



# Parametrization of PBL outer layer

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Irina Sandu

- Overview of models
- Bulk models
- Local K-closure
- ED/MF closure
- K-profile closure
- TKE closure
- Current closure in the ECMWF model



# Reynolds equations

$$\begin{aligned} \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} - f \bar{v} &= -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x} - \frac{\overline{\partial u' w'}}{\partial z} \\ \frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} + \bar{w} \frac{\partial \bar{v}}{\partial z} - f \bar{u} &= -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial y} - \frac{\overline{\partial v' w'}}{\partial z} \\ \frac{\partial \bar{q}}{\partial t} + \bar{u} \frac{\partial \bar{q}}{\partial x} + \bar{v} \frac{\partial \bar{q}}{\partial y} + \bar{w} \frac{\partial \bar{q}}{\partial z} &= -\frac{S_{qt}}{\rho} - \frac{\overline{\partial q' w'}}{\partial z} \\ \frac{\partial \bar{\theta}}{\partial t} + \bar{u} \frac{\partial \bar{\theta}}{\partial x} + \bar{v} \frac{\partial \bar{\theta}}{\partial y} + \bar{w} \frac{\partial \bar{\theta}}{\partial z} &= -\frac{1}{\rho c_p} \frac{\partial F}{\partial z} - \frac{L_v}{\rho c_p} - \frac{\overline{\partial \theta' w'}}{\partial z} \end{aligned}$$

$$u = \bar{u} + u'$$

  
Reynolds Terms

# Parametrization of PBL outer layer (overview)



<u>Parametrization</u>	<u>Application</u>	<u>Order and Type of Closure</u>	
Bulk models	Models that treat PBL top as surface	0 <sup>th</sup> order	non-local
K-diffusion	Models with fair resolution	1 <sup>st</sup> order	local
Mass-flux	Models with fair resolution	1 <sup>st</sup> order	non-local
ED/MF (K& M)	Models with fair resolution	1 <sup>st</sup> order	non-local
K-profile	Models with fair resolution	1 <sup>st</sup> order	non-local
TKE-closure	Models with high resolution	1.5 <sup>th</sup> order local	non-local
Higher order closure	Models with high resolution	2 <sup>nd</sup> or 3 <sup>rd</sup> order local	non-local



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# Local K closure

K-diffusion in analogy with molecular diffusion, but

$$\overline{u'w'} = -K_M \frac{\partial \bar{u}}{\partial z}, \quad \overline{v'w'} = -K_M \frac{\partial \bar{v}}{\partial z}$$

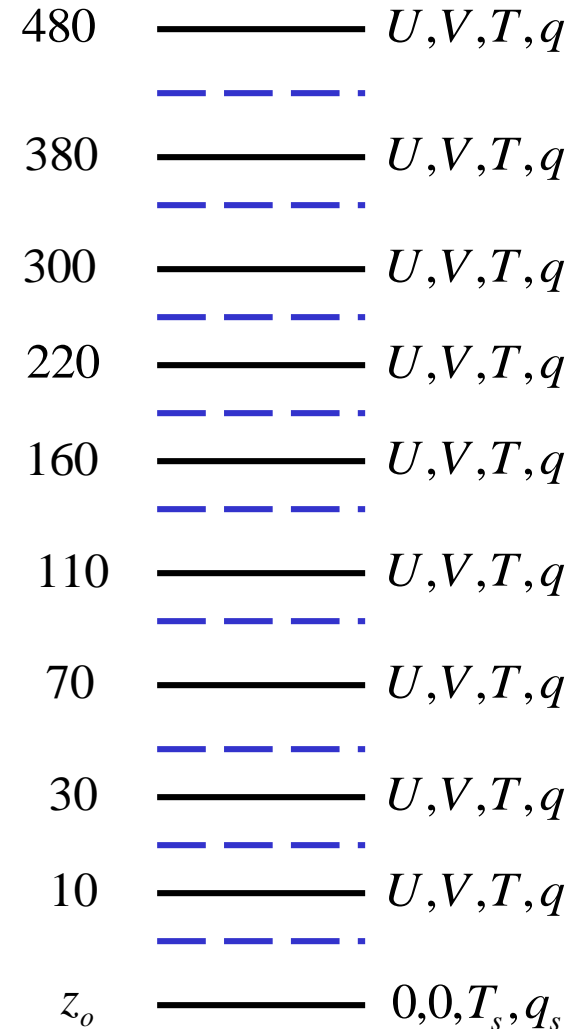
$$\overline{\theta'w'} = -K_H \frac{\partial \bar{\theta}}{\partial z}, \quad \overline{q'w'} = -K_H \frac{\partial \bar{q}}{\partial z}$$

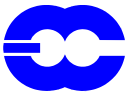
$$\frac{\partial \overline{\phi'w'}}{\partial z} \approx \frac{\partial}{\partial z} \left( -K \frac{\partial \bar{\phi}}{\partial z} \right) \approx -K \frac{\partial^2 \bar{\phi}}{\partial z^2}$$

Diffusion coefficients need to be specified as a function of flow characteristics (e.g. shear, stability, length scales).

## Levels in ECMWF model

91-level model





# Diffusion coefficients according to MO-similarity

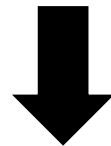
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$$K_M = \frac{\ell^2}{\phi_m^2} \left| \frac{dU}{dz} \right|, \quad K_H = \frac{\ell^2}{\phi_m \phi_h} \left| \frac{dU}{dz} \right|,$$

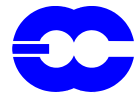
Use relation between  $Ri$  and  $z/L$

$$Ri = \frac{g}{\theta_v} \frac{d\theta_v / dz}{|dU / dz|^2} = \frac{g}{\theta_v} \frac{z \theta_* \phi_h}{u_*^2 \phi_m^2} = \frac{z}{\kappa L} \frac{\phi_h}{\phi_m^2}$$

to solve for  $z/L$ .



$$K_M = \ell^2 \left| \frac{dU}{dz} \right| f_M(R_i), \quad K_H = \ell^2 \left| \frac{dU}{dz} \right| f_H(R_i)$$



