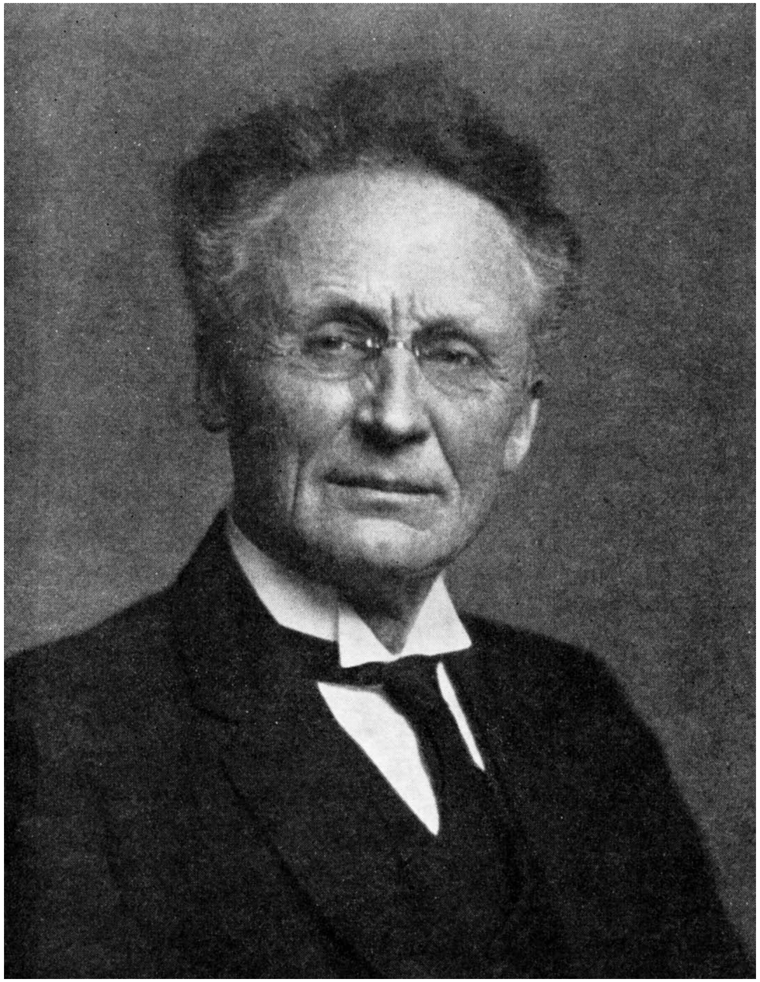
Introduction to chaos

Sarah Keeley

(giving lecture material prepared by Tim Palmer)

ECMWF

Vilhelm Bjerknes (1862-1951)

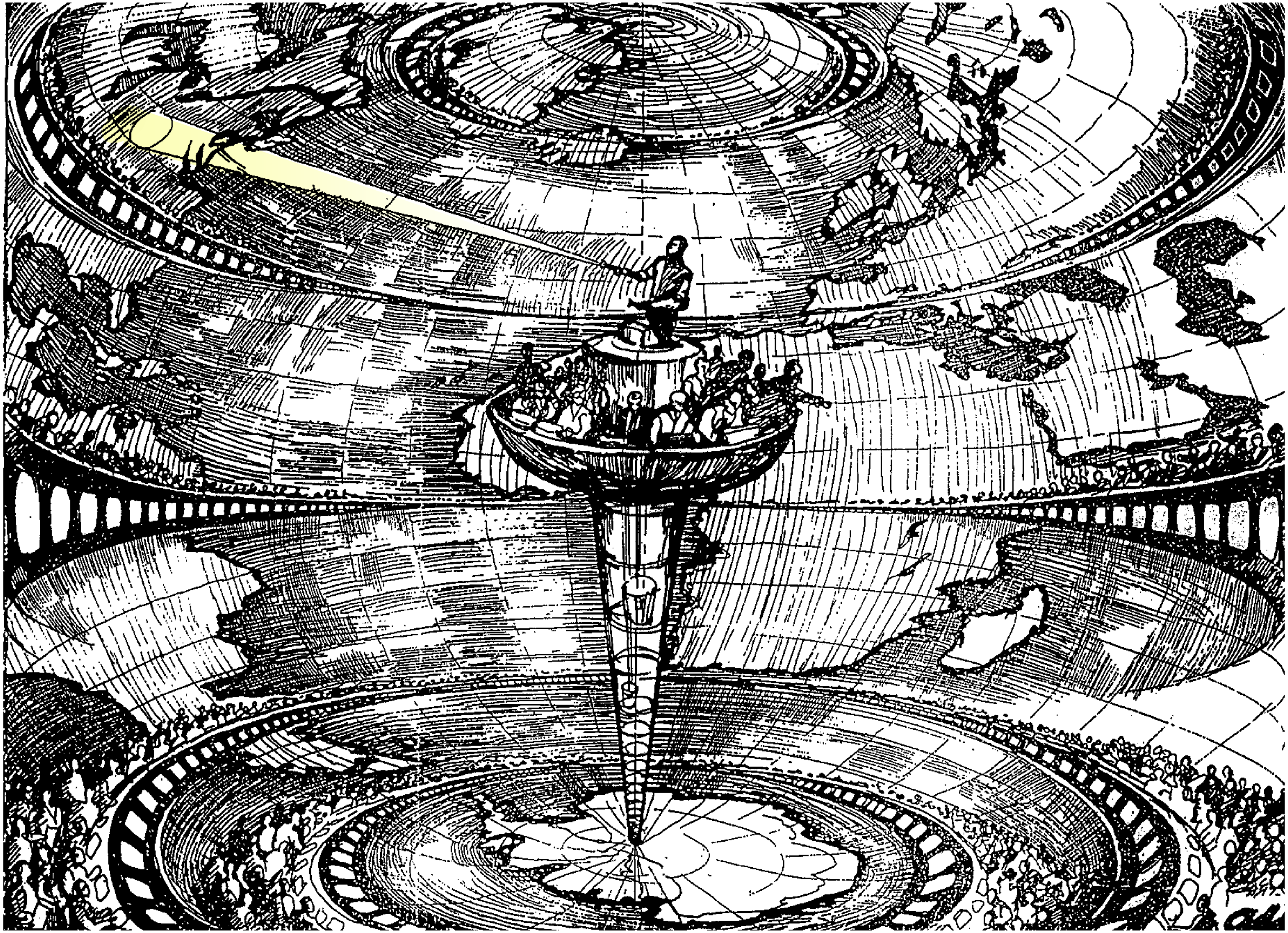


Proposed weather forecasting as a deterministic initial value problem based on the laws of physics

Lewis Fry Richardson (1881-1953)

Lewis Fry Richardson (1881-1953)

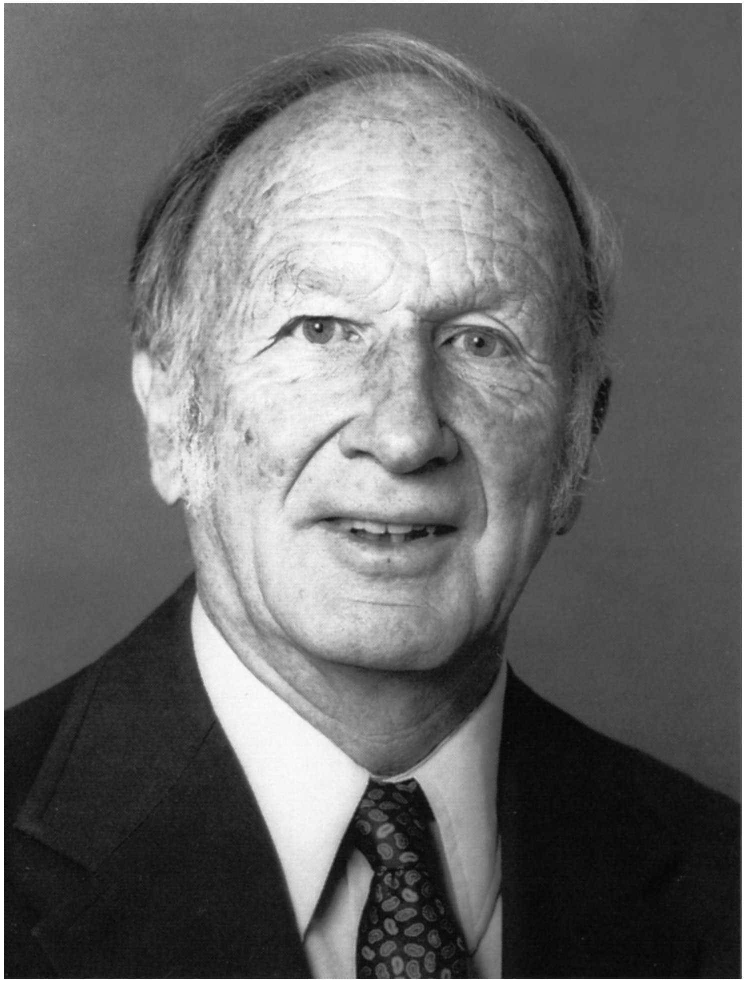
The first numerical weather forecast



“Why have meteorologists such difficulty in predicting the weather with any certainty? Why is it that showers and even storms seem to come by chance ... a tenth of a degree (C) more or less at any given point, and the cyclone will burst here and not there, and extend its ravages over districts that it would otherwise have spared. If (the meteorologists) had been aware of this tenth of a degree, they could have known (about the cyclone) beforehand, but the observations were neither sufficiently comprehensive nor sufficiently precise, and that is the reason why it all seems due to the intervention of chance”

