\frac{\partial u}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho\overline{u'w'}\right)$$

Stress rapidly changing; strong dissipation/wave breaking; $\partial u / \partial t \neq 0$

Stress largely unchanged; little dissipation/wave breaking; $\partial u / \partial t \approx 0$



Mean observed profile of momentum flux over Rocky mountains on 17 February 1970 (from Lilly and Kennedy 1973)

Wave breaking occurs:

1. **When wave perturbation leads to convective overturning**
2. **Due to shear instability when locally the Richardson number drops below a critical value**

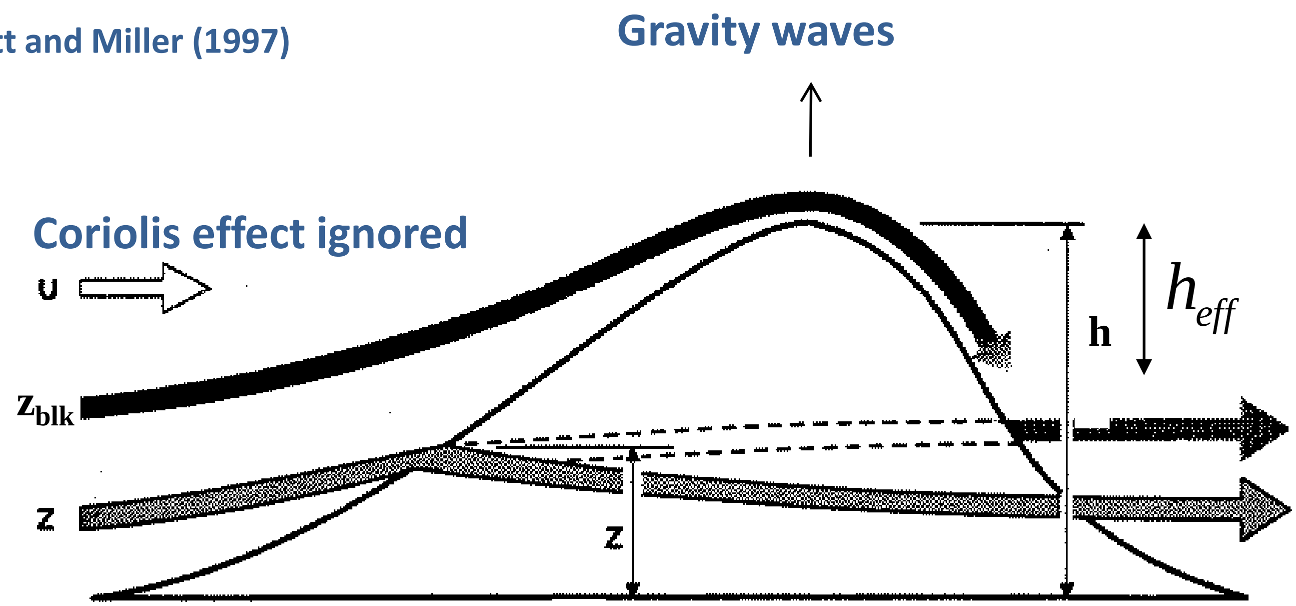
Single lenticular cloud

Figure 4: Single lenticular cloud over Laguna Verde, Bolivia. This cloud was probably formed by a vertically propagating mountain wave. (Copyright Bernhard Mühr, www.wolkenatlas.de)

Durran, 2003

What happens if height is not small compared to horizontal scale?

After Lott and Miller (1997)



- linear/flow-over regime (Nh/U small)
- non-linear/blocked regime (Nh/U large)

Blocking occurs if surface air has less kinetic energy than the potential energy barrier presented by the mountain

$$h_{eff} = H_c U / N$$
$$z_{blk} = h - h_{eff}$$

Height h_{eff} is such that the Froude number Nh_{eff}/U reaches its critical value H_c

See Hunt and Snyder (1980)

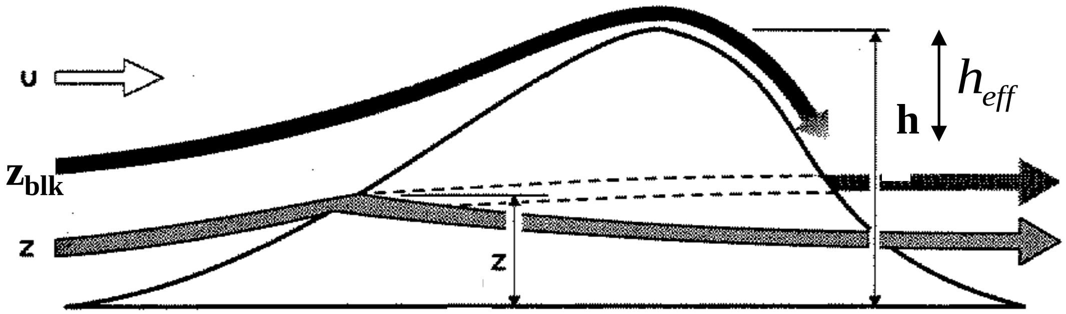
The ECMWF sub-grid orography scheme

- **Horizontal scales smaller than 5 km**: waves are evanescent and flow around steep orographic features will lead to form drag : Turbulent Orographic Form Drag (TOFD, see BL2)

- **Horizontal scales between 5 km and model resolution**: The subgrid orography scheme according to Lott and Miller (1997)

  - Blocking below the blocking height: Strong drag on model levels dependent on geometry of subgrid orography

  - Gravity wave generation by "effective" subgrid mountain height: gravity wave generation dependent on geometry of subgrid orography