# Stable boundary layer in the IFS: closure and caveats

$$K = \left| \frac{\partial U}{\partial z} \right| l^2 f(Ri)$$

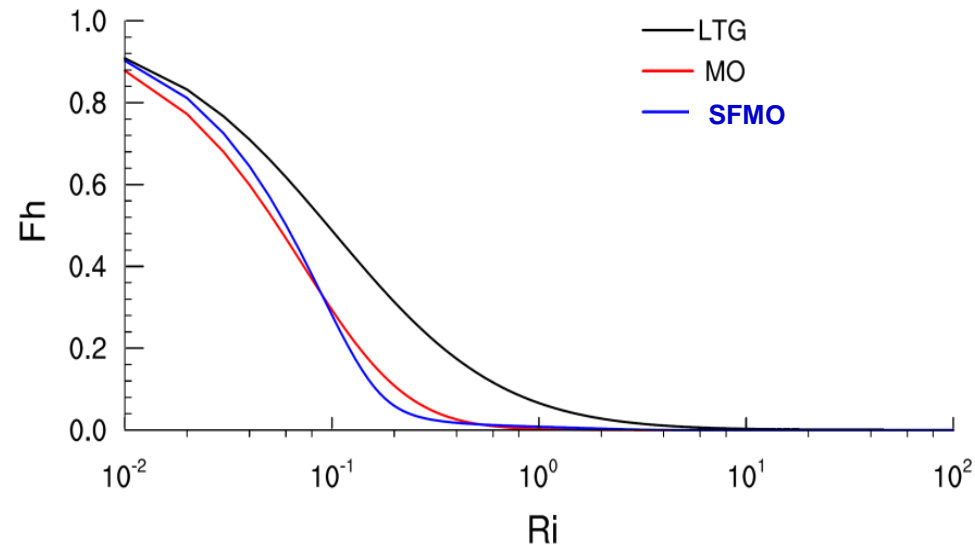
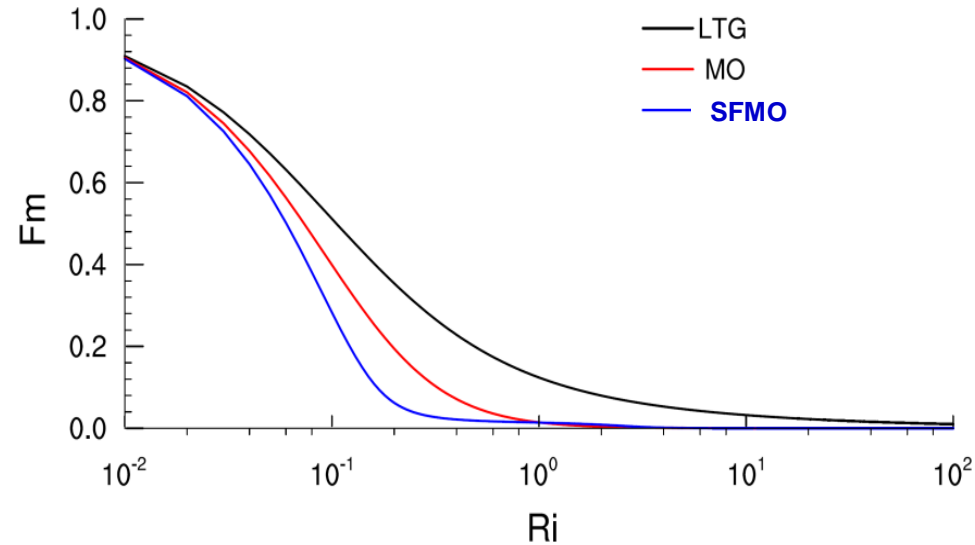
$$1/l = 1/kz + 1/\lambda, \lambda = 150\text{m}$$

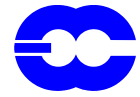
Recent years (36R4 – 38R2)

Surface layer – SFMO

$$\text{Above: } f = \alpha * f_{\text{LTG}} + (1 - \alpha) * f_{\text{MO}}$$
$$\alpha = \exp(-H/150)$$

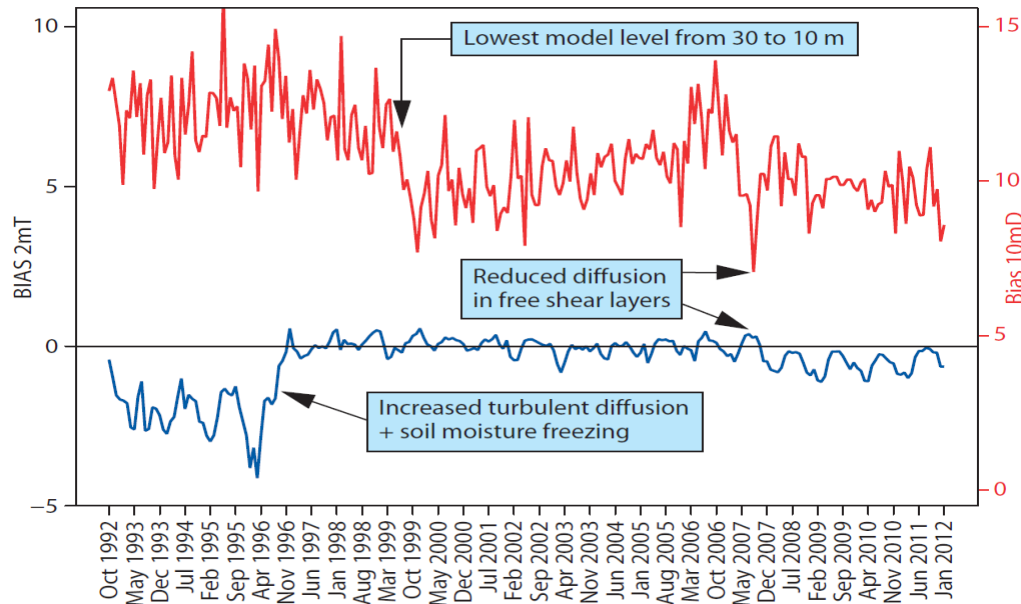
As in other NWP models the diffusion maintained in stable conditions is stronger than what LES or observations indicate