Poincaré, 1909

Edward Lorenz (1917 –2008)



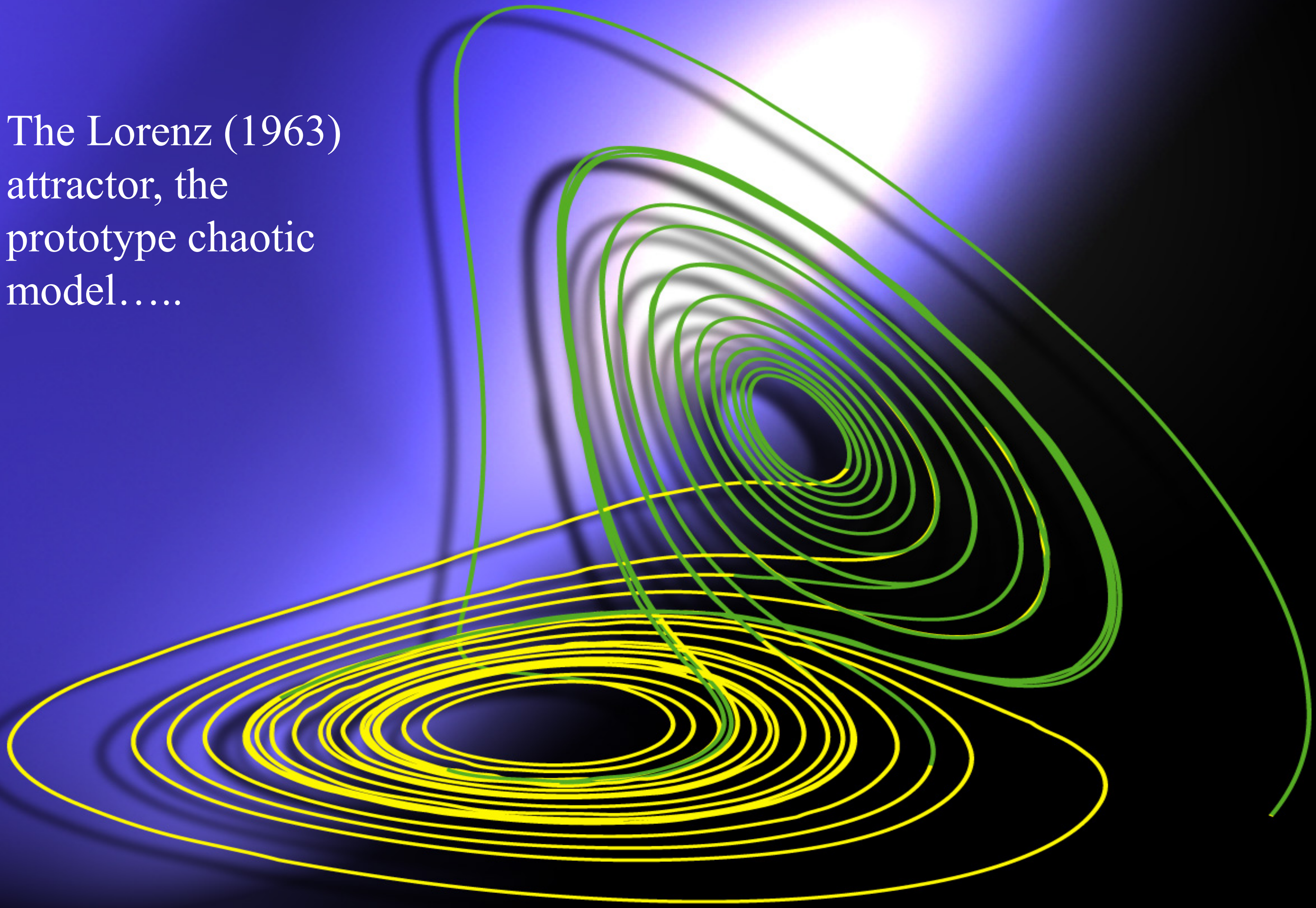
"… one flap of a sea-gull's wing may forever change the future course of the weather" (Lorenz, 1963)

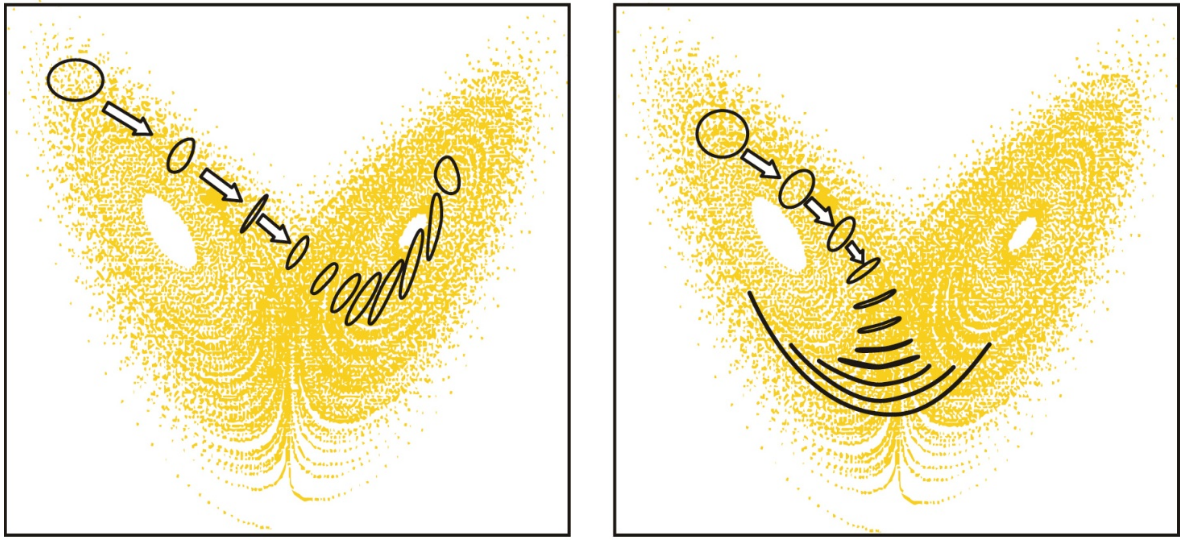
$$\dot{X} = -\sigma X + \sigma Y$$

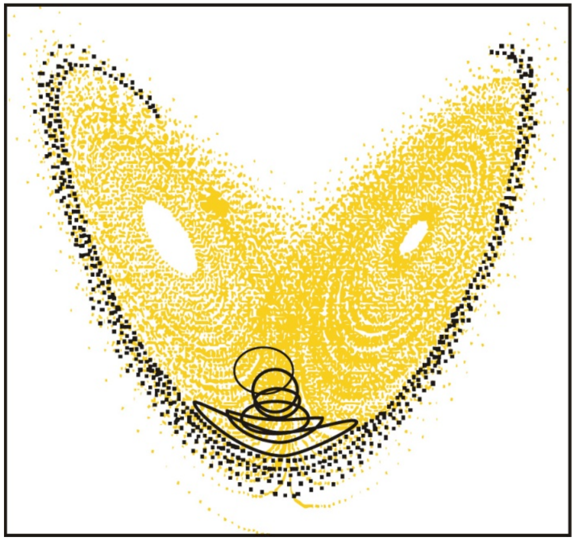
$$\dot{Y} = -XZ + rX - Y$$

$$\dot{Z} = XY - bZ$$

The Lorenz (1963) attractor, the prototype chaotic model…..







