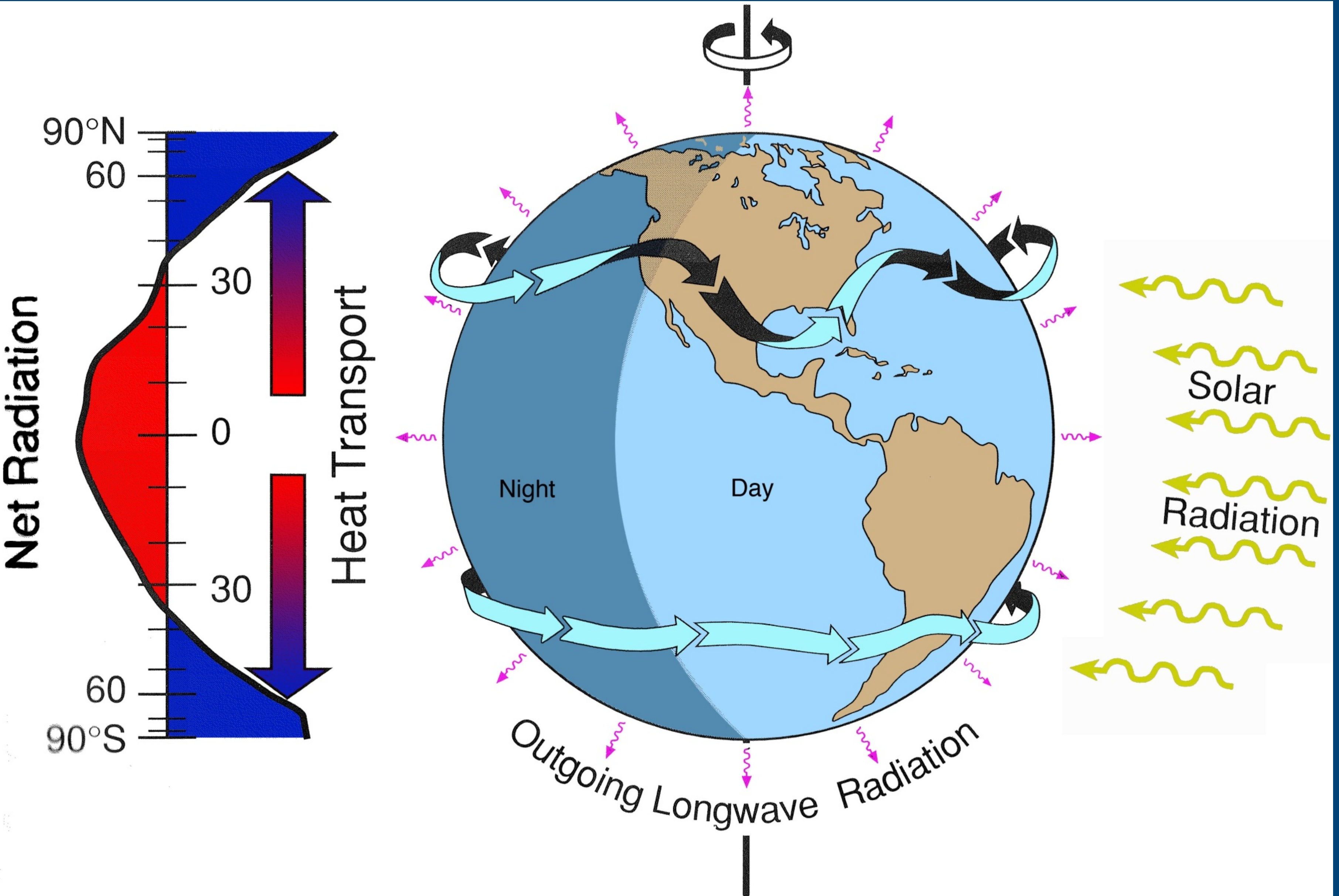
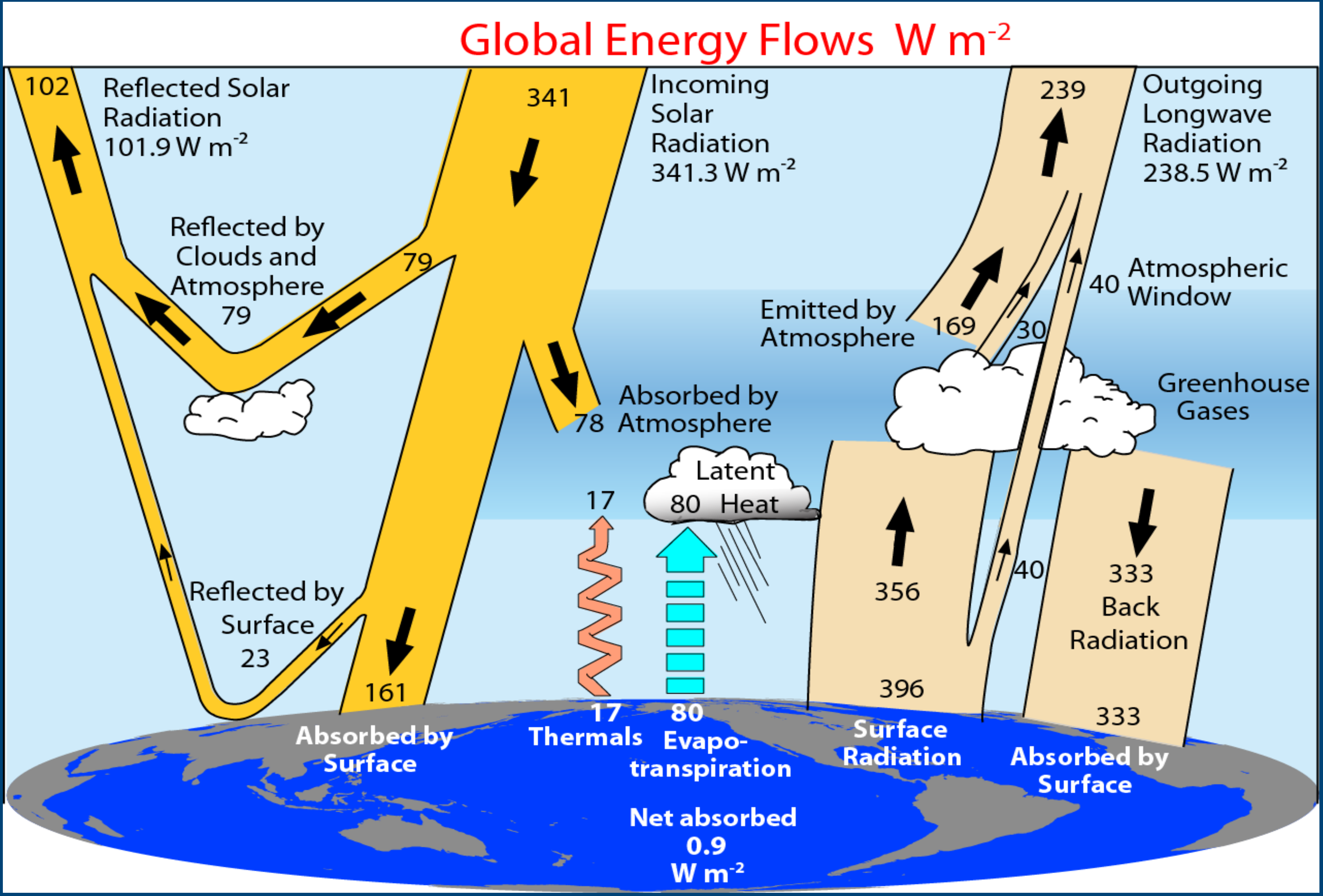
General circulation refresher