# Stable boundary layer in the IFS: closure and caveats

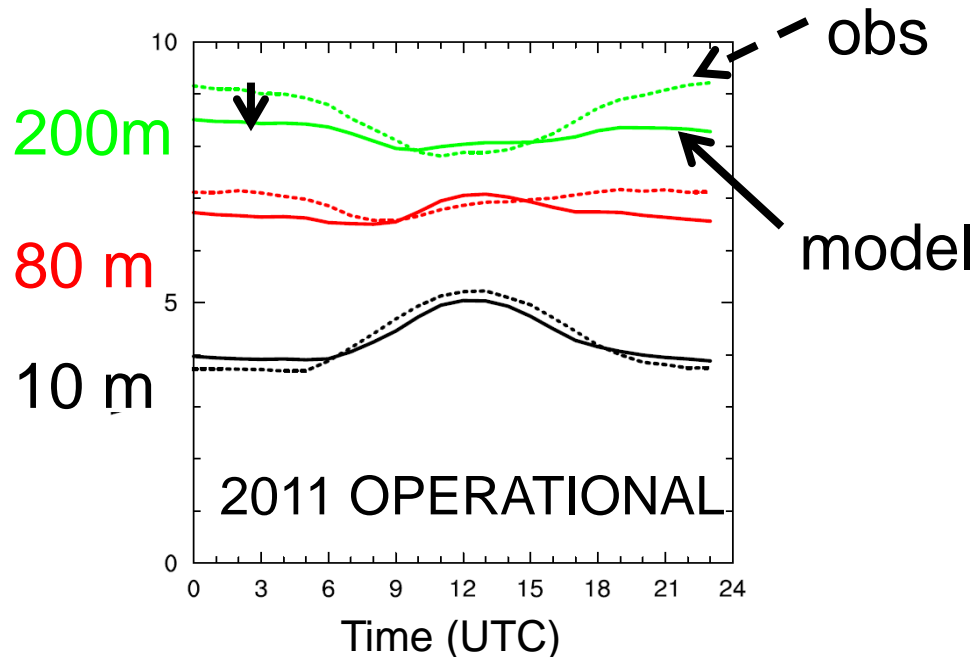
Mean nocturnal bias over Europe



Wind turning is underestimated

2m T is too low despite too strong diffusion

Mean annual wind speed at Cabaw





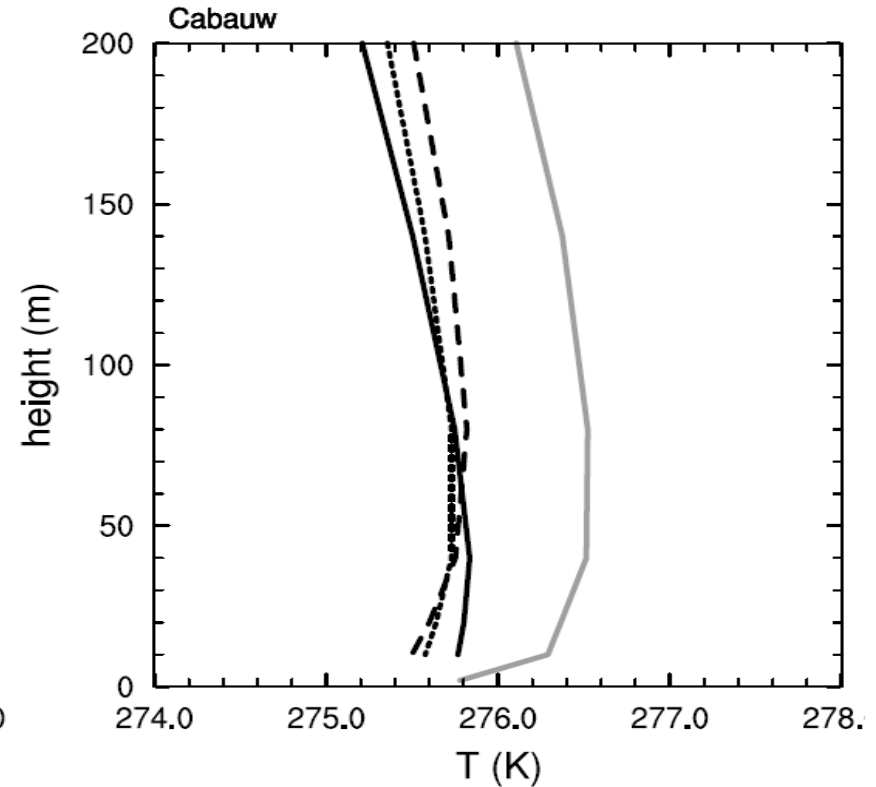
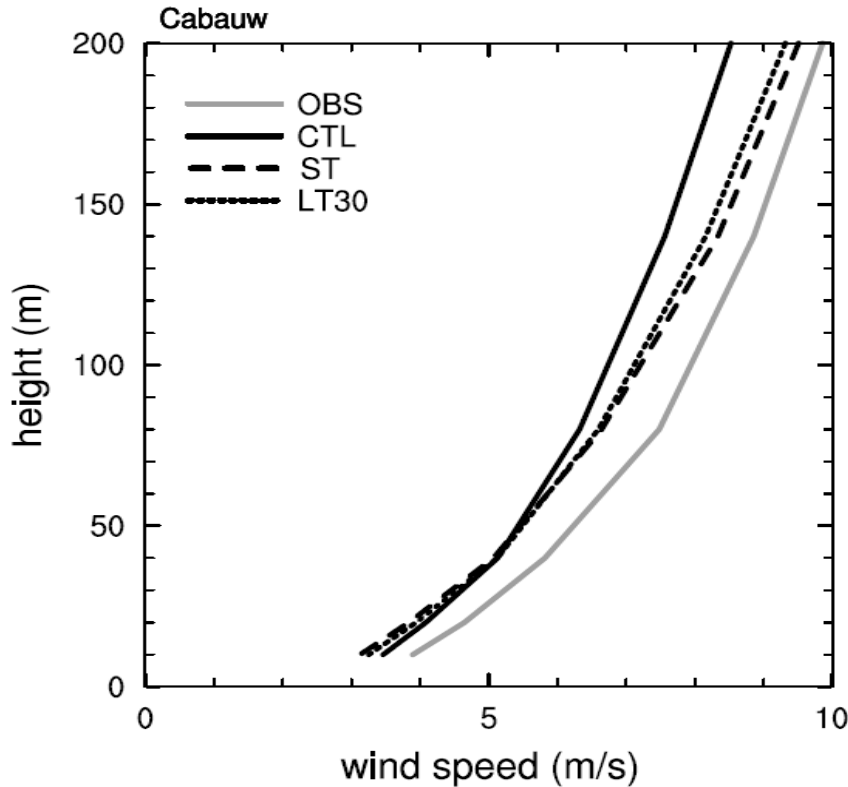