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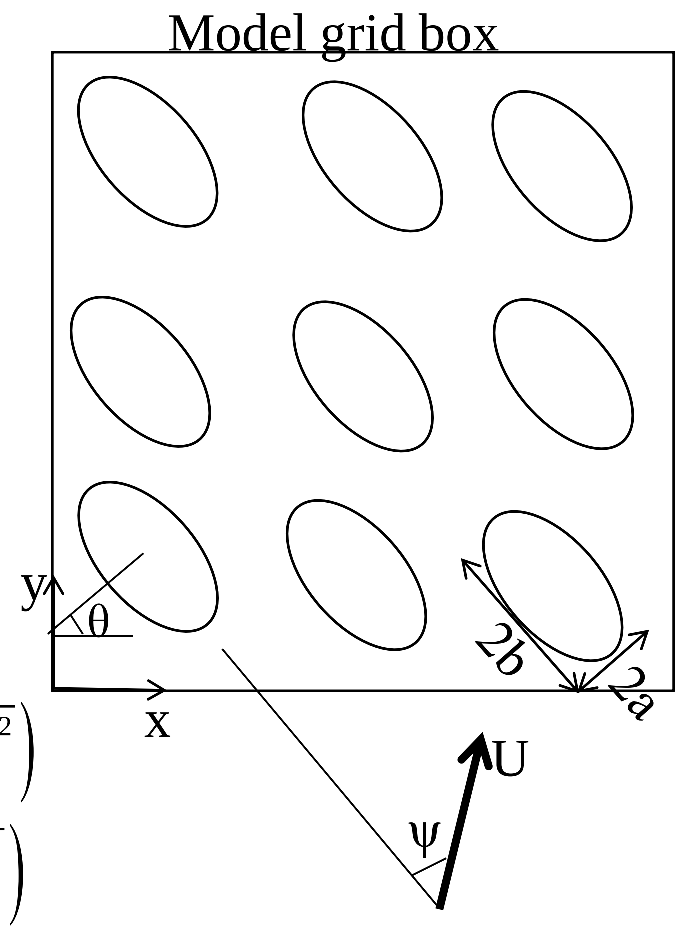
The ECMWF sub-grid orography scheme

- Elliptically shaped mountains are assumed with aspect ratio **a/b**, and orientation **ψ** with respect to the wind

- Elliptic mountains are equally spaced

- Subgrid orography is characterized by:
    - Standard deviation **μ**
    - Slope **σ**
    - Orientation **θ**
    - Anisotropy **γ (1:circular; 0: ridge)**



$$\gamma^2 = \frac{K-(L^2+M^2)^{1/2}}{K+(L^2+M^2)^{1/2}}$$

$$\theta = 0.5\tan^{-1}(M/L)$$

$$\sigma^2 = K+(L^2+M^2)^{1/2}$$

$$\mu^2 = \overline{h^2} - \left(\overline{h}\right)^2$$

$$K = 0.5\left(\overline{(\partial h/\partial x)^2} + \overline{(\partial h/\partial y)^2}\right)$$

$$L = 0.5\left(\overline{(\partial h/\partial x)^2} - \overline{(\partial h/\partial y)^2}\right)$$

$$M = \overline{(\partial h/\partial x)(\partial h/\partial y)}$$

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Preparation of the data sets to characterize the sub-grid orography

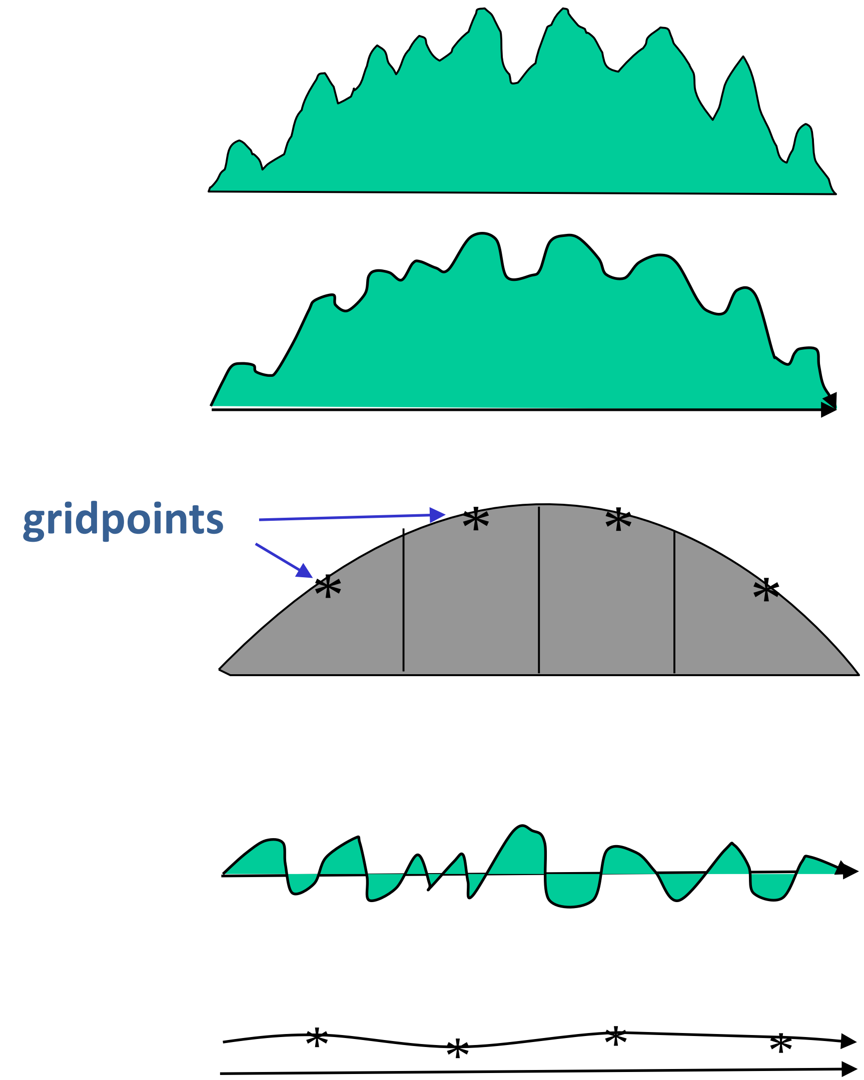
1. Global 1km resolution surface elevation data