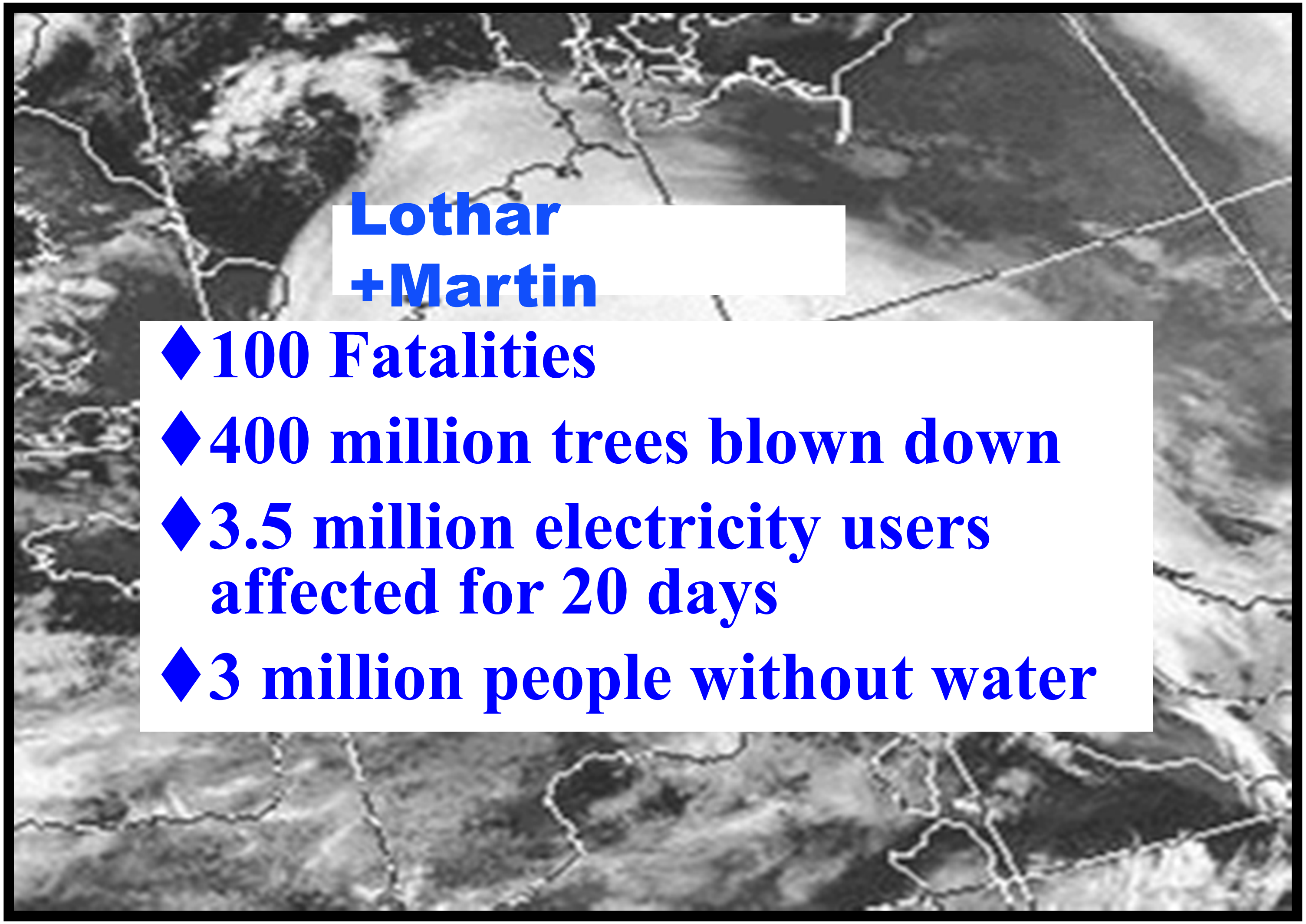
$$\frac{dX}{dt} = F[X] \quad \text{is a nonlinear system}$$

$$\Rightarrow \quad \frac{d\delta X}{dt} = \frac{dF}{dX} \delta X \equiv J \ \delta X$$

Since F is a nonlinear function of X

$$\Rightarrow \quad J = J(X)$$

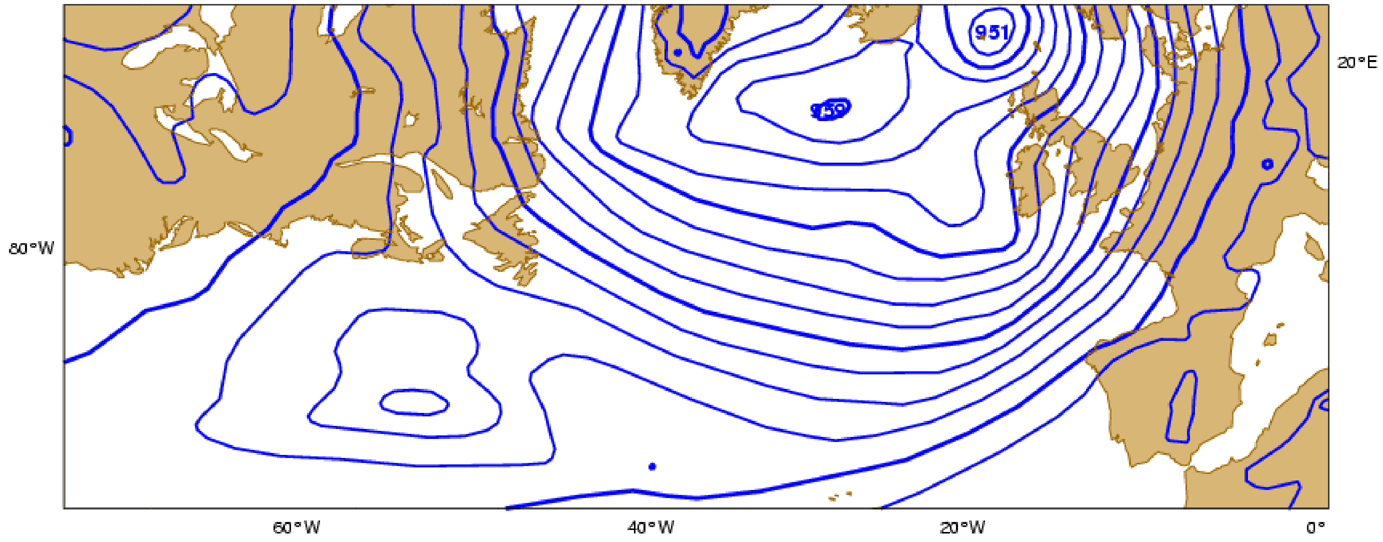
Lothar: 08Z, 26 Dec. 1999



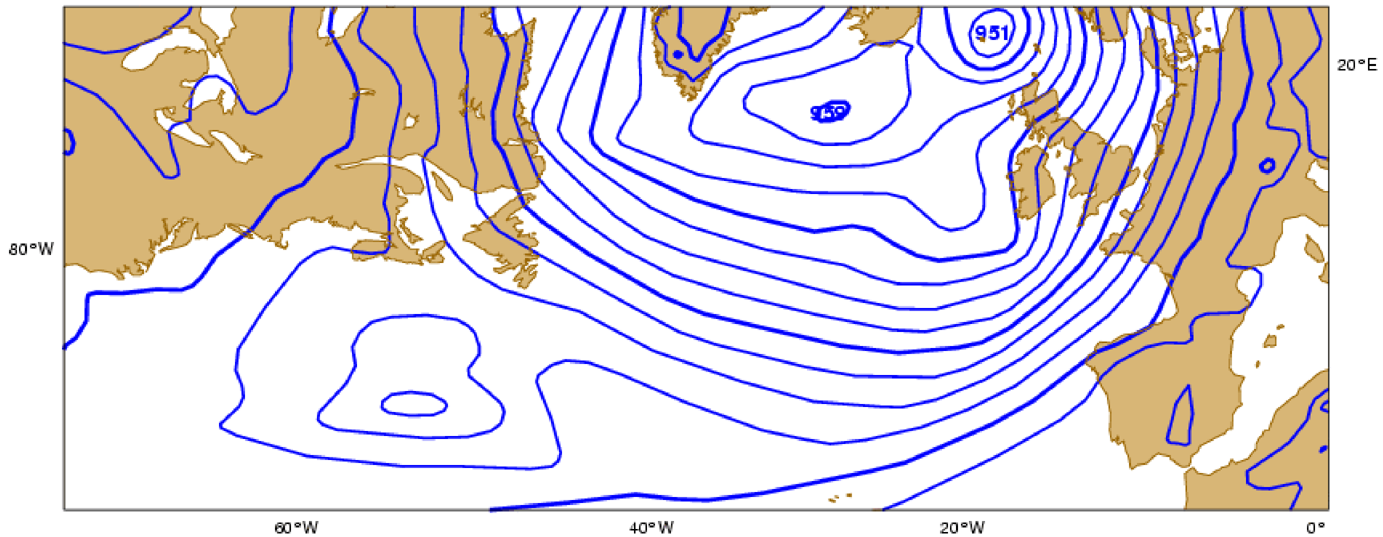
Lothar +Martin

- 100 Fatalities
- 400 million trees blown down
- 3.5 million electricity users affected for 20 days
- 3 million people without water