# Impact of reducing the diffusion in stable conditions

**ST:** long tails  $\rightarrow$  short tails  
**LT30:**  $\lambda=150\text{m}$   $\rightarrow$   $\lambda=30\text{m}$

$$K = \left| \frac{\partial U}{\partial z} \right| l^2 f(Ri)$$

$$1/l = 1/kz + 1/\lambda, \lambda = 150\text{m}$$

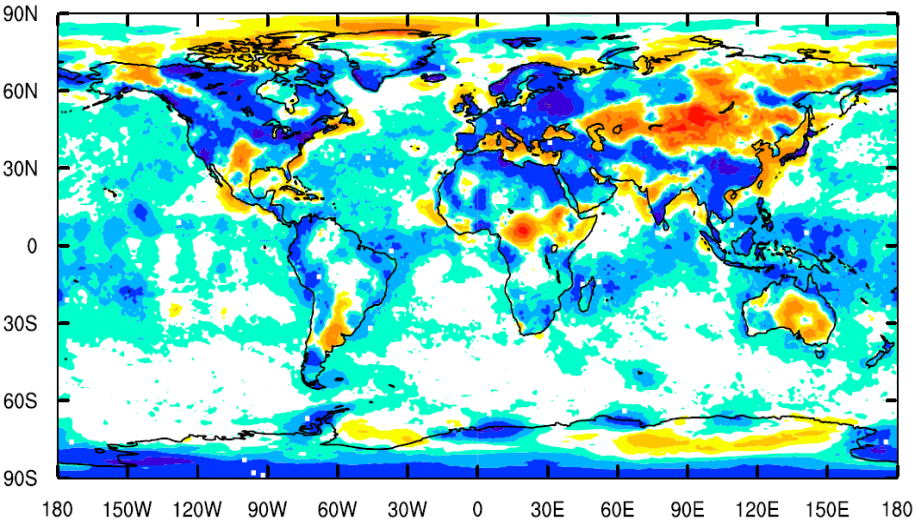


Almost halves the errors in low level jet, also increases the wind turning

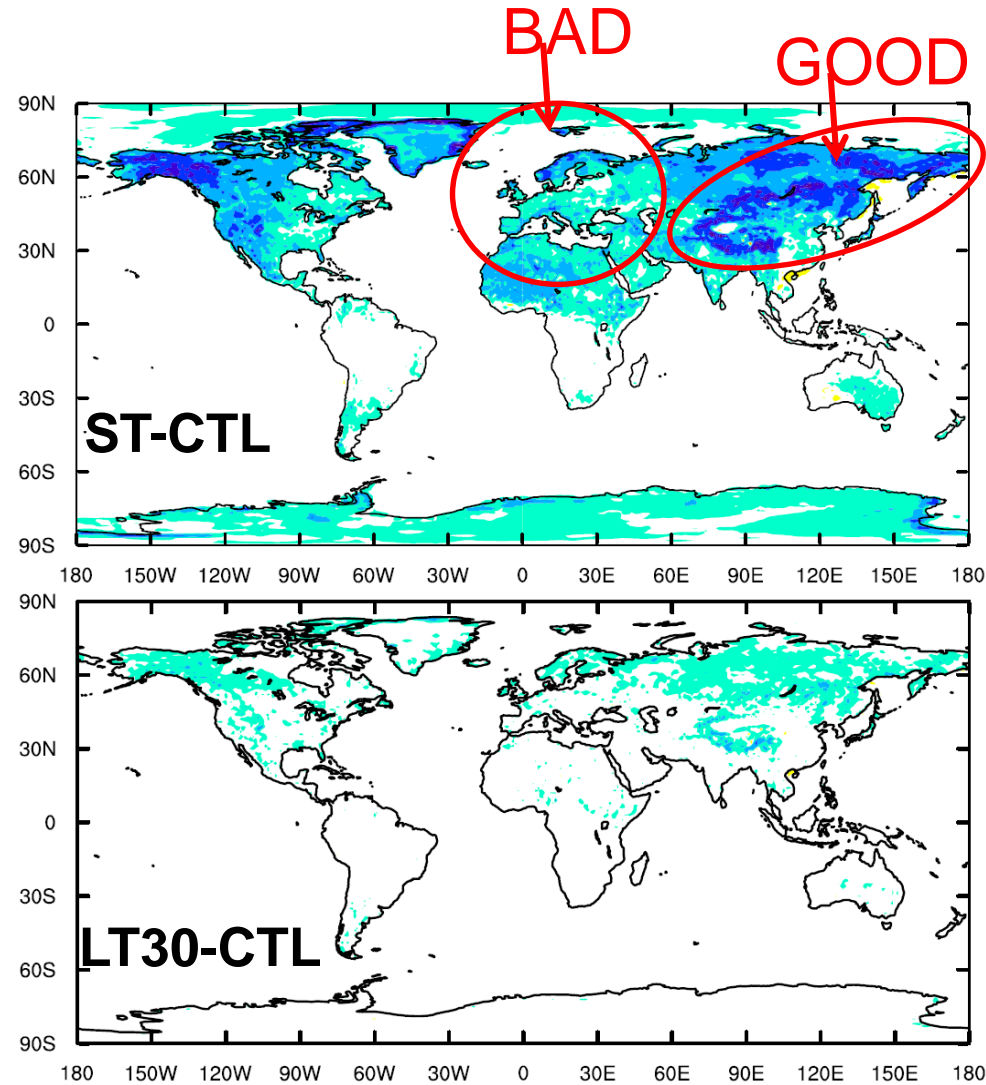


# Impact of reducing the diffusion in stable conditions

## Bias (FC-AN) T2m CTL



**ST:** long tails  $\rightarrow$  short tails  
**LT30:**  $\lambda=150m$   $\rightarrow$   $\lambda=30m$





# Stable boundary layer : changes to closure in 40R1 (Nov. 2013)

Turbulence closure for stable conditions:  $K_{M,H} = \left| \frac{\partial U}{\partial Z} \right| l^2 f_{M,H}(R_i), \quad \frac{1}{l} = \frac{1}{kz} + \frac{1}{\lambda}$

## Up to 38R2

- long tails near surface, short tails above PBL
- $\lambda = 150\text{m}$
- non-resolved shear term, with a maximum at 850hPa