2. Reduce to 5 km resolution by smoothing

3. Compute mean orography at model resolution

4. Subtract model orography (3) from 5km orography (2)

5. Compute standard deviation, slope, orientation and anisotropy for every grid box



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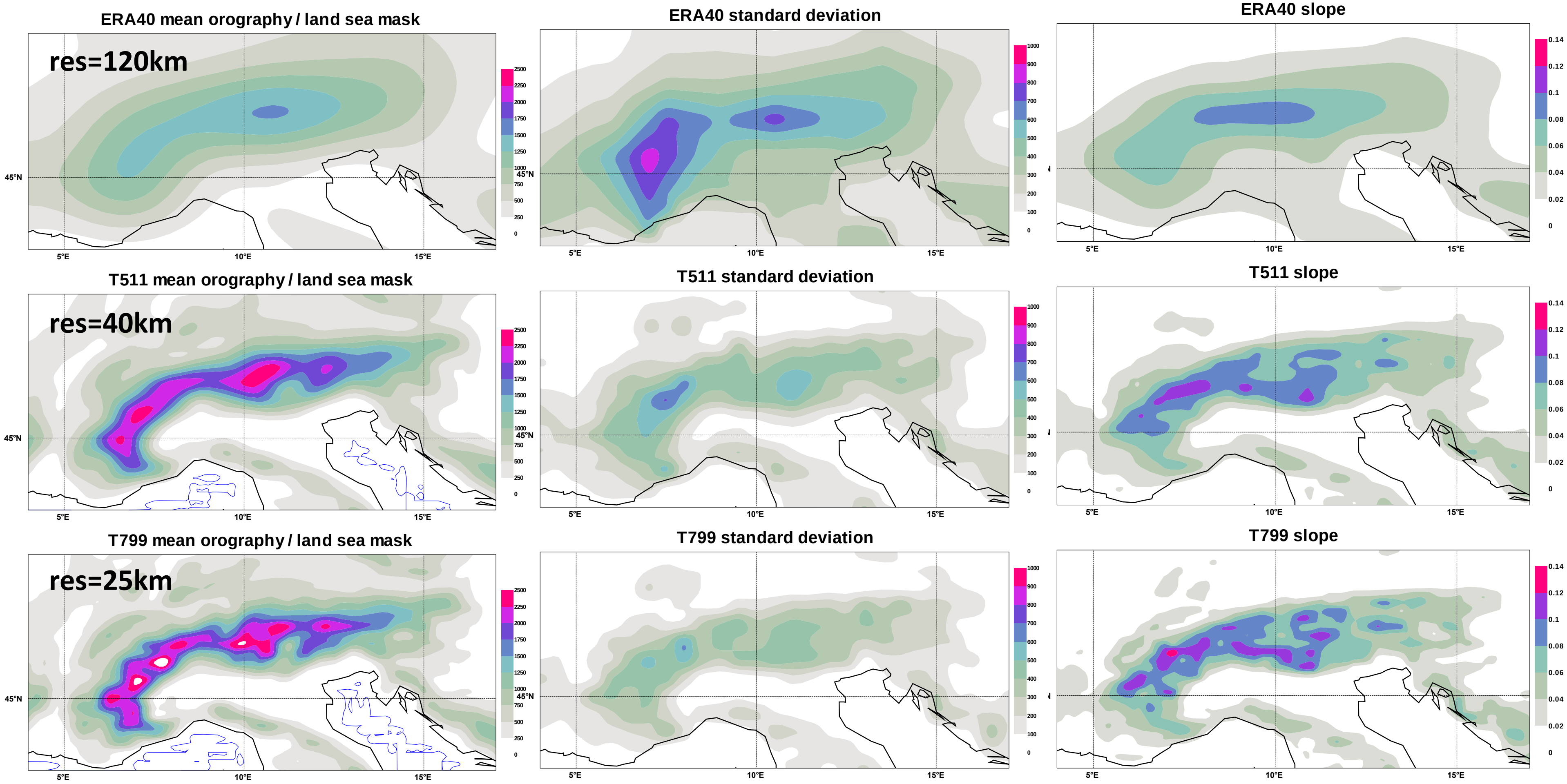
Resolution sensitivity of sub-grid fields

mean orography / land sea mask

standard deviation (μ)

slope (σ)



Horizontal resolutions: ERA40~120km; T511~40km; T799~25km

The surface drag due to blocking and gravity wave generation

Drag at height z below blocking height applied on model levels:

$$D_{blk}(z) = \rho C_d \max\left(2 - \frac{1}{r}, 0\right) \frac{\sigma}{2\mu} \left(\frac{z_{blk} - z}{z + \mu}\right)^{1/2} \left(B\cos^2\psi + C\sin^2\psi\right) \frac{U\,|U\,|}{2}$$

$$\textit{with} \quad r = \frac{\cos^2\psi + \gamma\sin^2\psi}{\gamma\cos^2\psi + \sin^2\psi}$$

Gravity wave stress above blocking height:

$$\tau_{gwd} = \rho_H U_H N_H {h_{eff}}^2 \frac{\sigma}{4\mu} G(B\cos^2\psi_H + C\sin^2\psi_H, (B-C)\sin\psi_H \cos\psi_H)$$

- B,C,G are constants
- Index H indicates the characteristic height (2μ)
- Ψ is computed from θ and wind direction
- Density of ellipses per grid box is characterized by μ/σ

μ : Standard deviation
σ : Slope
θ : Orientation
γ : Anisotropy

Gravity wave dissipation

•Strongest dissipation occurs in regions where the wave becomes unstable and breaks down into turbulence, referred to as wave breaking:

- Convective instability: where the amplitude of the wave becomes so large that it causes relatively cold air to rise over less dense, warm air

$$N_{\min}^2 = N^2\left\{1+\frac{N\,\delta h}{U}\right\}$$

δ : amplitude of wave

N : mean Brunt-Vaisala frequency

- Kelvin-Helmholtz instability also important: associated with shear zones. Amplitude of wave is reduced such that $\mathbf{Ri_{min}}$ reaches critical value of 0.25 (saturation hypothesis; Lindzen 1981)

$$Ri_{\min} = \frac{N^2}{\eta^2} = Ri\left\{\frac{1-\alpha}{\left(1+Ri^{1/2}\alpha^2\right)^2}\right\}$$

$$\alpha = N\left|\delta h\right|/U$$

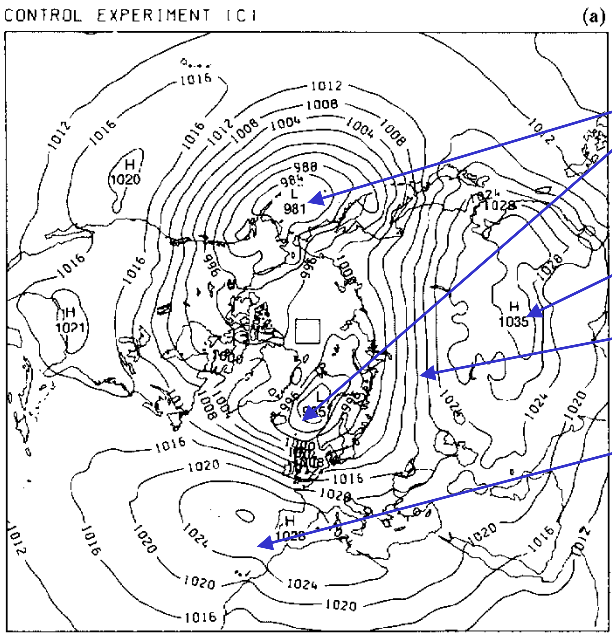
$$\eta = \partial U / \partial z$$

δh : amplitude of wave

Ri : mean Richardson number

Impact of scheme

Alleviation of systematic westerly bias in low resolution model (2.5°x3.75°) in 1985

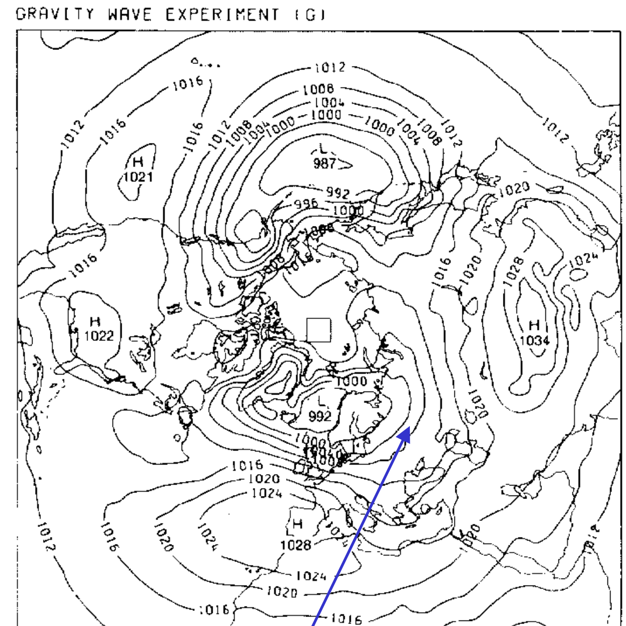


Icelandic/Aleutian lows are too deep

Siberian high too weak and too far south

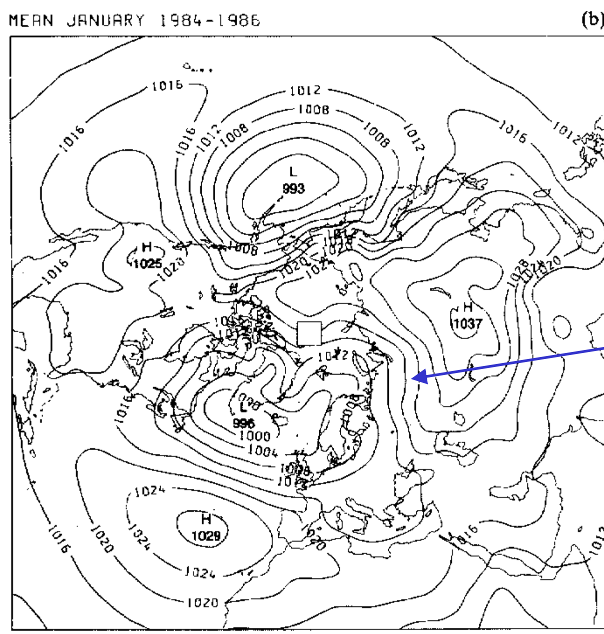
Flow too zonal / westerly bias

Azores anticyclone too far east



Without GWD scheme

With GWD scheme



Mean January sea level pressure (mb) for years 1984 to 1986

alleviation of westerly bias

Analysis (best guess)

better agreement

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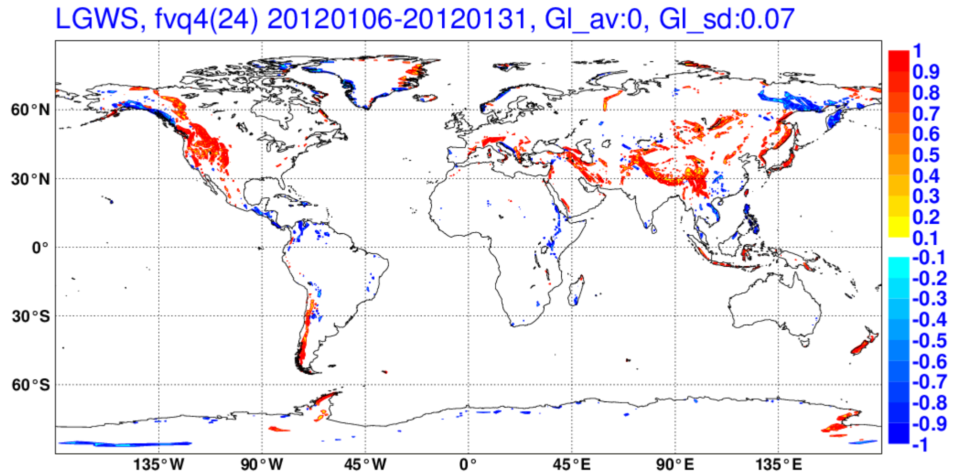
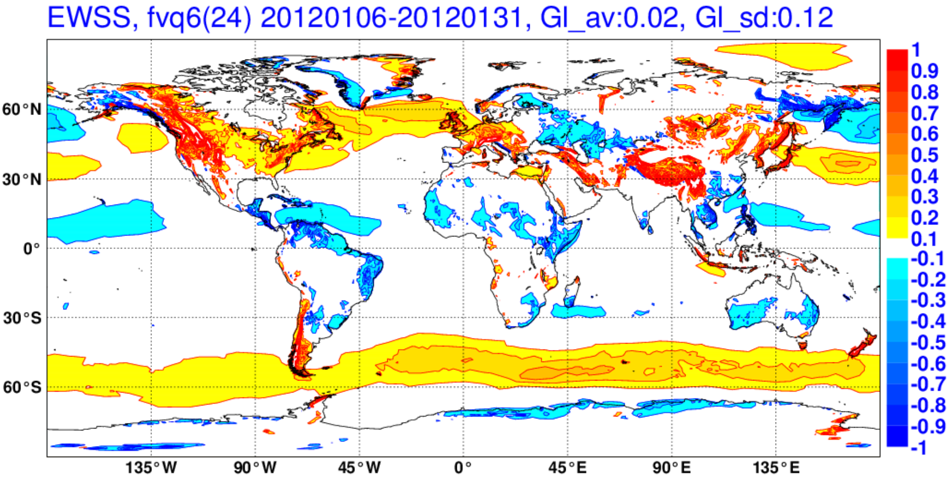
From Palmer et al. 1986