Erland Källén

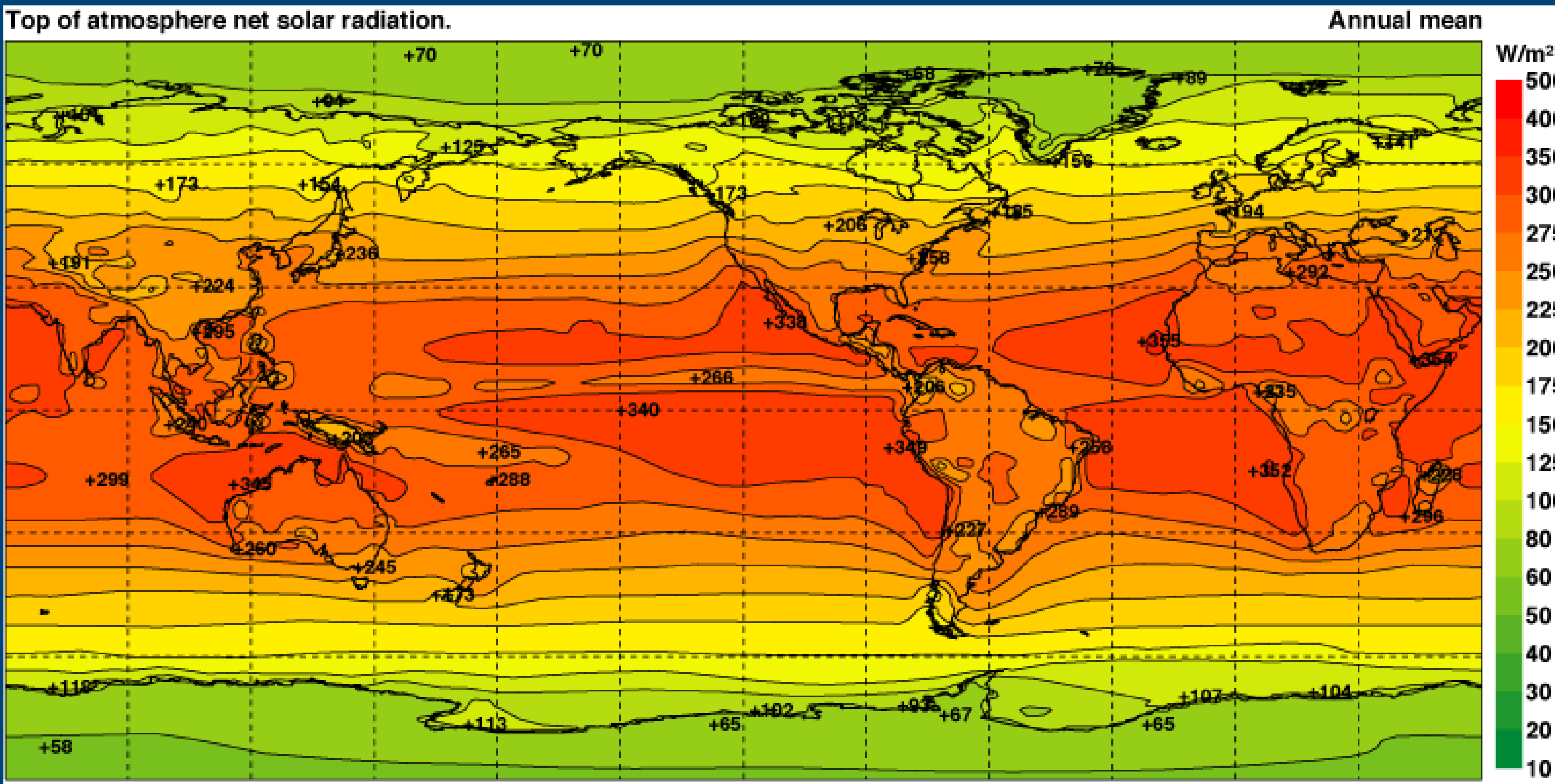
Physical climate system



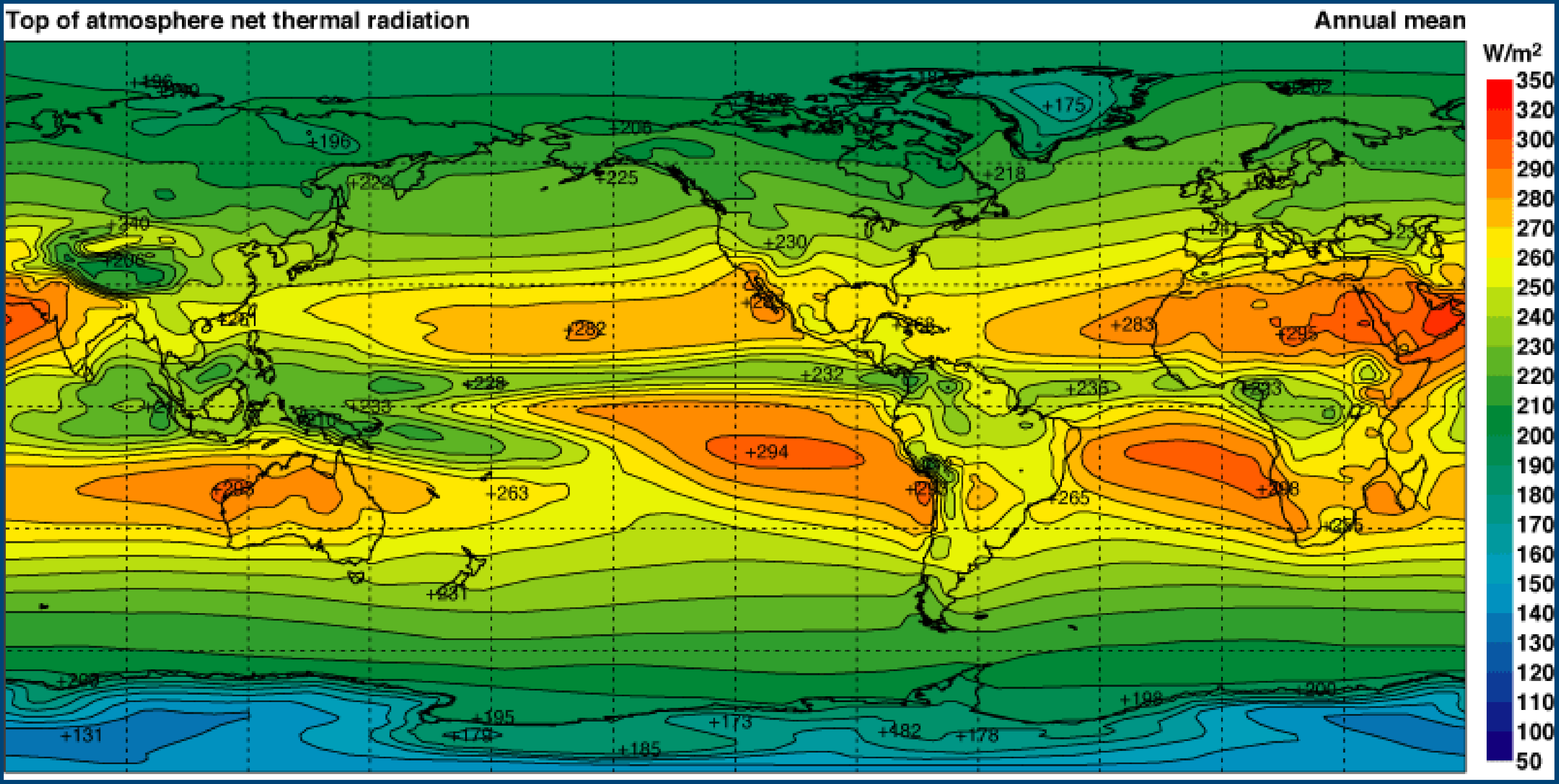
Radiation balance (Trenberth, 2009)