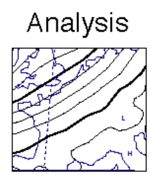
forecast 24 December 1999, 12UTC +0 h   - mem no. 13 of 17

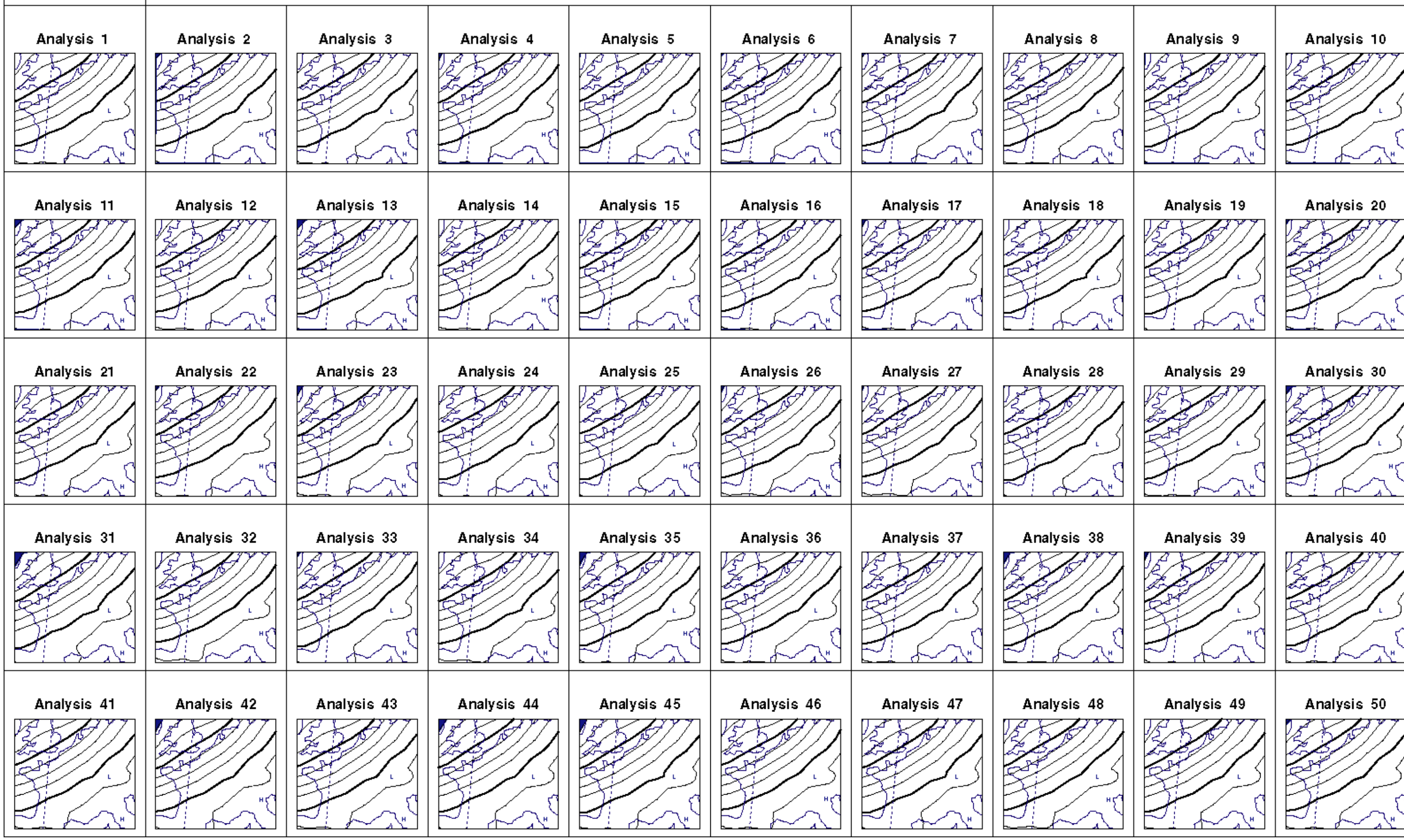


forecast 24 December 1999, 12UTC +0 h   - mem no. 14 of 17

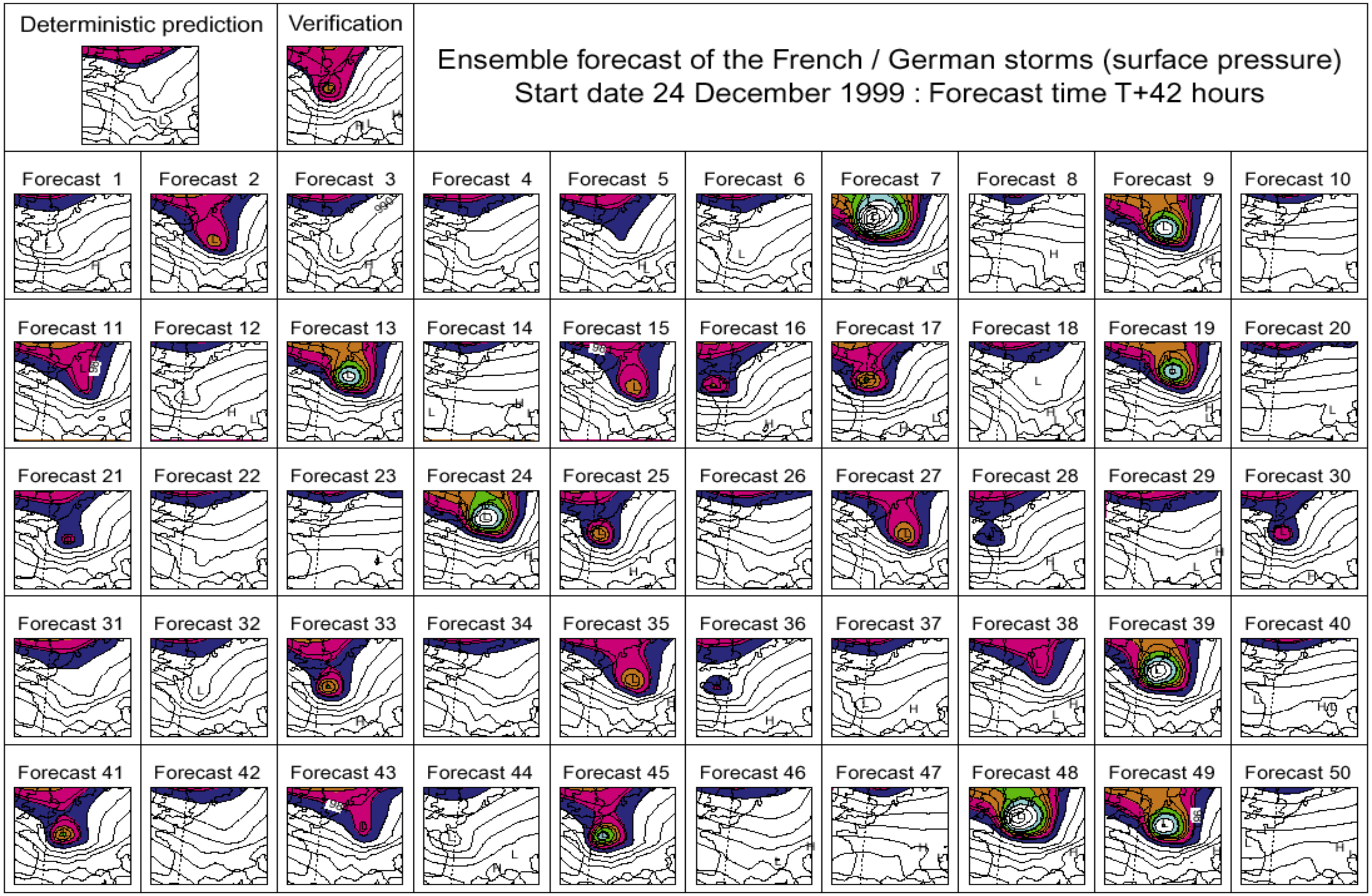


Ensemble Initial Conditions 24 December 1999



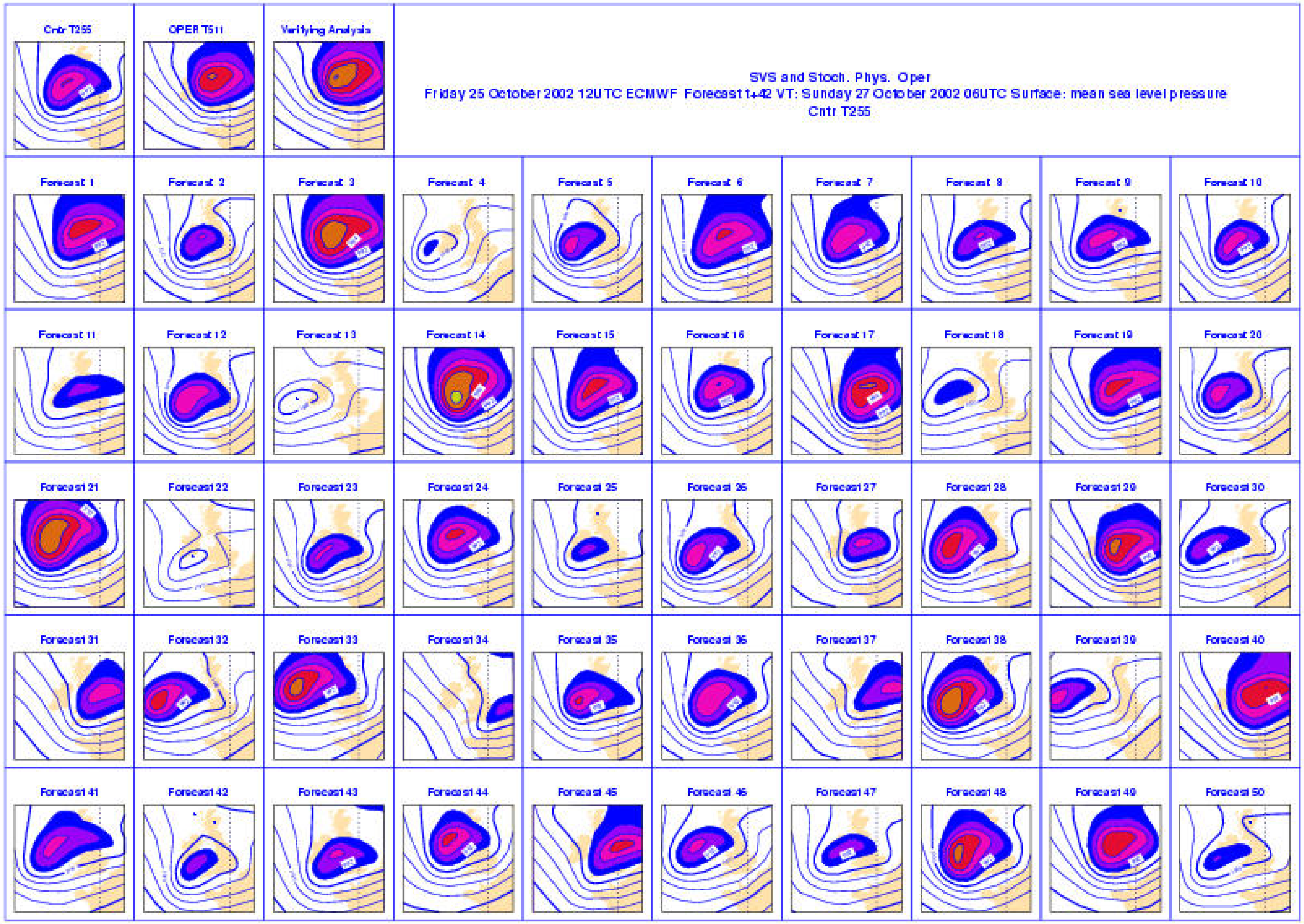


Lothar (T+42 hours)



Headlines: Oct 28 2002

Thanks to Rob Hine


SVS and Stoch. Phys. Oper
Friday 25 October 2002 12UTC ECMWF Forecast t+42 VT: Sunday 27 October 2002 06UTC Surface: mean sea level pressure
Cntr T255
Cntr T255
OPER T511
Verifying Analysis
Forecast 1
Forecast 2
Forecast 3