Surface stresses averaged over of 26 days (T511L91); Jan 2012

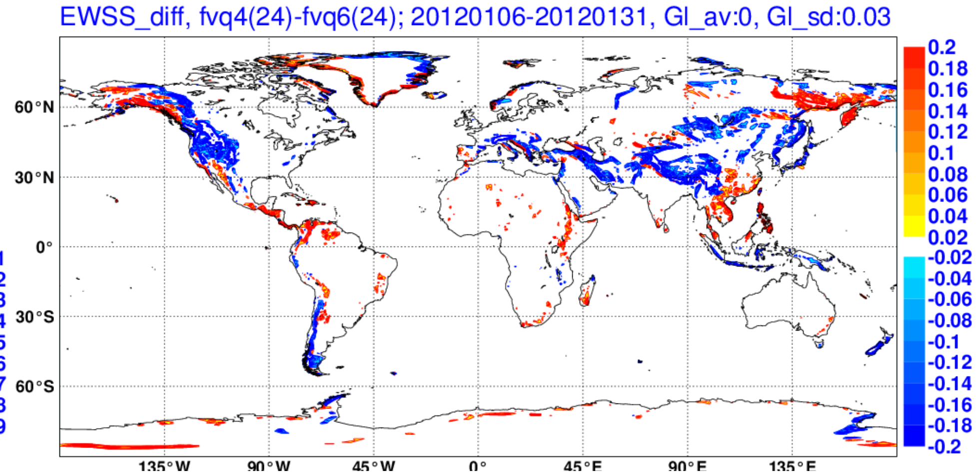
East/West SO stress



East/West turbulent stress

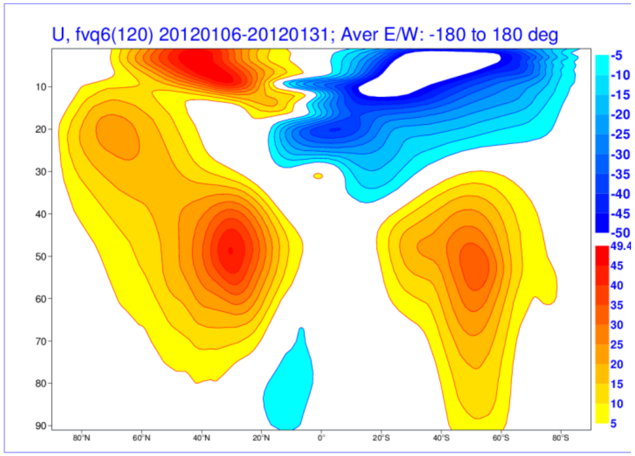


East/West turbulent stress difference (SO – no SO)

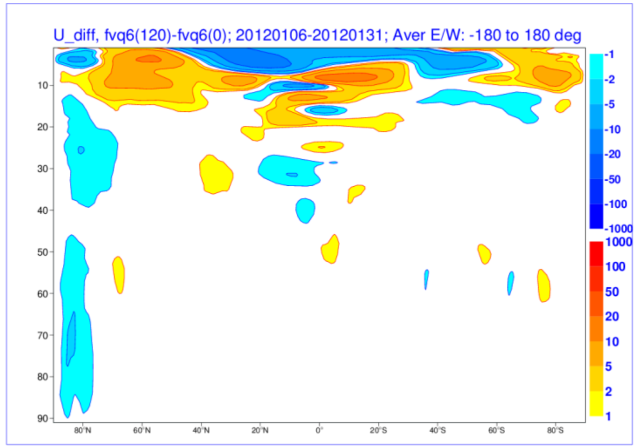


North/South cross section 90N to 90S (averaged over 180W to 180E) averaged of 26 5-day forecasts