Top of atmosphere net solar radiation



Top of atmosphere net thermal radiation



Equations of motion (hydrostatic)

$$\frac{\partial \mathbf{V_h}}{\partial t} + \mathbf{V_h} \cdot \nabla \mathbf{V_h} + \omega \frac{\partial \mathbf{V_h}}{\partial p} + f\,\mathbf{k} \times \mathbf{V_h} = -\nabla \Phi, \quad \frac{\partial \Phi}{\partial p} = -\frac{RT}{p}$$

Thermodynamic equation

$$\left(\frac{\partial}{\partial t} + \mathbf{V_h} \cdot \nabla \right) T - S_p \omega = J / c_p$$

Continuity equation

$$\nabla \cdot \mathbf{V_h} + \frac{\partial \omega}{\partial p} = 0$$

Moisture equations

$$\frac{\partial q}{\partial t} = -\mathbf{V_h} \cdot \nabla q - \omega \frac{\partial q}{\partial p} + P_q + K_q$$

$$\frac{\partial q_l}{\partial t} = -\mathbf{V_h} \cdot \nabla q_l + P_{q_l}$$

Steady state, zonal mean (Held and Hou, JAS 1980)

$$
\left.
\begin{aligned}
0 &= -\nabla\cdot(\mathbf{v}u) + fv + \frac{uv\tan\theta}{a} + \frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right)\\
0 &= -\nabla\cdot(\mathbf{v}v) - fu - \frac{u^2\tan\theta}{a} - \frac{1}{a}\frac{\partial\Phi}{\partial\theta}\\
&\quad + \frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right)\\
0 &= -\nabla\cdot(\mathbf{v}\Theta) - (\Theta-\Theta_E)\tau^{-1} + \frac{\partial}{\partial z}\left(\nu\frac{\partial\Theta}{\partial z}\right)\\
0 &= -\nabla\cdot\mathbf{v}\\
\frac{\partial\Phi}{\partial z} &= g\Theta/\Theta_0
\end{aligned}
\right\}
$$

with boundary conditions

$$
\left.
\begin{aligned}
&\text{at}\quad z=H:\quad w=0;\quad \frac{\partial u}{\partial z}=\frac{\partial v}{\partial z}=\frac{\partial\Theta}{\partial z}=0\\
&\text{at}\quad z=0:\quad w=0;\quad \frac{\partial\Theta}{\partial z}=0;\\
&\qquad\qquad \nu\frac{\partial u}{\partial z}=Cu;\quad \nu\frac{\partial v}{\partial z}=Cv
\end{aligned}
\right\}
$$

Steady, zonal mean state

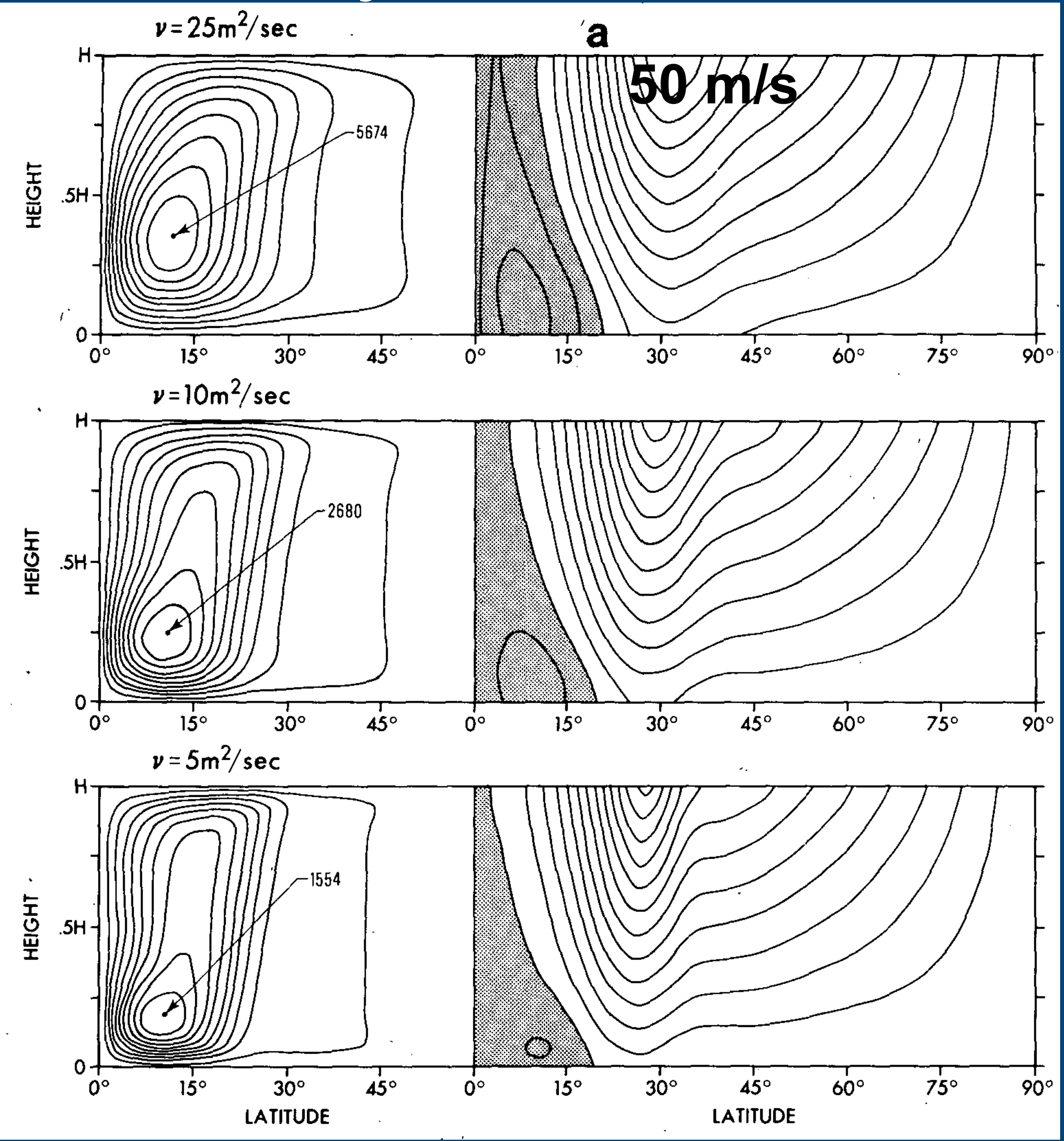
$$\frac{\Theta_E(\theta,z)}{\Theta_0} = 1 - \frac{2}{3}\Delta_H P_2(\sin\theta) + \Delta_v\left(\frac{z}{H} - \frac{1}{2}\right)$$