Forecast 4
Forecast 5
Forecast 6
Forecast 7
Forecast 8
Forecast 9
Forecast 10
Forecast 11
Forecast 12
Forecast 13
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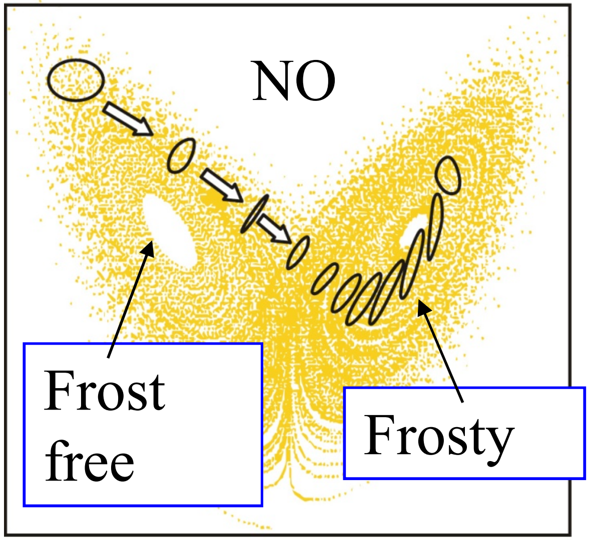
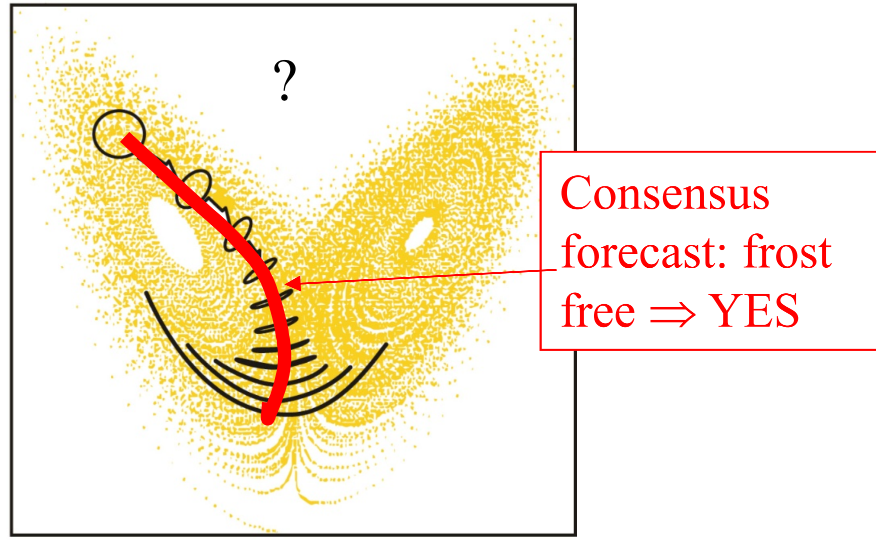
Forecast 49
Forecast 50

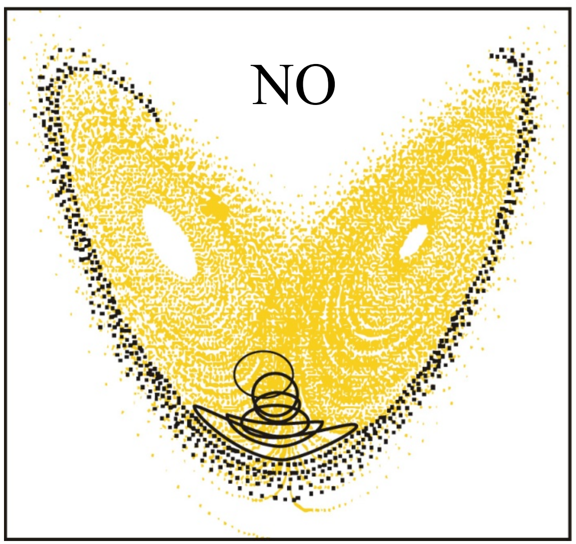
Charlie is planning to lay concrete tomorrow.





Should he?

Let p denote the probability of frost



Charlie loses L if concrete freezes.

•But Charlie has fixed (eg staff) costs

•There may be a penalty for late completion of this job.

•By delaying completion of this job, he will miss out on other jobs.