## From 40R1

- long tails everywhere
- $\lambda = 10\%$  PBL height in stable boundary layers
- $\lambda = 30\text{ m}$  in free shear layers



Increase in drag over orography  
Increase in atm/surf coupling

Consequence: net reduction in diffusion in stable boundary layers, not much change in free-shear layers, except at 850 hPa



## Stable boundary layer : changes to closure in 40R1 (Nov. 2013)

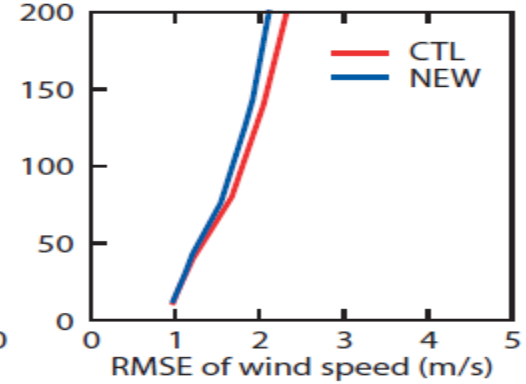
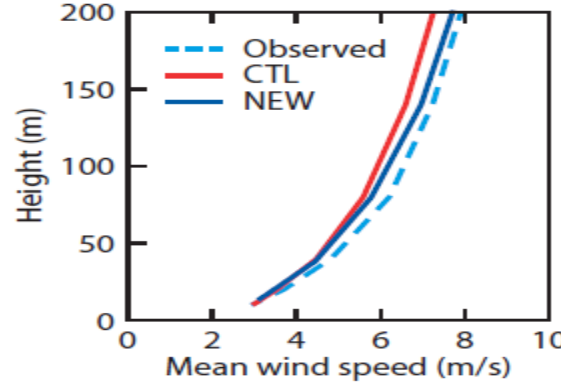
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- small changes in 2m temperature during night time in winter ( $\sim 0.1$  K over Europe)
- Reduction of wind direction bias over Europe by  $3^\circ$  in winter,  $1^\circ$  in summer (out of  $10^\circ$ )
- Improvement in low level jets (next slide)
- Improvement of the large-scale performance of the model in winter N.Hemisphere
- Deterioration of tropical wind scores (against own analysis, not against observations)

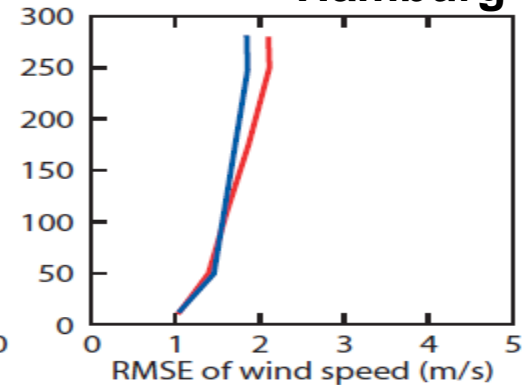
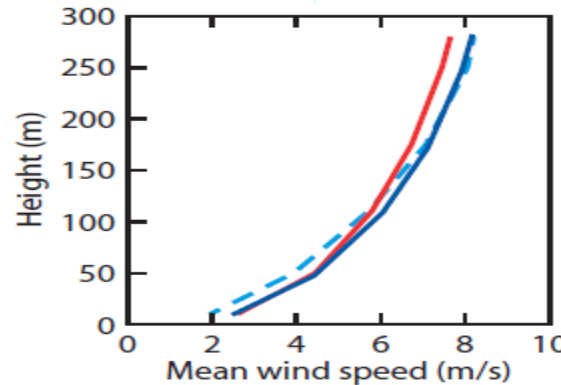


# Improvement of low level winds

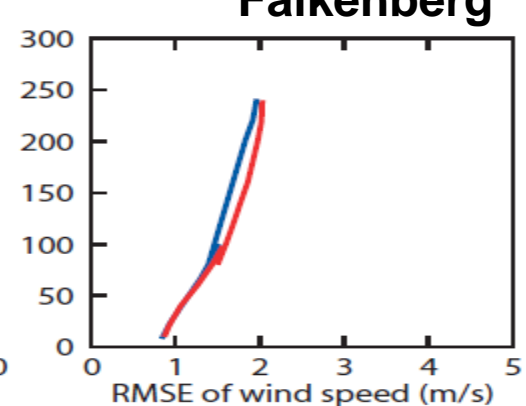
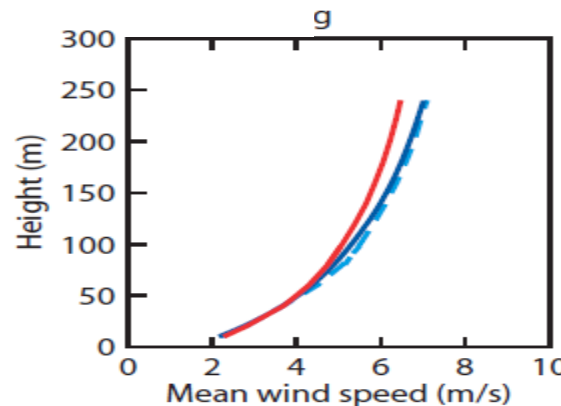
## Cabauw



## Hamburg



## Falkenberg



Comparison with tower data  
T511L137 analysis runs  
JJA 2012, 0 UTC, step 24h

Improvement in both mean  
and RMSE in the upper part  
of stable boundary layers

# K-closure with local stability dependence (summary)

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- ☞ Scheme is simple and easy to implement.
- ☞ Fully consistent with local scaling for stable boundary layer.
- ☞ A sufficient number of levels is needed to resolve the BL i.e. to locate inversion.
- ☞ Entrainment at the top of the boundary layer is not represented

$$K = \left| \frac{\partial U}{\partial z} \right| \cdot l^2 \cdot f(Ri)$$



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# K-diffusion versus Mass flux method

K-diffusion method - used to describe the small-scale turbulent motions:

$$\overline{\phi' w'} \approx -K \frac{\partial \bar{\phi}}{\partial z}$$

$$\frac{\partial \overline{\phi' w'}}{\partial z} \approx \frac{\partial}{\partial z} \left( -K \frac{\partial \bar{\phi}}{\partial z} \right) \approx -K \frac{\partial^2 \bar{\phi}}{\partial z^2} \quad \text{analogy to molecular diffusion}$$