$(v = w = 0)$

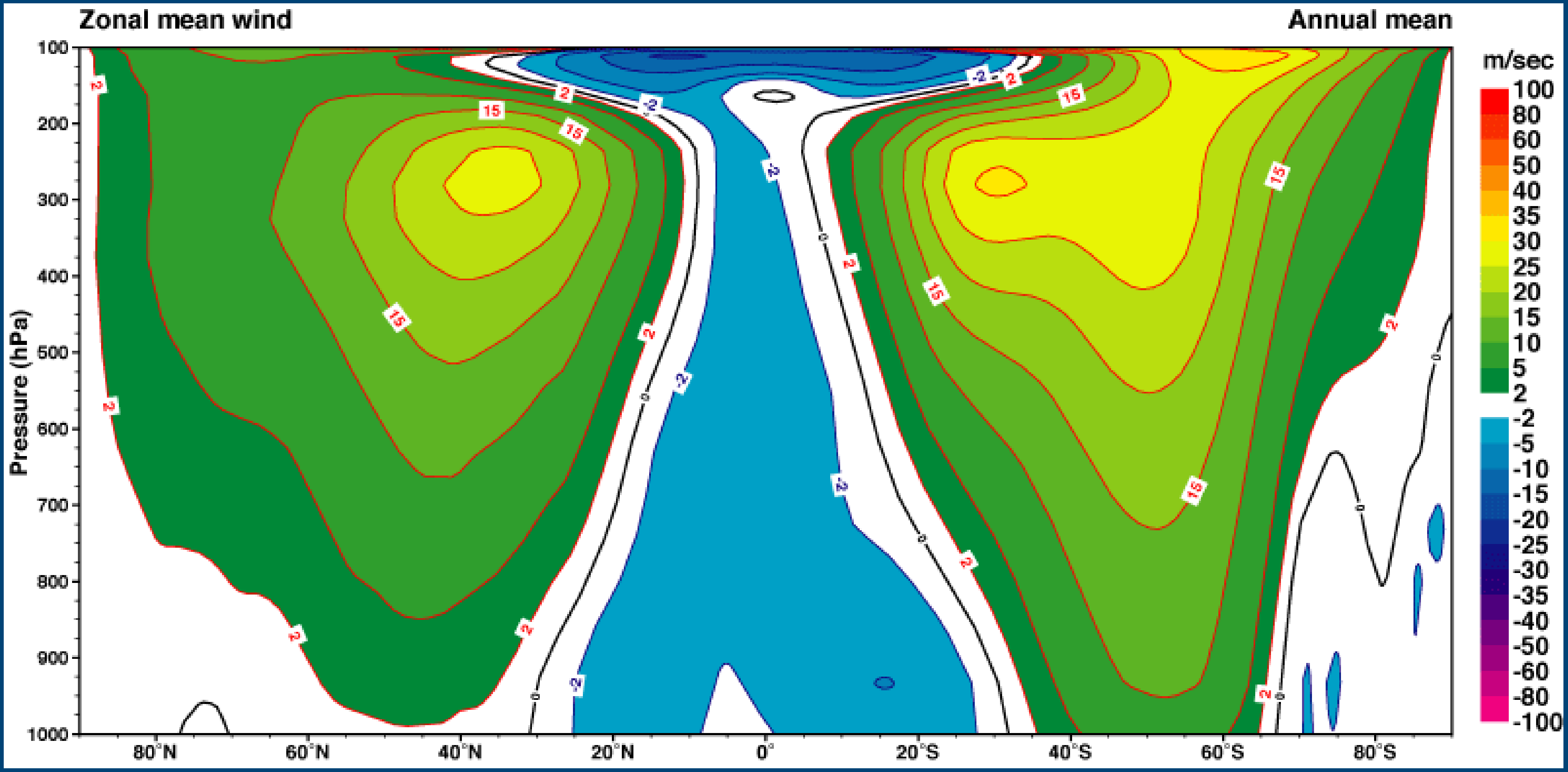
$(\Theta = \Theta_E)$

$$\frac{\partial}{\partial z}\left(f u_E + \frac{u_E{}^2 \tan\theta}{a}\right) = -\frac{g}{a\Theta_0}\frac{\partial \Theta_E}{\partial \theta}$$