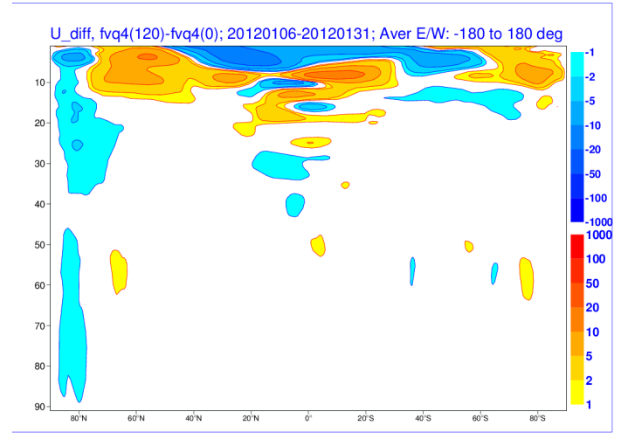
U



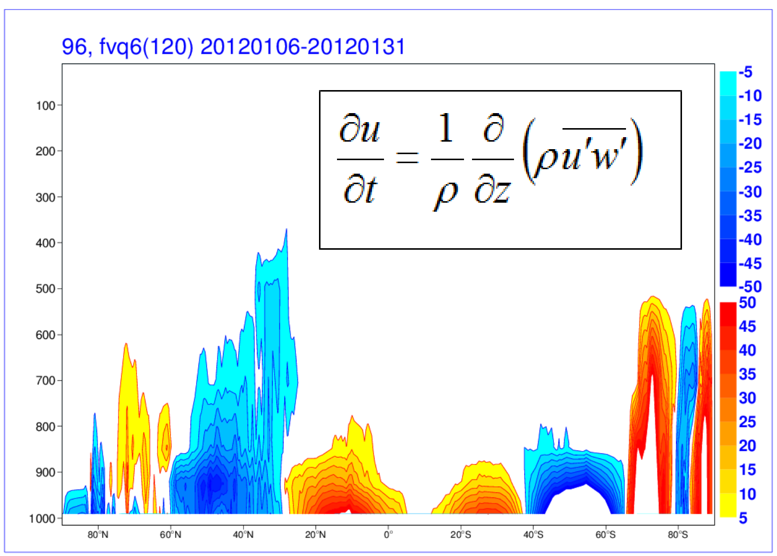
Day-5 U-err without SO



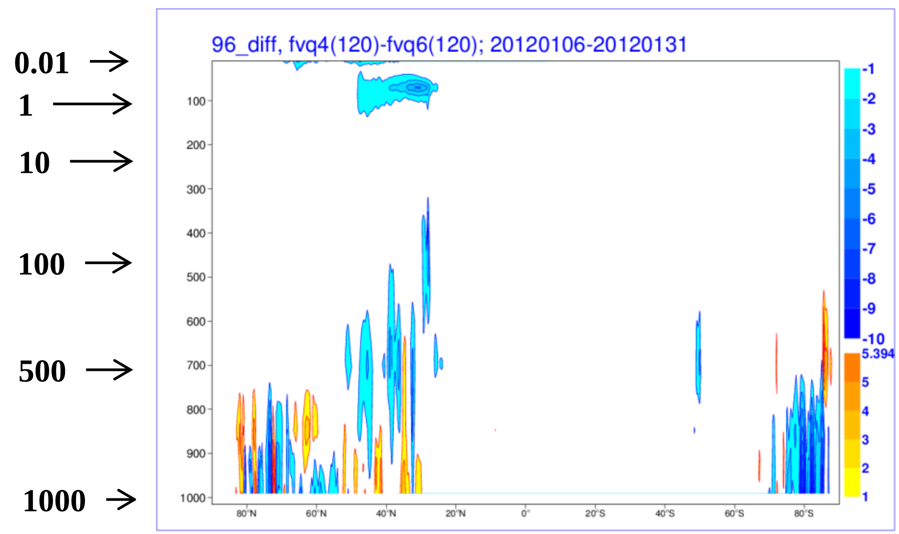
Day-5 U-err with SO



U-tendency from Turb (m/s/5-days)



U-tend difference: Turb&SO - Turb



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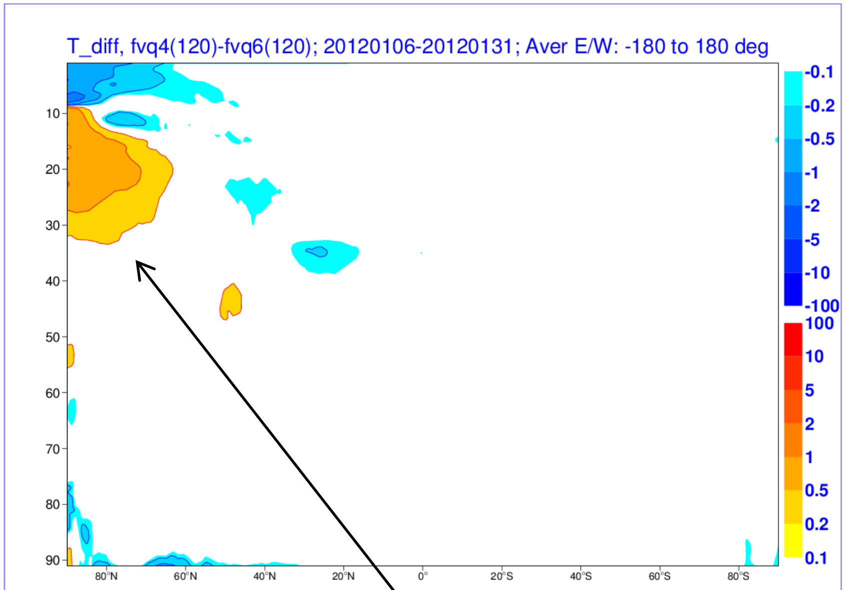
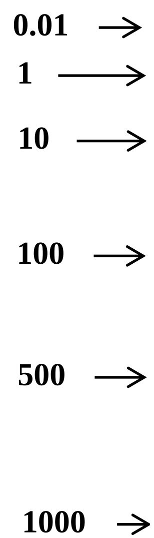
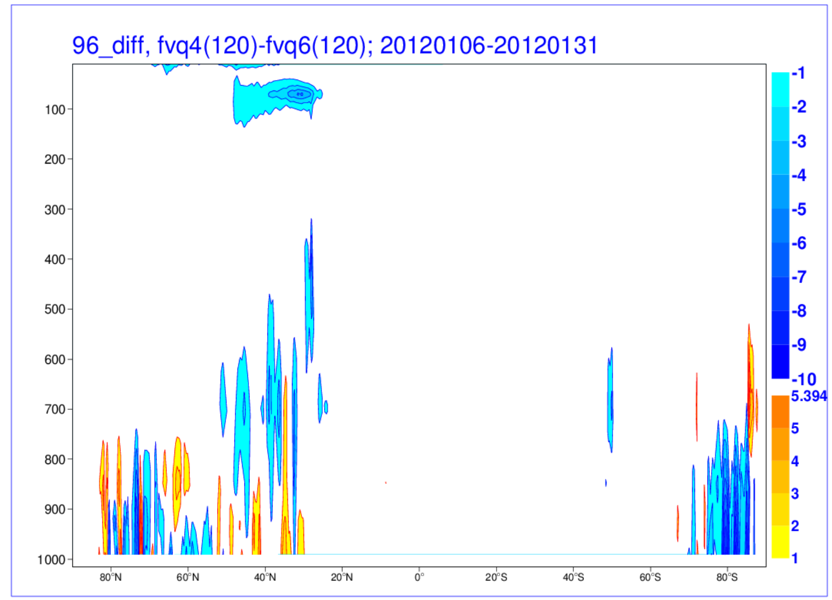
North/South cross section 90N to 90S (averaged over 180W to 180E) averaged of 26 5-day forecasts