•These cost C

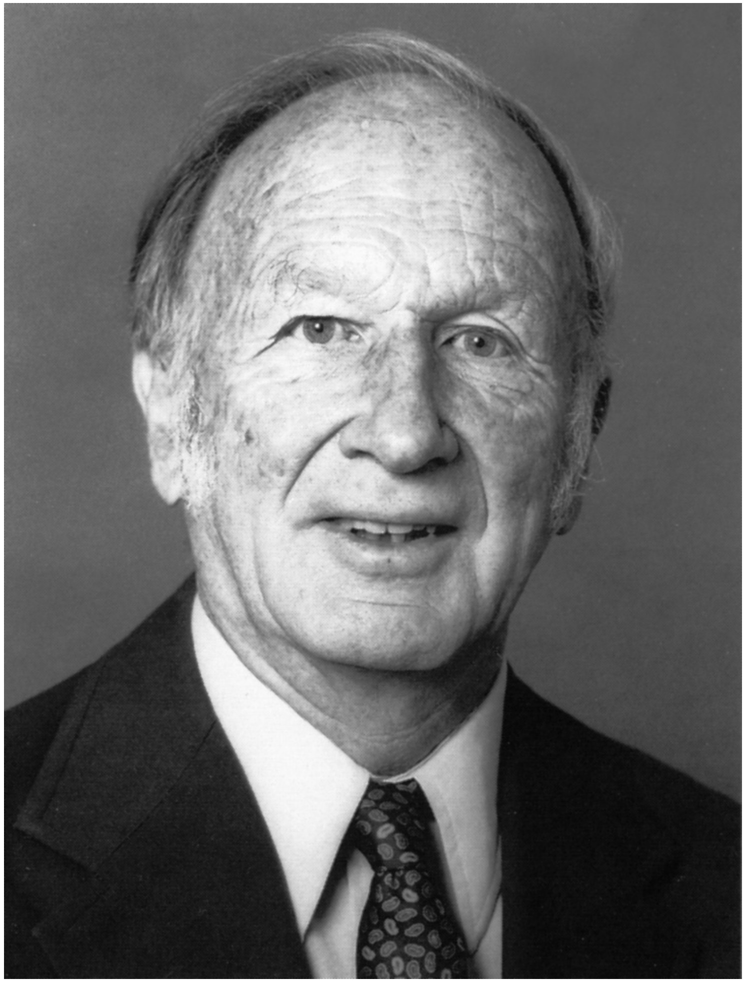
Is $Lp>C$? If $p > C/L$ don't lay concrete!

Introduction to chaos for:

Seasonal climate prediction

Atmospheric predictability arises from slow variations in lower-boundary forcing

Edward Lorenz (1917 – 2008)