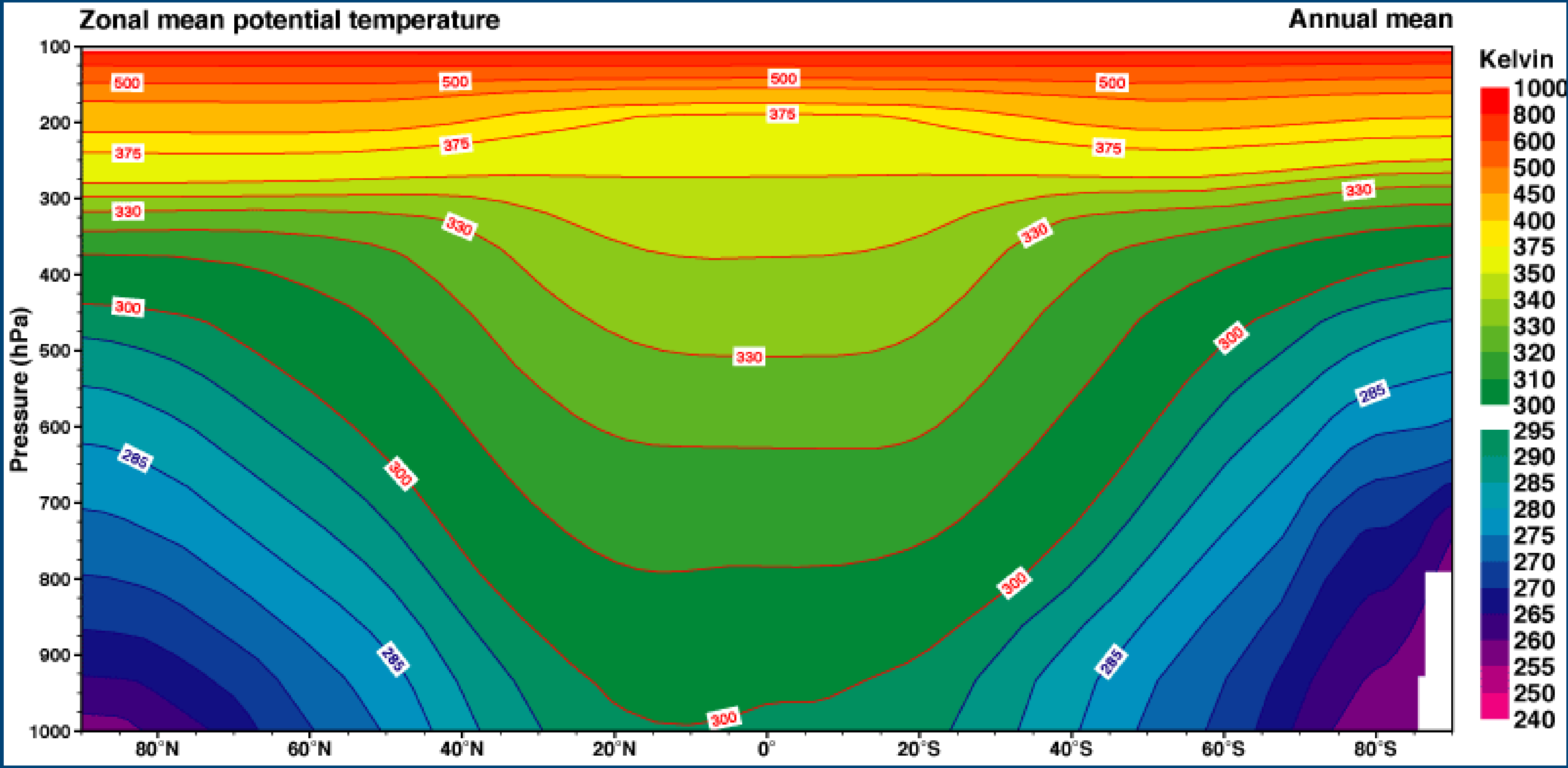
Hadley cell solutions



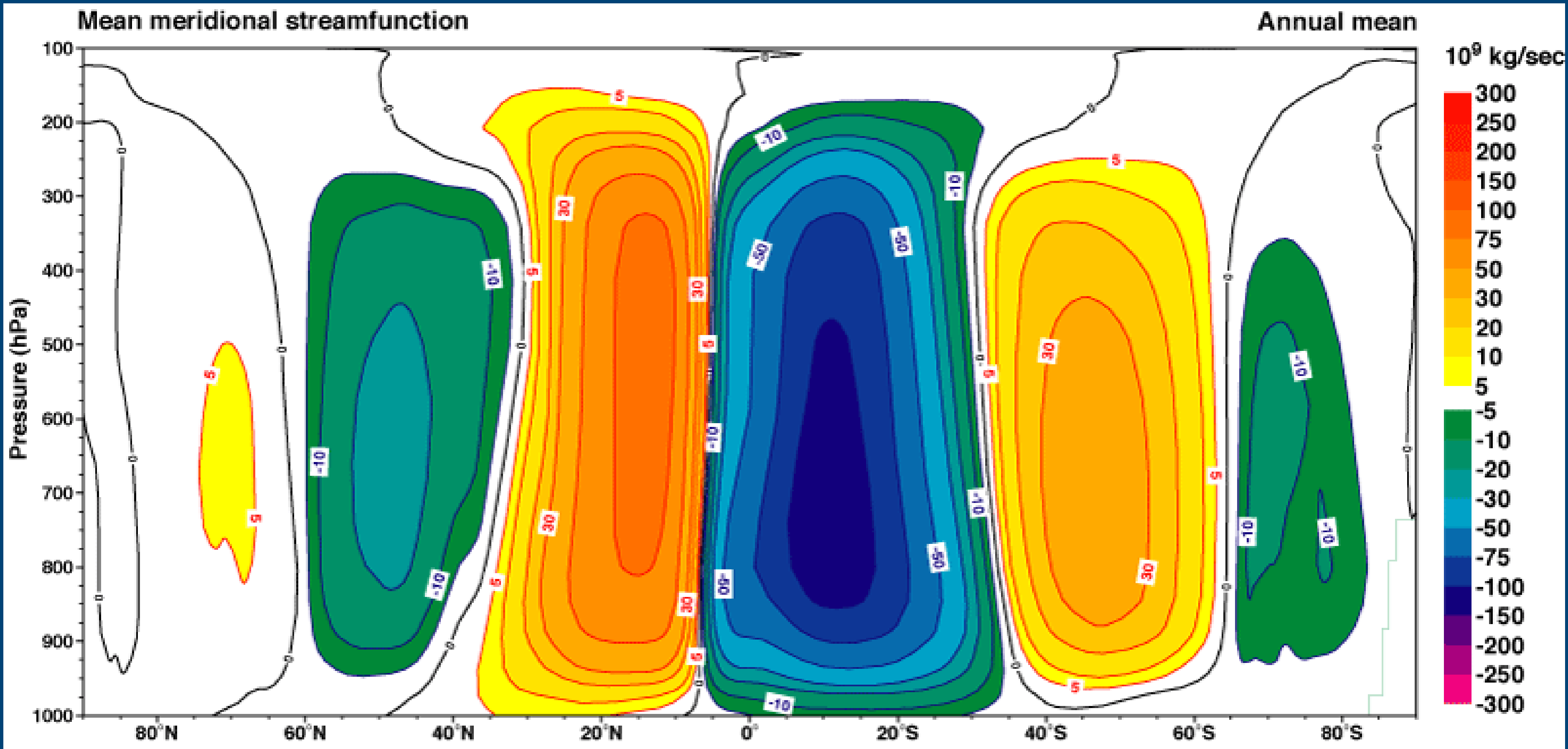
Zonal mean winds



Zonal mean potential temperature



Mean meridional streamfunction



Baroclinic instability theory

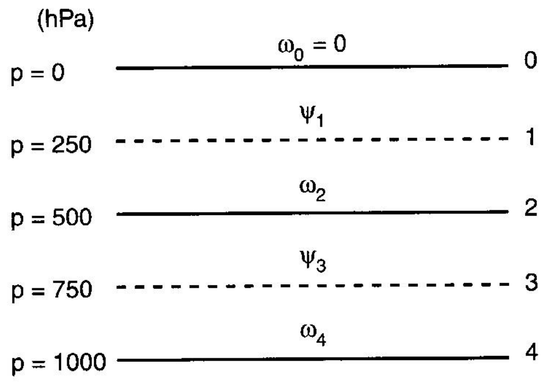


Fig. 8.2 Arrangement of variables in the vertical for the two-level baroclinic model.

$$\mathbf{V}_\psi = \mathbf{k} \times \boldsymbol{\nabla}\psi, \quad \zeta_\mathbf{g} = \nabla^2\psi \tag{8.1}$$

The quasi-geostrophic vorticity equation (6.19) and the hydrostatic thermodynamic energy equation (6.13) can then be written in terms of ψ and ω as

$$\frac{\partial}{\partial t}\nabla^2\psi + \mathbf{V}_\psi \cdot \boldsymbol{\nabla}\left(\nabla^2\psi\right) + \beta\frac{\partial\psi}{\partial x} = f_0\frac{\partial\omega}{\partial p} \tag{8.2}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial p}\right) = -\mathbf{V}_\psi \cdot \boldsymbol{\nabla}\left(\frac{\partial\psi}{\partial p}\right) - \frac{\sigma}{f_0}\omega \tag{8.3}$$

Dispersion relation

$$c = U_m - \frac{\beta\left(k^2+\lambda^2\right)}{k^2\left(k^2+2\lambda^2\right)} \pm \delta^{1/2}$$

$$\delta \equiv \frac{\beta^2\lambda^4}{k^4\left(k^2+2\lambda^2\right)^2} - \frac{U_T^2\left(2\lambda^2-k^2\right)}{\left(k^2+2\lambda^2\right)}$$