Day-5 T-difference: Turb&SO - Turb





U-tend difference: Turb&SO - Turb



Gravity wave stress in the stratosphere causes a-geostrophic meridional circulation which results in warming in polar stratosphere

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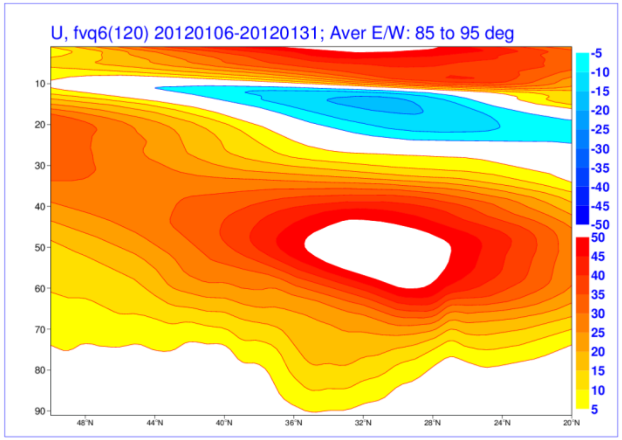
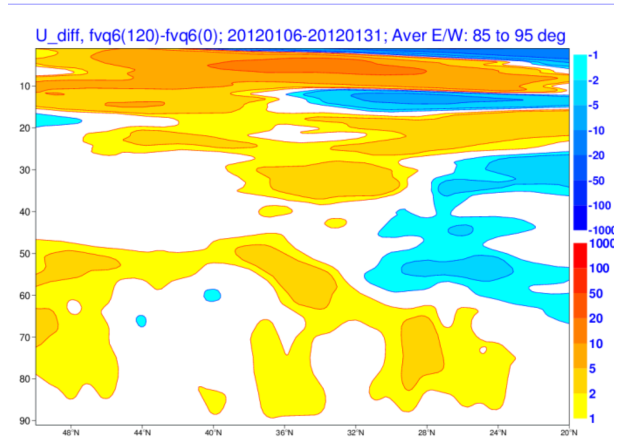
North/South cross section 50N to 20N (averaged over 85E to 95E) averaged of 26 5-day forecasts

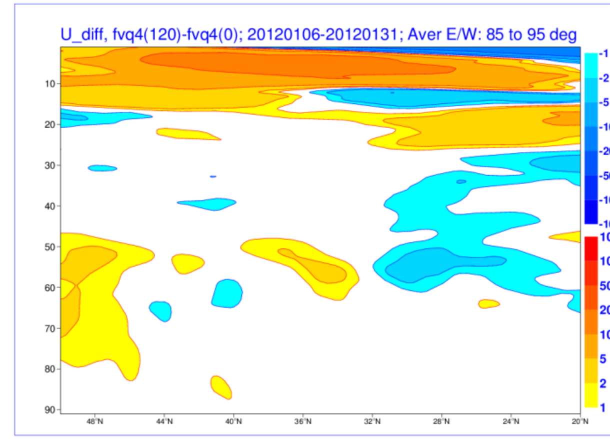
U



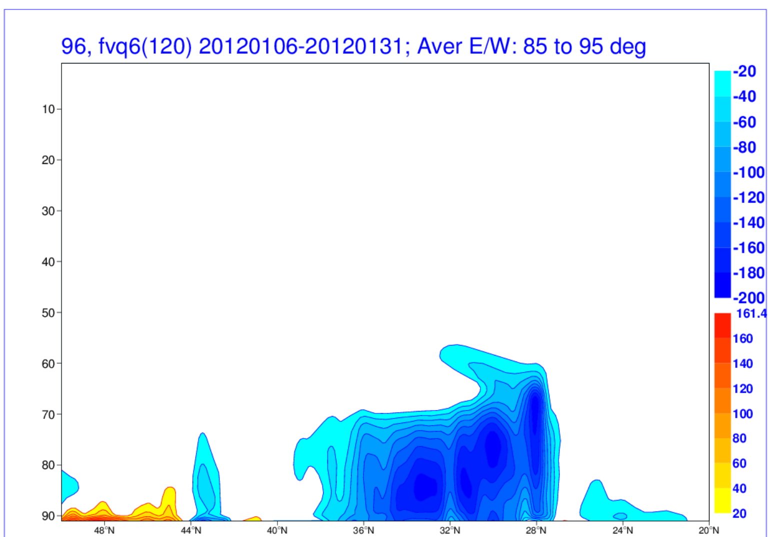
Day-5 U-err without SO



Day-5 U-err with SO



U-tendency from Turb (m/s/5-days)