Mass-flux method – used to describe the strong large-scale updraughts:

$$\overline{\phi' w'} \approx M (\phi^{up} - \bar{\phi}) \quad \text{mass flux}$$

$$\frac{\partial}{\partial z} \phi^{up} = -\varepsilon (\phi^{up} - \bar{\phi}) \quad \text{entraining plume model}$$

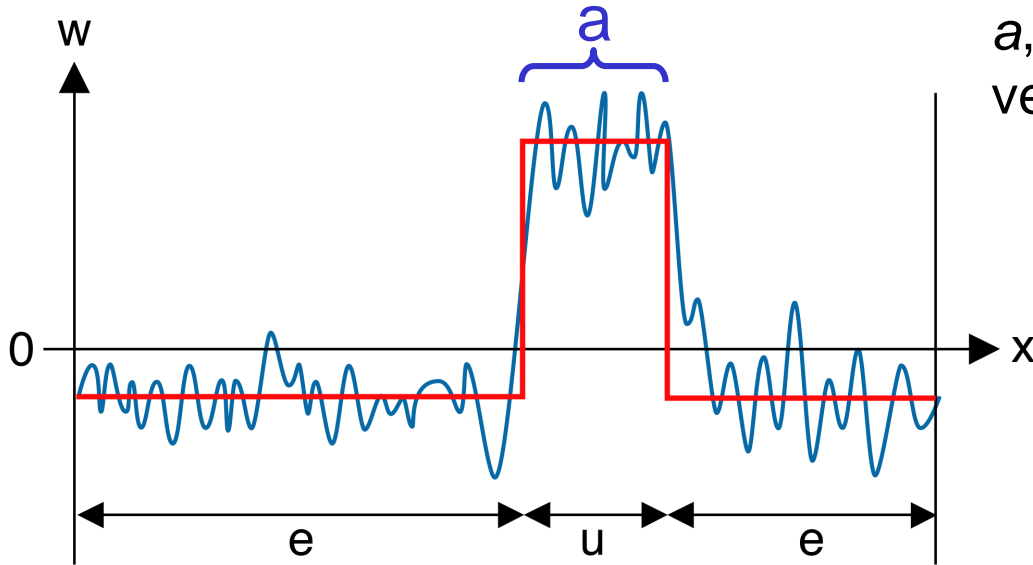
$$\frac{\partial M}{\partial z} = (\varepsilon - \delta) M \quad \text{detrainment rate}$$





# ED/MF framework

The updraught: small fractional area  $a$ , containing the strongest upward vertical motions



$$\phi_u = \phi'_u + \overline{\phi}_u^u$$

$$\phi_e = \phi'_e + \overline{\phi}_e^e$$

$$\overline{\phi} = a\overline{\phi}_u^u + (1-a)\overline{\phi}_e^e$$

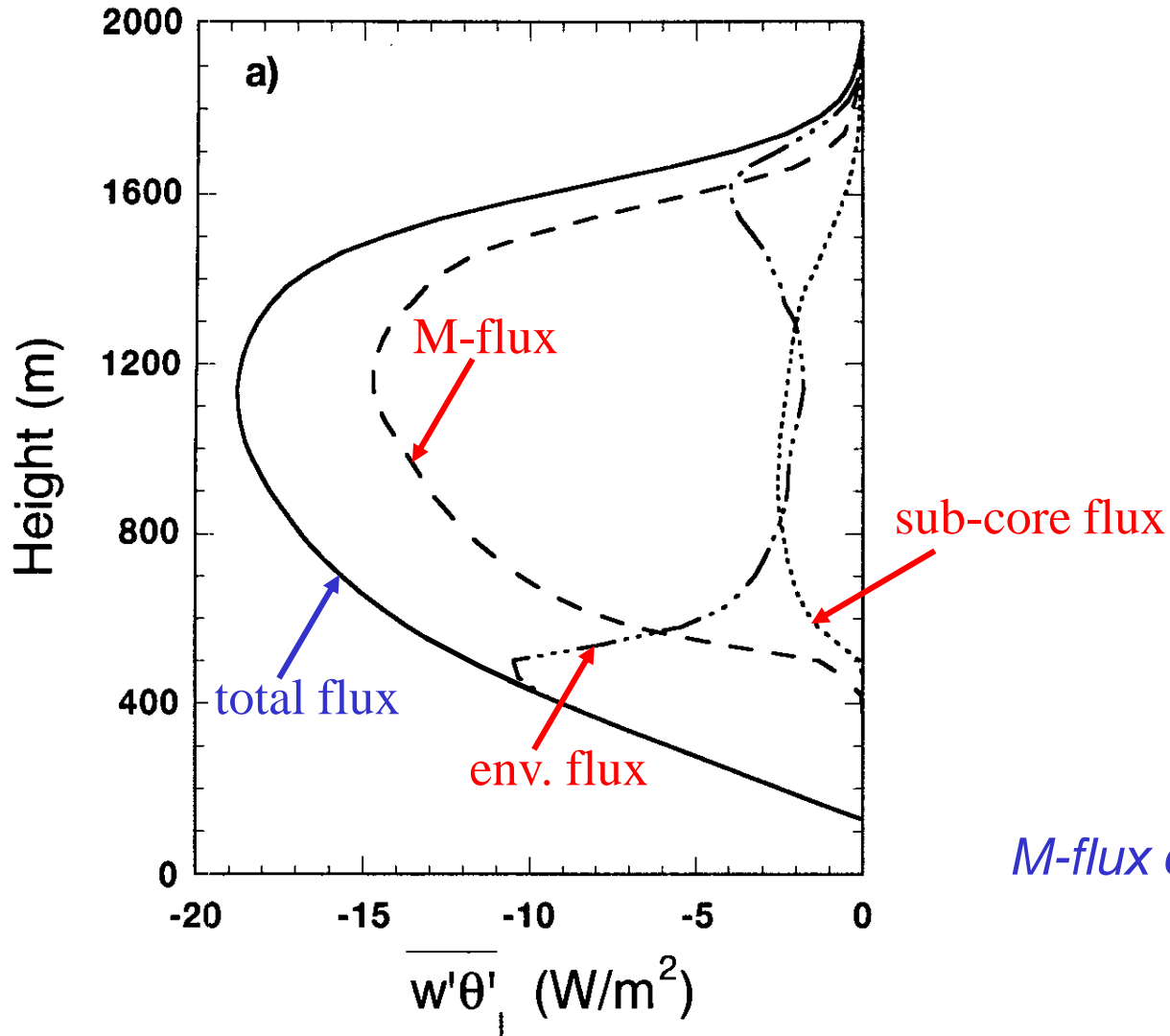
$$a \ll 1$$

$$\overline{w' \phi'} = \underbrace{\overline{w' \phi'_u}}_{\text{sub-core flux (neglected)}} + \underbrace{(1-a)\overline{w' \phi'_e}}_{\text{env. flux}} + \underbrace{\frac{M}{\rho} (\phi_u - \overline{\phi})}_{\text{M-flux}}, \quad M = \rho a w_u$$