Instability diagram

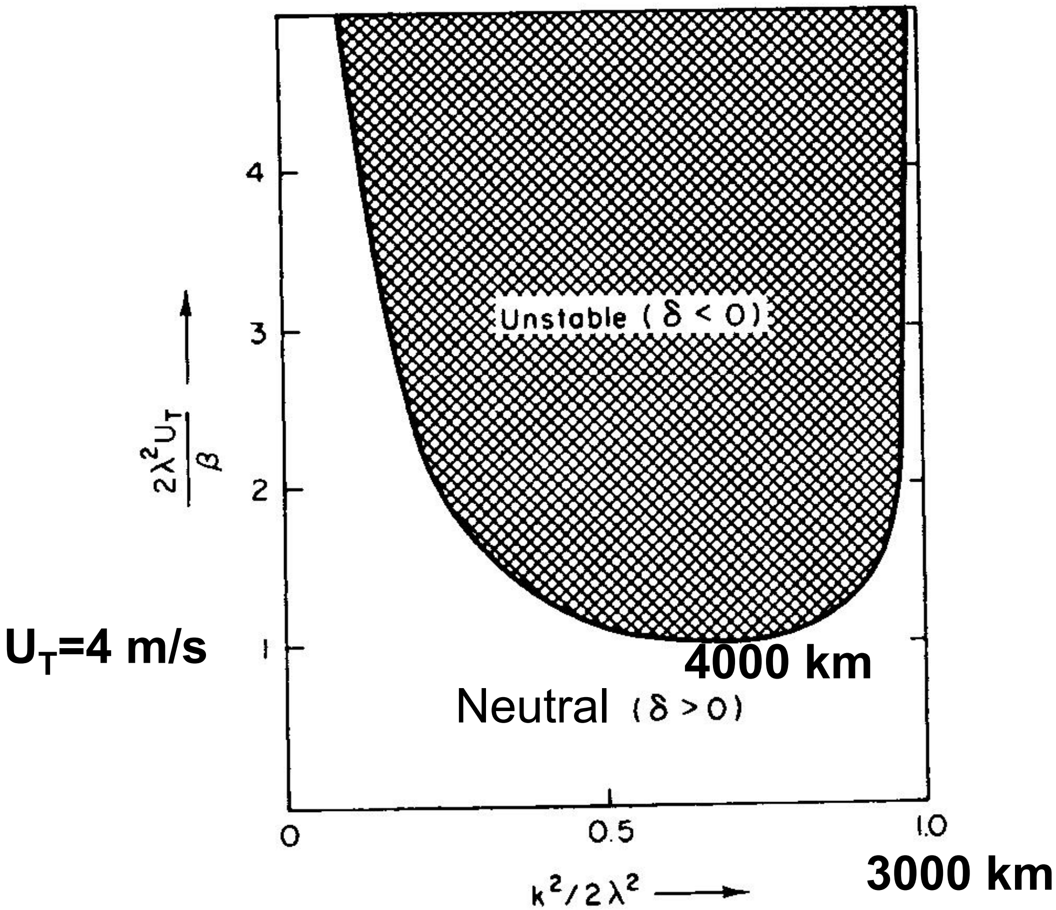
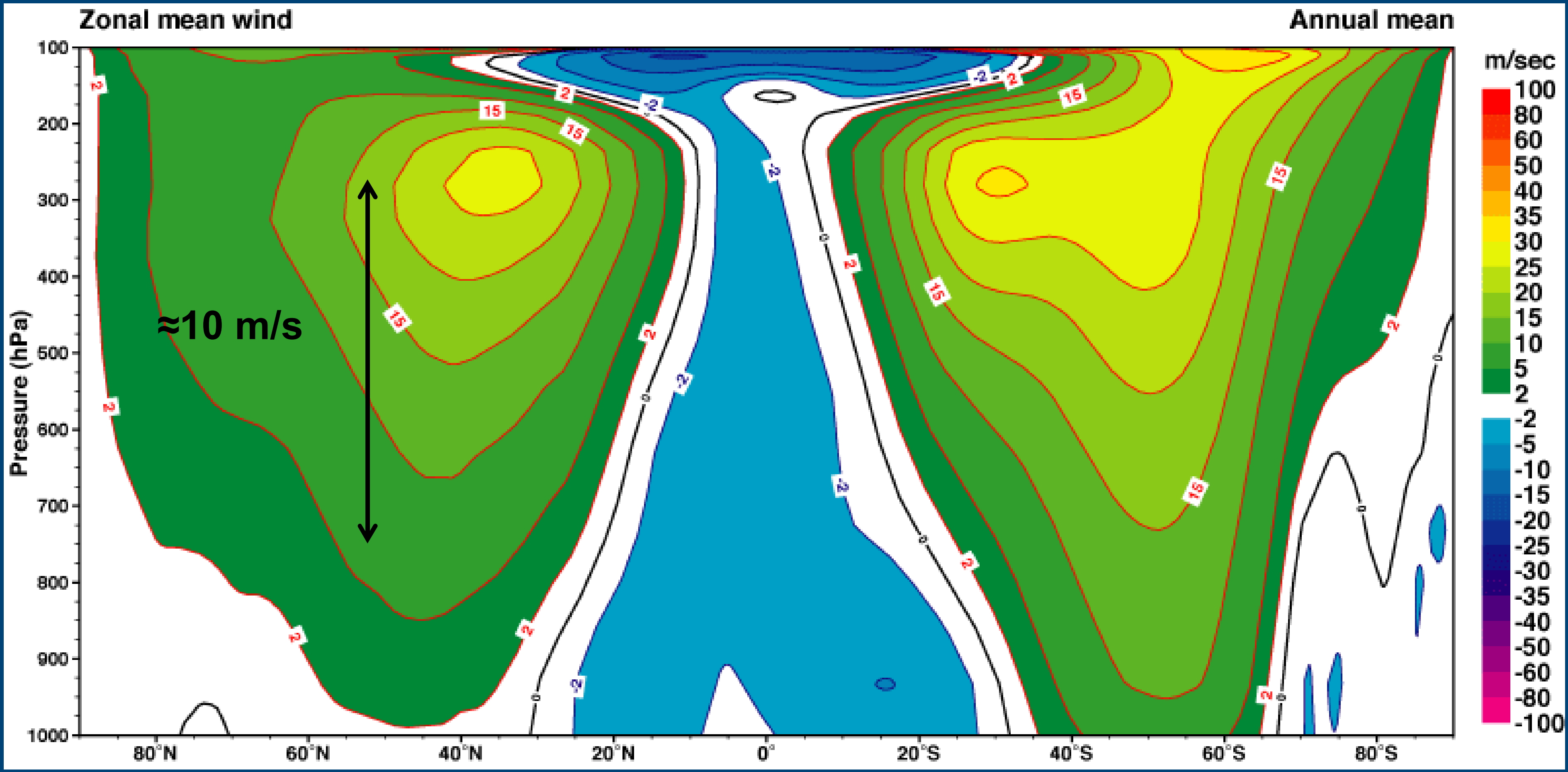
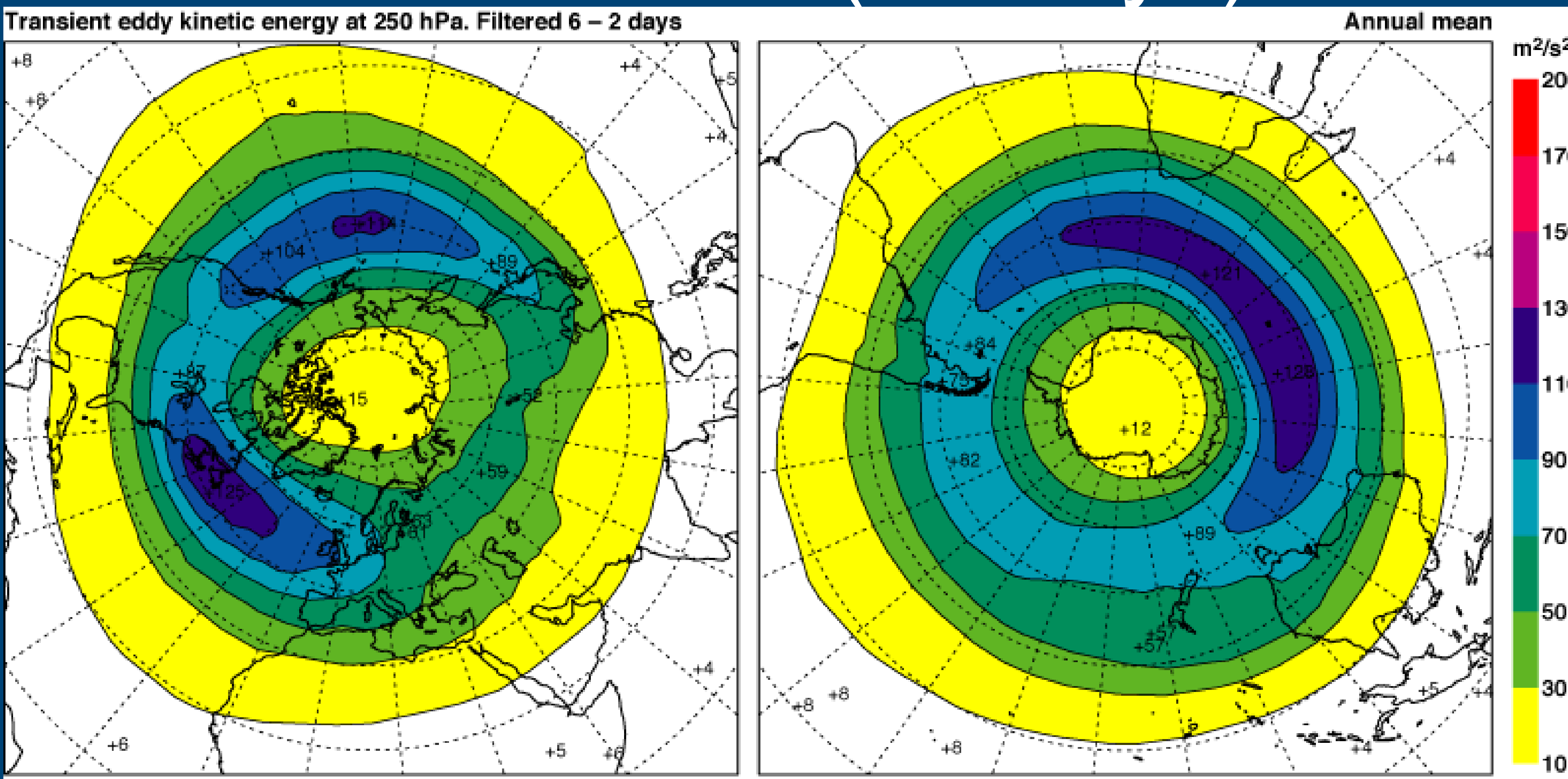