0.01 →

1 →

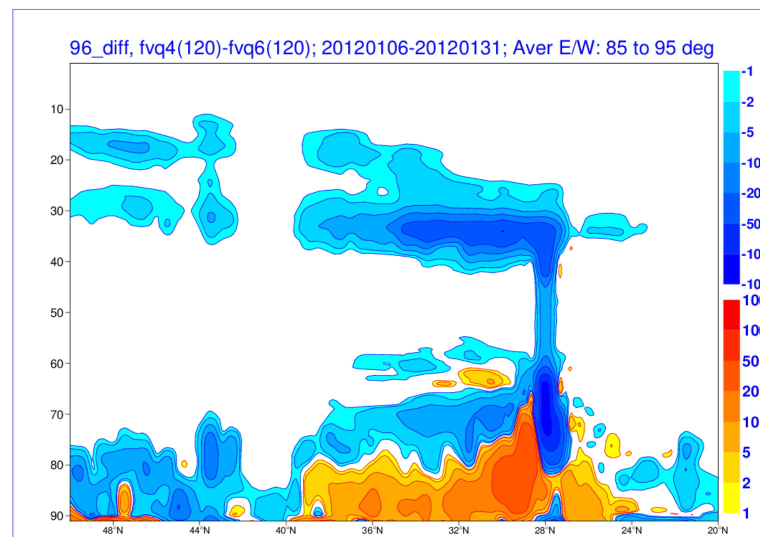
10 →

100 →

500 →

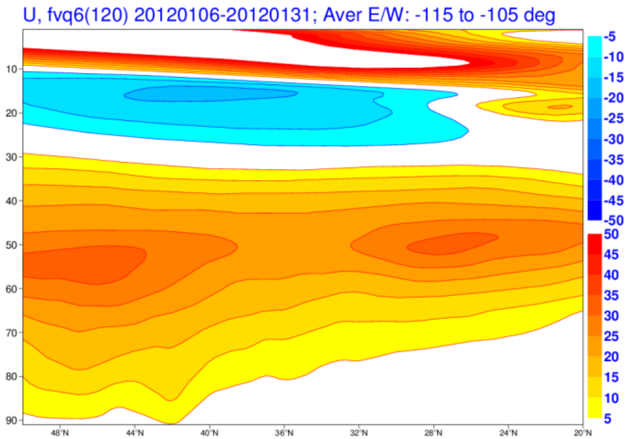
1000 →

U-tend difference: Turb&SO - Turb

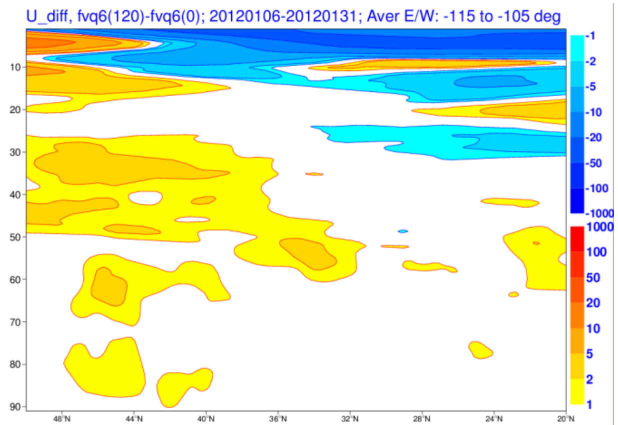


North/South cross section 50N to 20N (averaged over 105W to 115W) averaged of 26 5-day forecasts

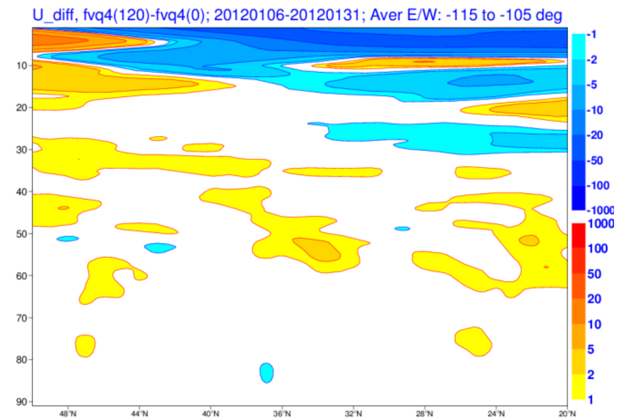
U



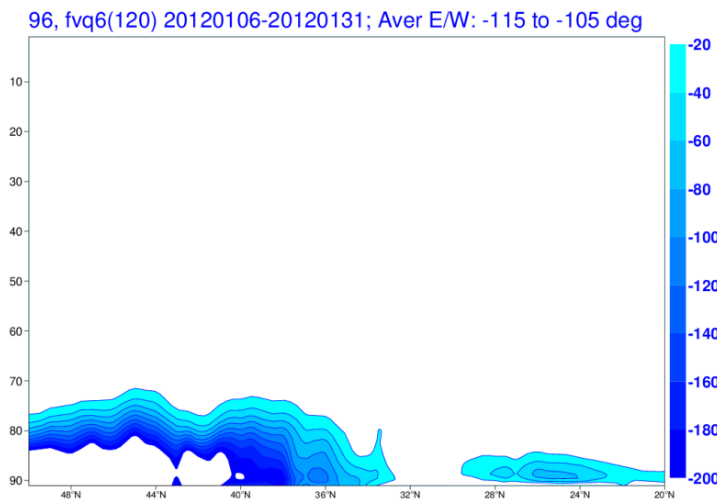
Day-5 U-err without SO



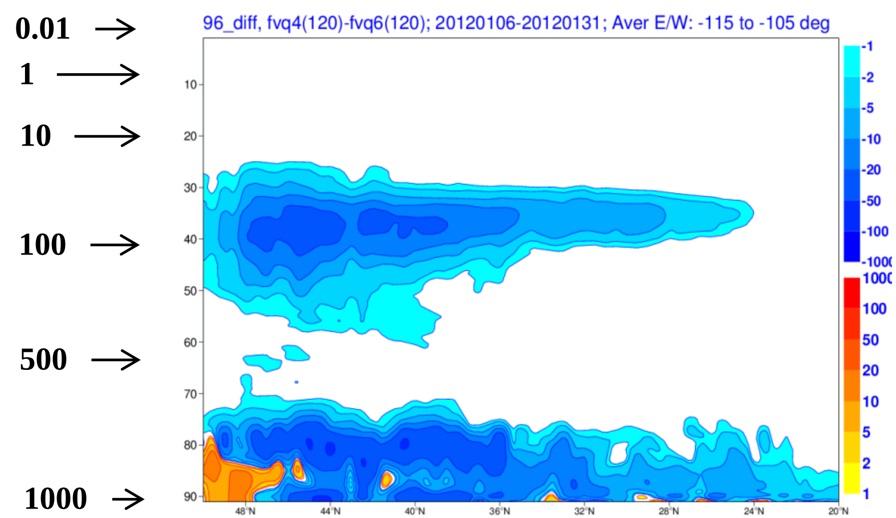
Day-5 U-err with SO



U-tendency from Turb (m/s/5-days)



U-tend difference: Turb&SO - Turb



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