$-K \frac{\partial \overline{\phi}}{\partial z}$



# BOMEX LES decomposition



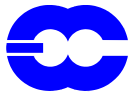
*M-flux covers 80% of flux*



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# K-profile closure Troen and Mahrt (1986)

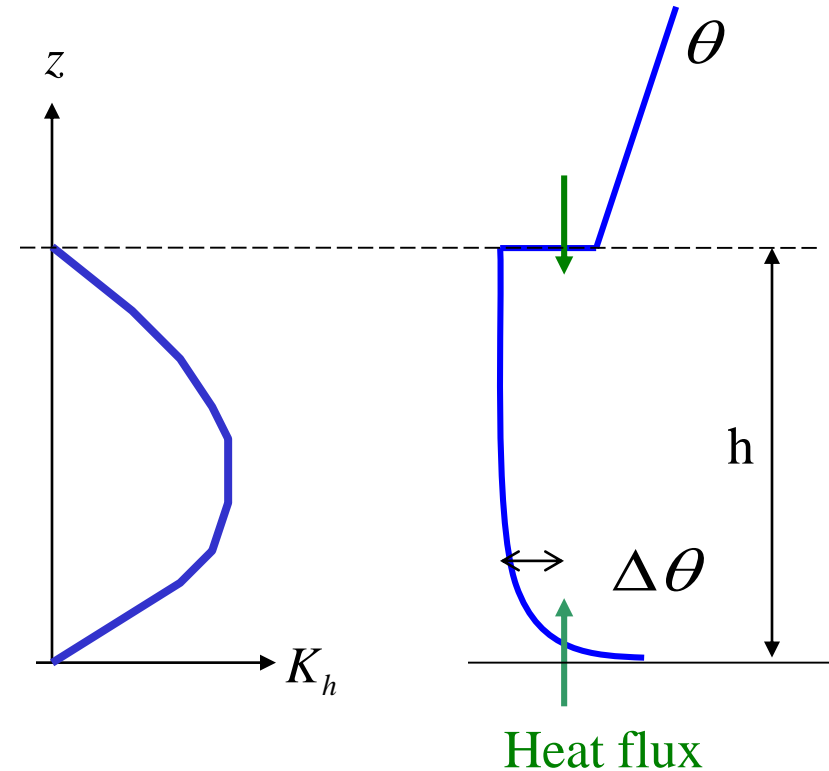
$$\overline{\theta'w'} = -K_H \left( \frac{\partial \theta}{\partial z} - \gamma_\theta \right)$$

Profile of diffusion coefficients:

$$K_H = w_s \kappa z (1 - z/h)^2$$

$$w_s = \left( u_*^3 + C_1 w_*^3 \right)^{1/3}$$

$$\gamma_\theta = C \overline{\theta'w'}^s / w_s h$$



Find inversion by parcel lifting  
with T-excess:

$$\theta_{vs} = \theta_s + \Delta\theta, \quad \Delta\theta = D \overline{w'\theta_v'}^s / w_s$$

such that:

$$Ri_c = h \frac{g}{\theta_v} \frac{\theta_{vh} - \theta_{vs}}{U_h^2 + V_h^2 - U_s^2 - V_s^2} = 0.25$$



# K-profile closure (summary)

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- ☞ Scheme is simple and easy to implement.
- ☞ Numerically robust.
- ☞ Scheme simulates realistic mixed layers.
- ☞ Counter-gradient effects can be included (might create numerical problems).
- ☞ Entrainment can be controlled rather easily.
- ☞ A sufficient number of levels is needed to resolve BL e.g. to locate inversion.



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## TKE closure (1.5 order)

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Eddy diffusivity approach:

$$\overline{u'w'} = -K_M \frac{\partial \bar{u}}{\partial z}, \quad \overline{v'w'} = -K_M \frac{\partial \bar{v}}{\partial z}$$
$$\overline{\theta'w'} = -K_H \frac{\partial \bar{\theta}}{\partial z}, \quad \overline{q'w'} = -K_H \frac{\partial \bar{q}}{\partial z}$$

With diffusion coefficients related to kinetic energy:

$$K_M = C_K \ell_K E^{1/2}, \quad K_H = \alpha_H K_M$$



# Closure of TKE equation

TKE from prognostic equation:

$$\frac{\partial E}{\partial t} = \underbrace{-\overline{u'w'}}_{\text{Shear production}} \frac{\partial U}{\partial z} - \underbrace{\overline{v'w'}}_{\text{Shear production}} \frac{\partial V}{\partial z} \underbrace{- \frac{g}{\rho_0} \overline{\rho'w'}}_{\text{Buoyancy}} + \underbrace{\frac{\partial}{\partial z} (\overline{E'w'})}_{\text{Turbulent transport}} + \underbrace{\frac{\overline{p'w'}}{\rho}}_{\text{Pressure correlation}} - \underbrace{\varepsilon}_{\text{Dissipation}}$$

Storage      Shear production      Buoyancy      Turbulent transport      Pressure correlation      Dissipation

with closure:

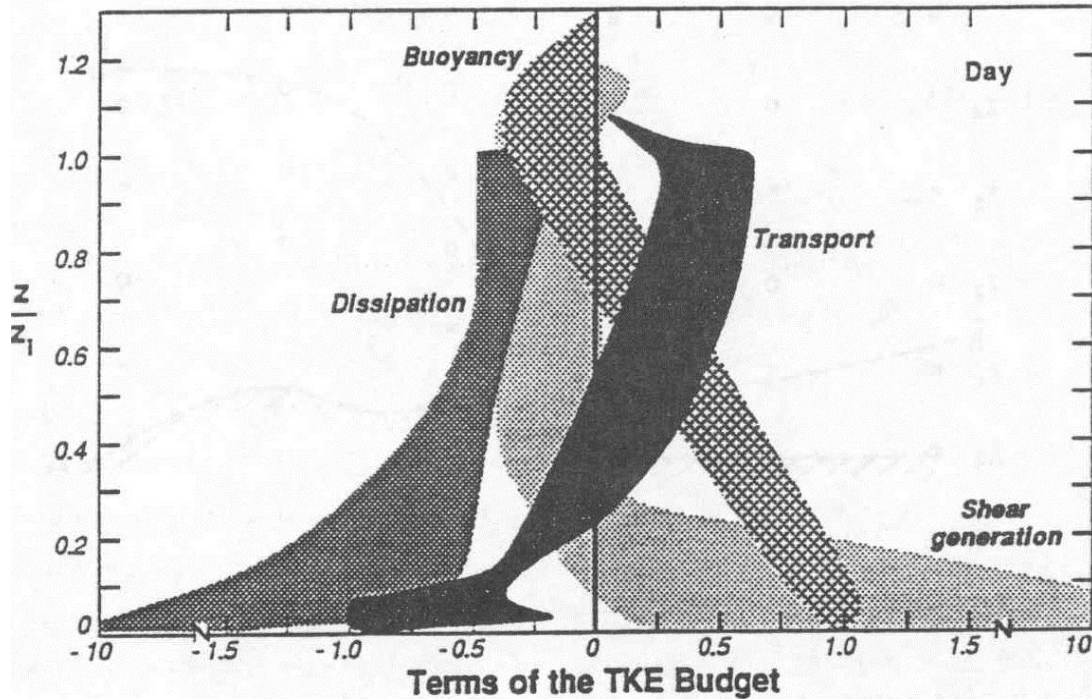
$$\varepsilon = C_\varepsilon \frac{E^{3/2}}{\ell_\varepsilon}, \quad \left( \overline{E'w'} + \frac{\overline{p'w'}}{\rho} \right) = -K_E \frac{\partial E}{\partial z}$$

Main problem is specification of length scales, which are usually a blend of  $\kappa z$ , an asymptotic length scale  $\lambda$  and a stability related length scale in stable situations.





# TKE (summary)



- TKE has natural way of representing entrainment.
- TKE needs more resolution than first order schemes.
- TKE does not necessarily reproduce MO-similarity.
- Stable boundary layer may be a problem.



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# Current closure in the ECMWF model

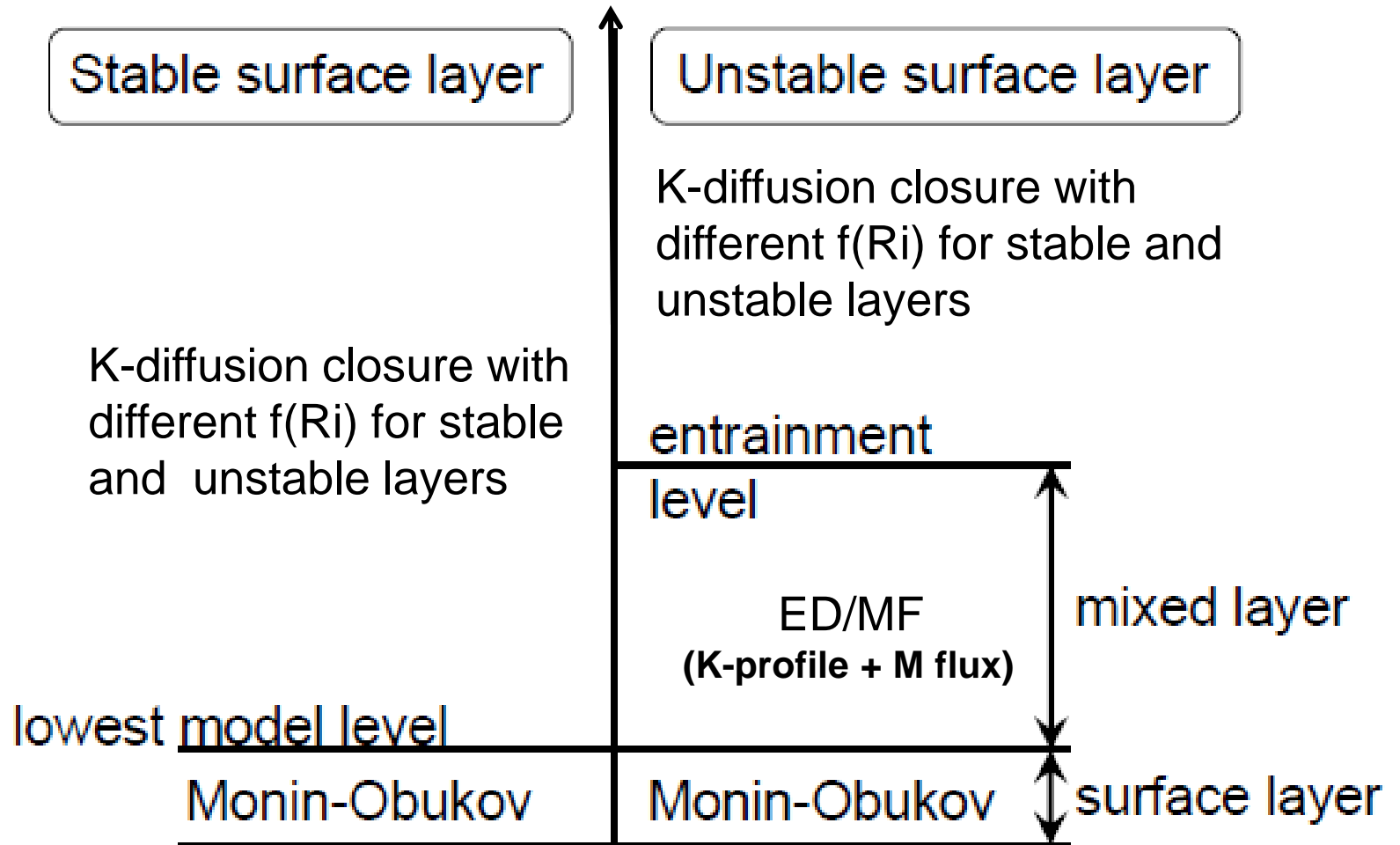


Figure 3.1 Schematic diagram of the different boundary layer regimes.