Fig. 8.3 Neutral stability curve for the two-level baroclinic model.

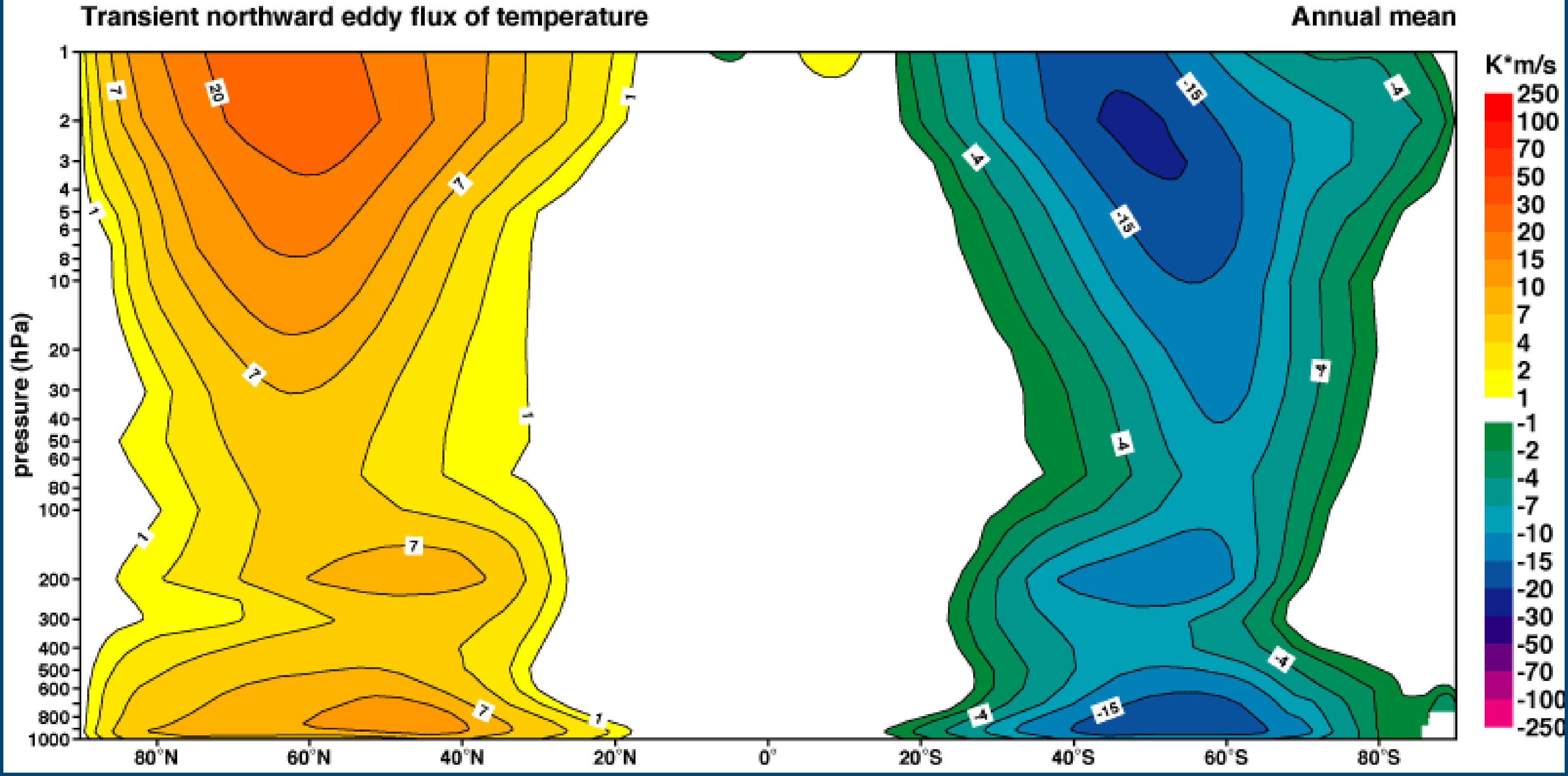
Zonal mean winds



Transient eddy kinetic energy at 250 hPa (2-6days)



Transient northward eddy flux of temperature



Stationary Rossby waves (Charney and Eliasen, 1949)