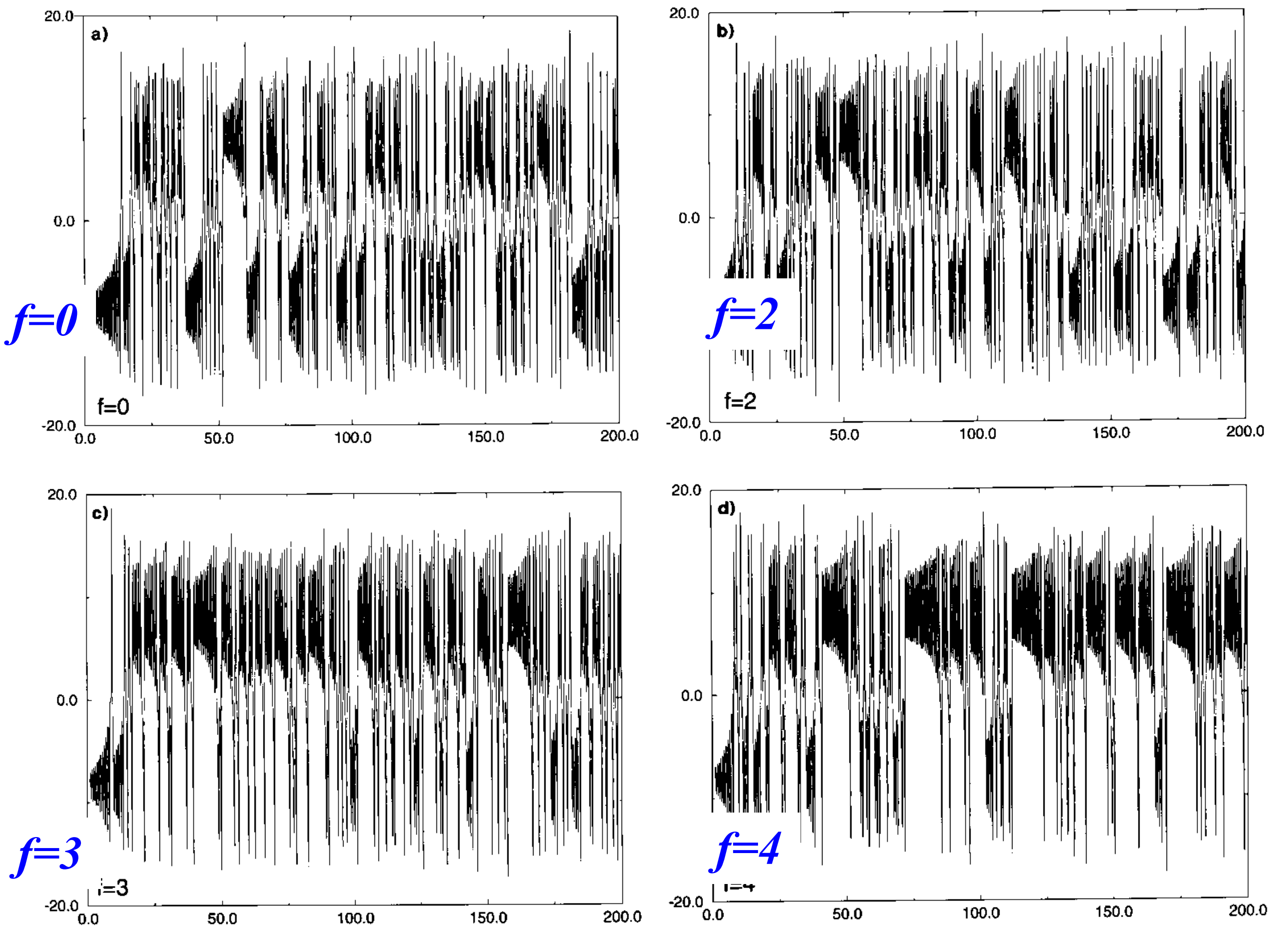
$$\dot{X} = -\sigma X + \sigma Y + f$$

$$\dot{Y} = -XZ + rX - Y + f$$

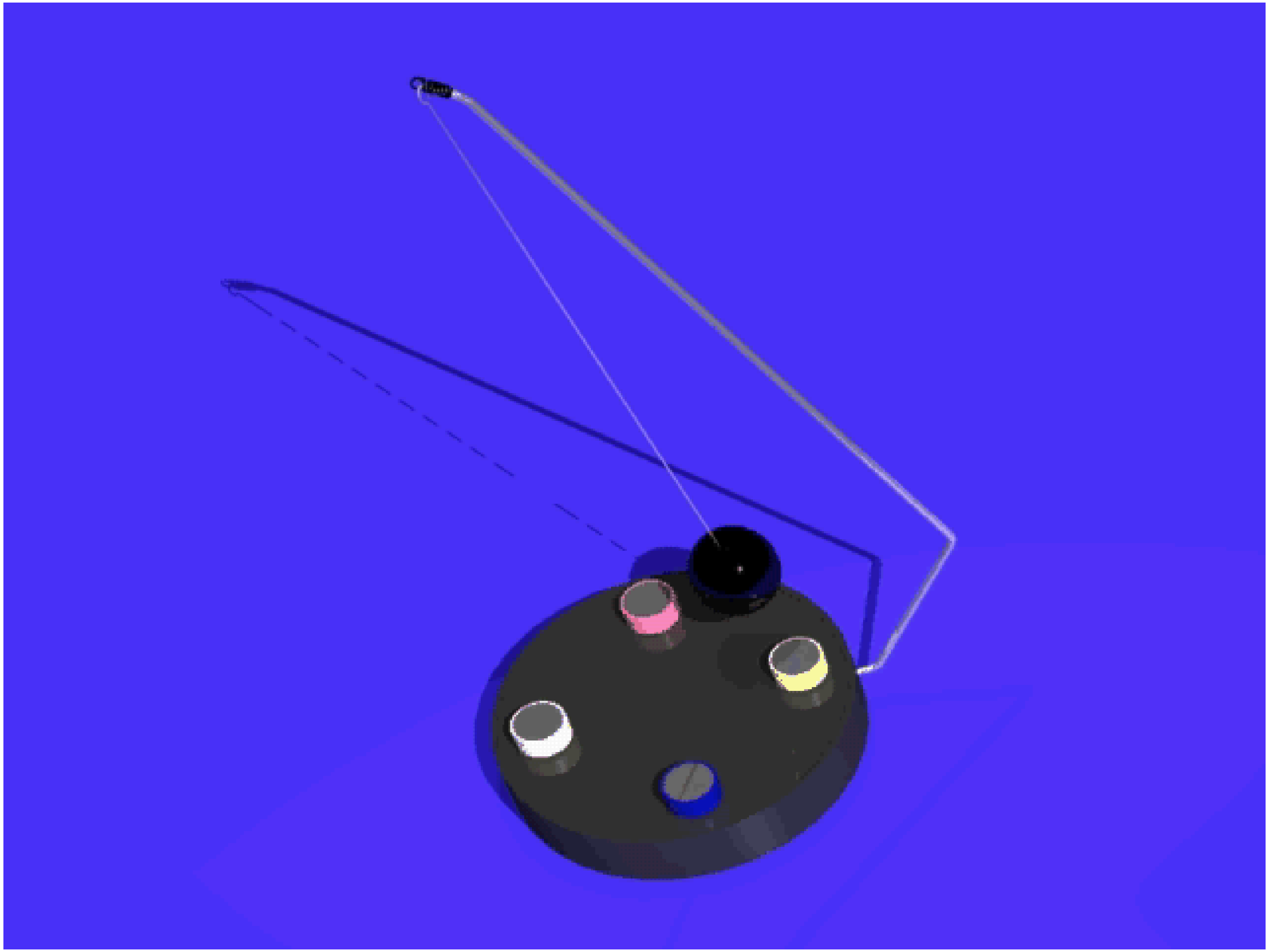
$$\dot{Z} = XY - bZ$$

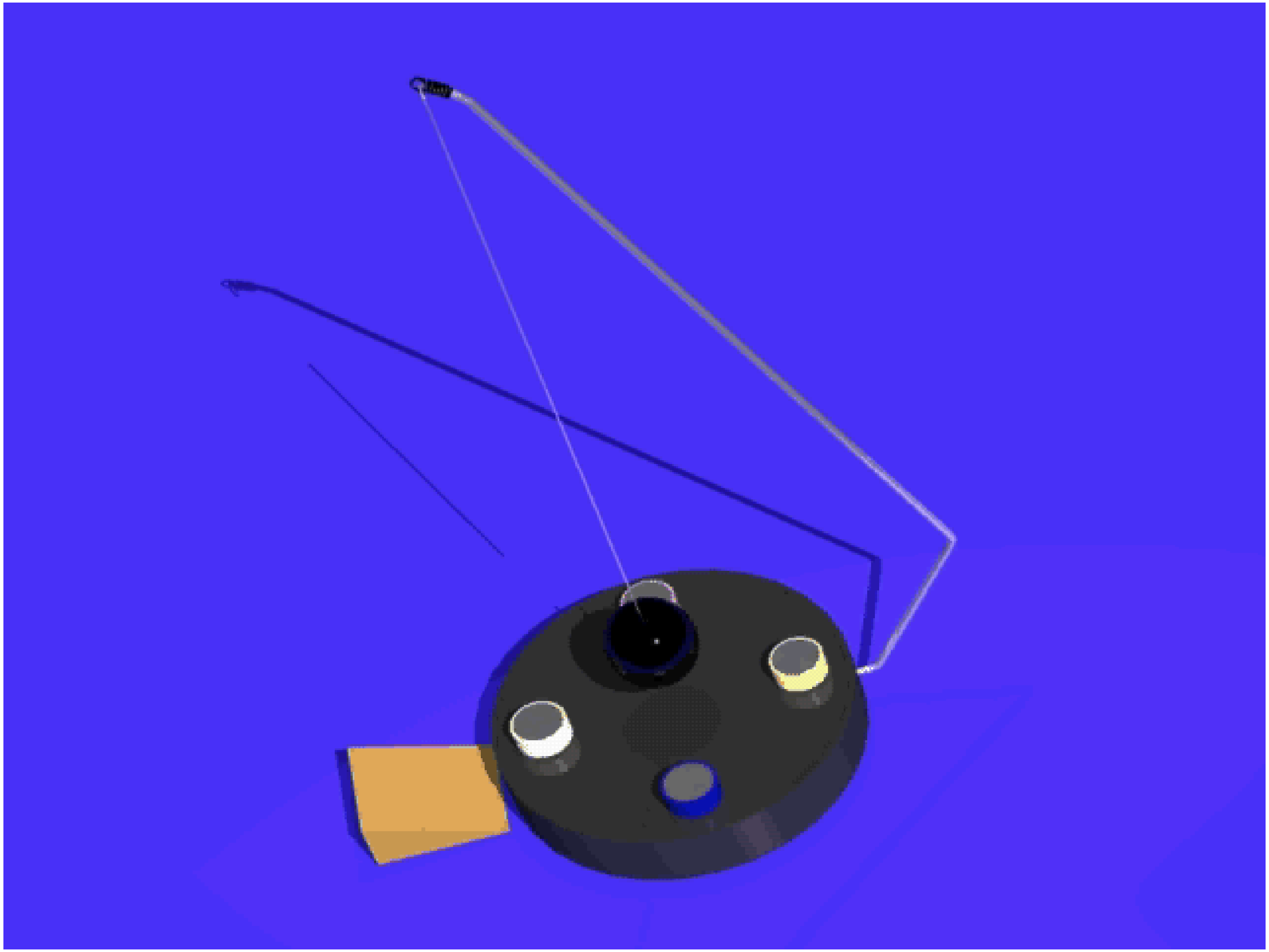
What is the impact of *f* on the attractor?

Add external steady forcing *f* to the Lorenz (1963) equations

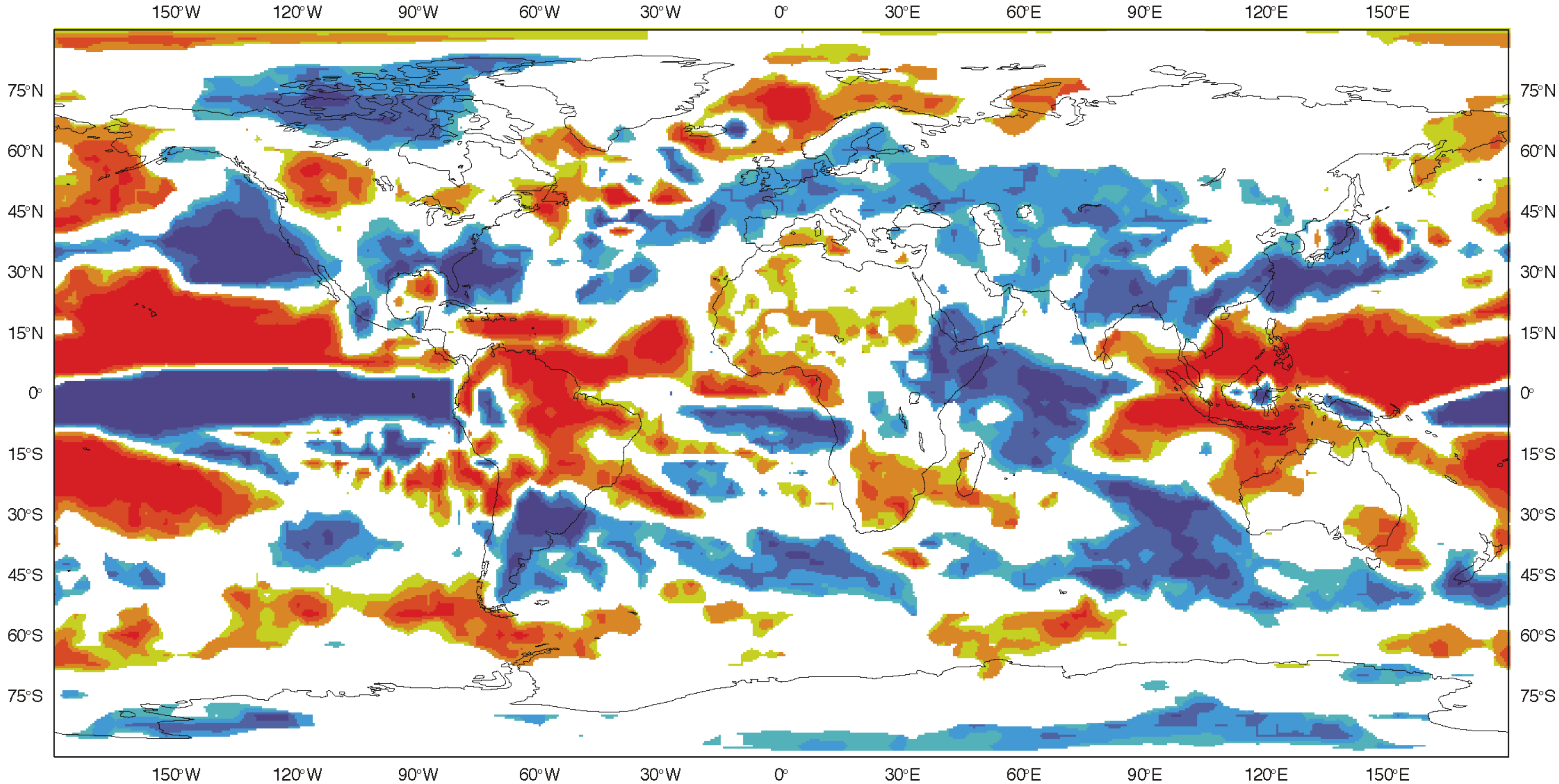


The influence of *f* on the state vector probability function is itself predictable.



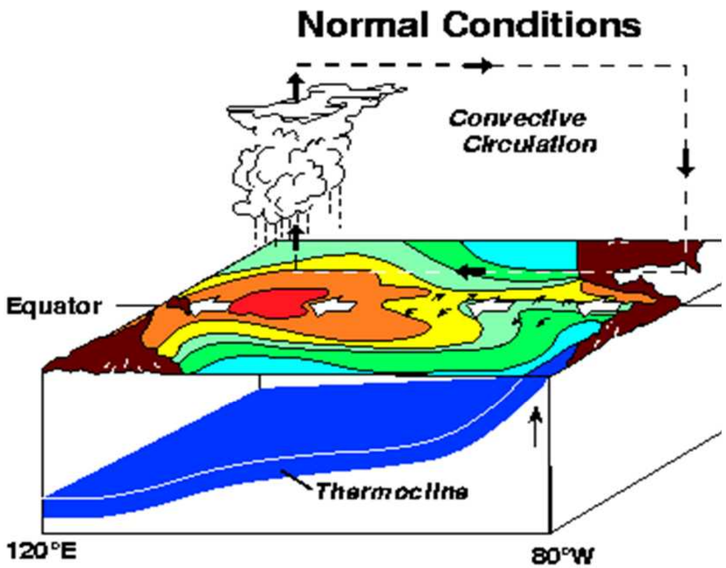


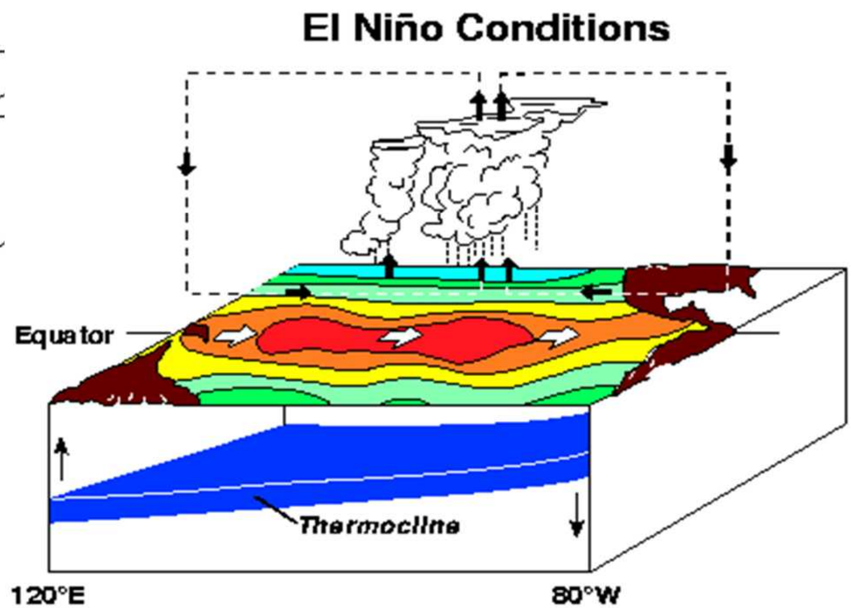
Seasonal Probability Forecasts (ECMWF / HOPE coupled model)
Winter 1997/98 from October 1997



Blue: More likely to be wetter than normal, **Red:** More likely to be drier than normal

The tropical Pacific ocean/atmosphere





Introduction to chaos for:

Stochastic parametrisation

Eg 2) Lorenz(1963) in an EOF

$$\dot{a}_1 = 2.3a_1 - 6.2a_3 - 0.49a_1a_2 - 0.57a_2a_3$$

$$\dot{a}_2 = -62 - 2.7a_2 + 0.49a_1^2 - 0.49a_3^2 + 0.14a_1a_3$$

$$\dot{a}_3 = -0.63a_1 - 13a_3 + 0.43a_1a_2 + 0.49a_2a_3$$

3rd EOF only explains 4% of variance (Selten, 1995) .

Parametrise it?

Lorenz(1963) in a truncated EOF basis with parametrisation of a_3

$$\dot{a}_1 = 2.3a_1 - 6.2a_3 - 0.49a_1a_2 - 0.57a_2a_3$$

$$\dot{a}_2 = -62 - 2.7a_2 + 0.49a_1^2 - 0.49a_3^2 + 0.14a_1a_3$$

$$a_3 = f(a_1, a_2)$$

Good as a short-range forecast model (using L63 as truth), but exhibits major systematic errors compared with L63, as, by Poincaré-Bendixon theorem, the system cannot exhibit chaotic variability – system collapses onto a point attractor.

Stochastic-Lorenz(1963) in a truncated EOF basis

$$\dot{a}_1 = 2.3a_1 - 6.2a_3 - 0.49a_1a_2 - 0.57a_2a_3$$

$$\dot{a}_2 = -62 - 2.7a_2 + 0.49a_1^2 - 0.49a_3^2 + 0.14a_1a_3$$

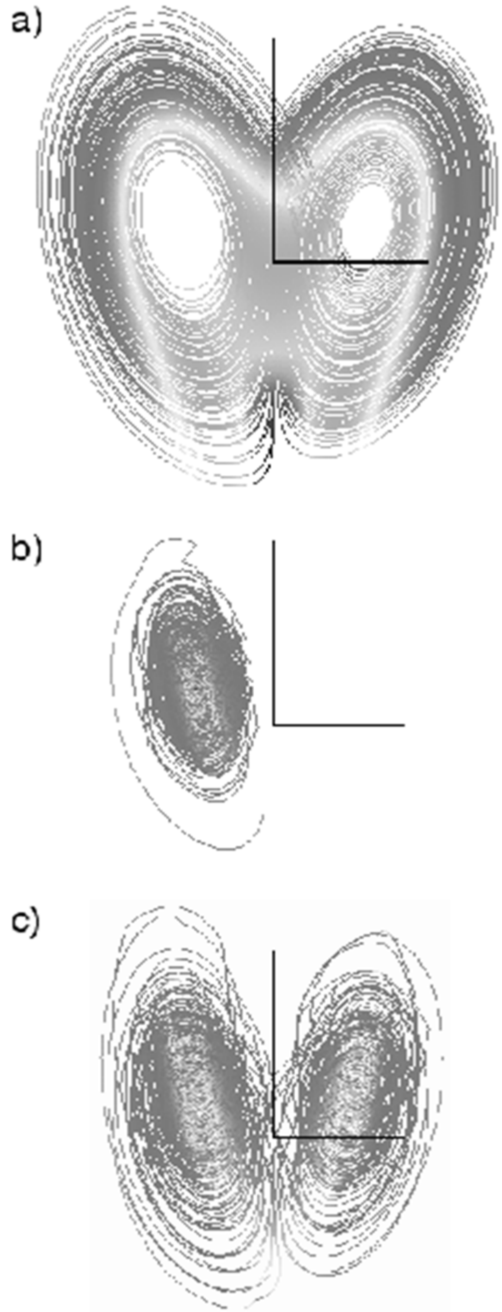
$$a_3 = \beta$$

Stochastic noise

Lorenz attractor

Truncated Stochastic-Lorenz attractor –weak noise

Truncated Stochastic-Lorenz attractor



Error in mean and variance

Palmer, 2001 (acknowledgment to Frank Selten)