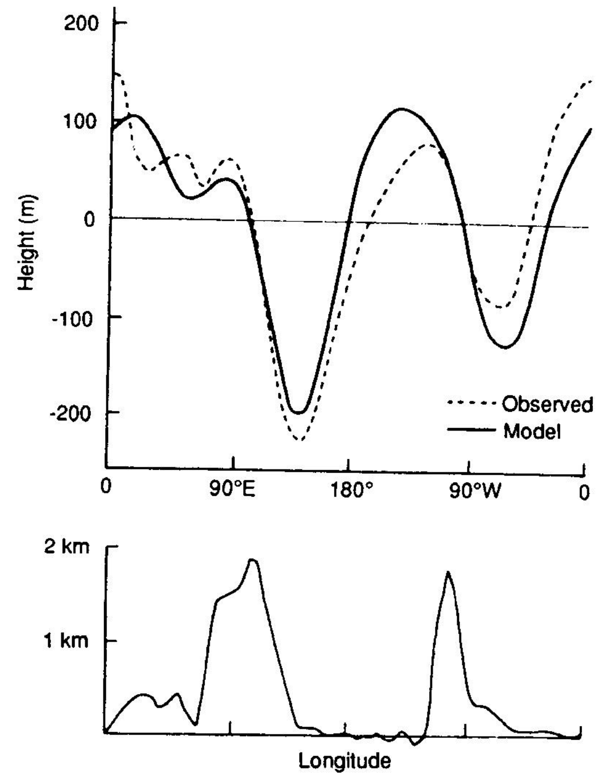
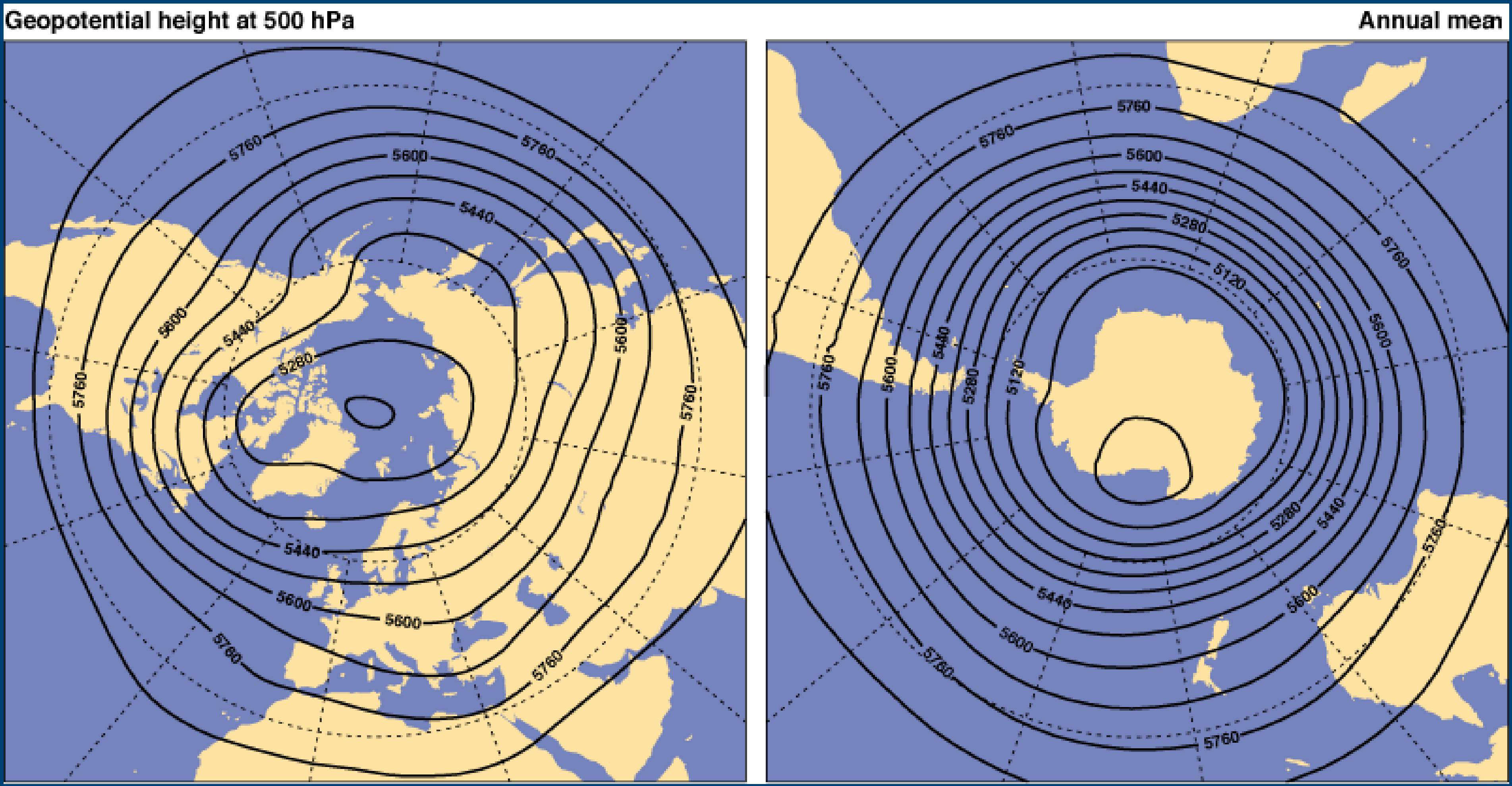
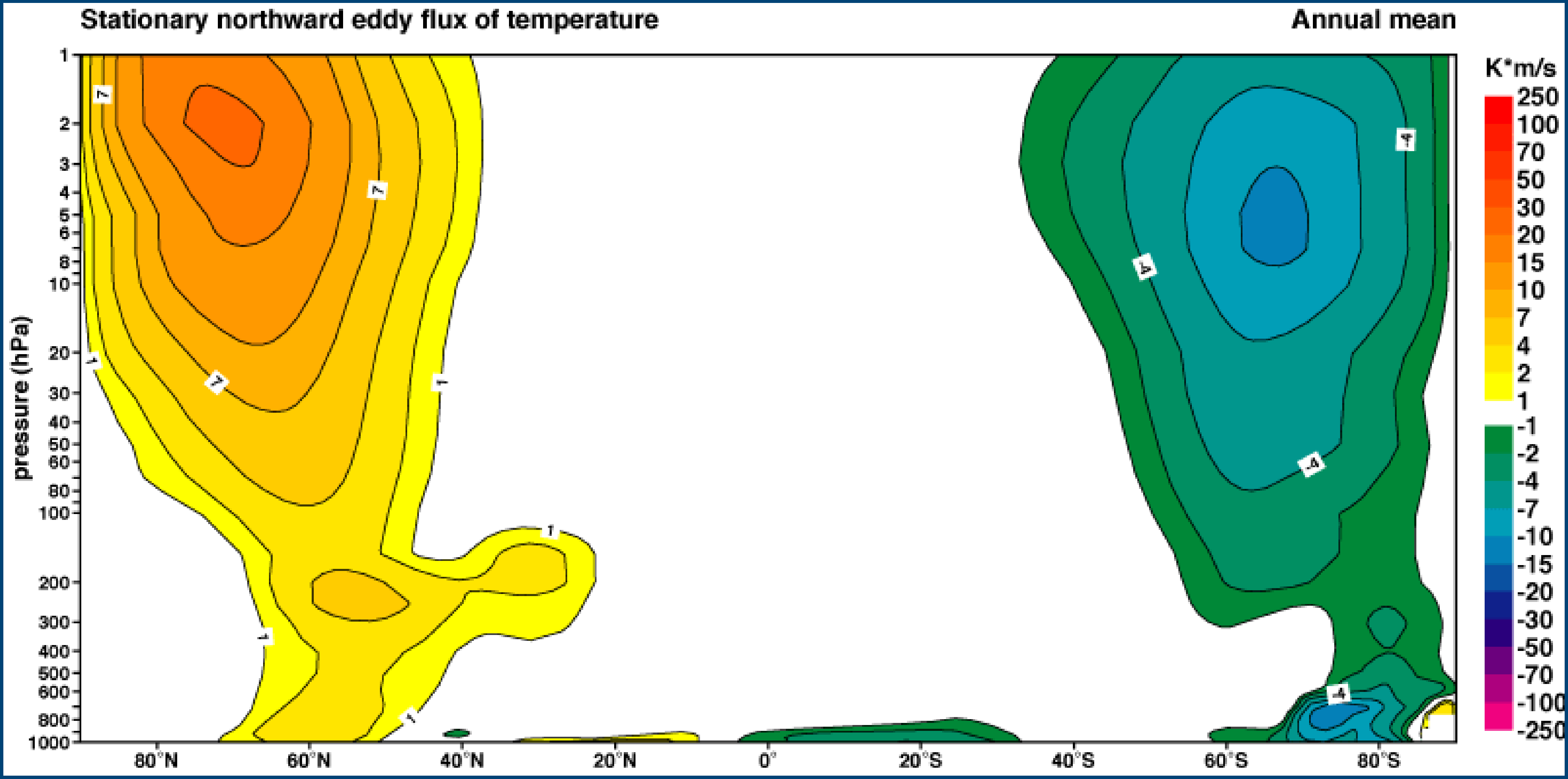


Fig. 7.15 (Top) Longitudinal variation of the disturbance geopotential height ($\equiv f_0\Psi/g$) in the Charney–Eliassen model for the parameters given in the text (solid line) compared with the observed 500-hPa height perturbations at 45°N in January (dashed line). (Bottom) Smoothed profile of topography at 45°N used in the computation. (After Held, 1983.)

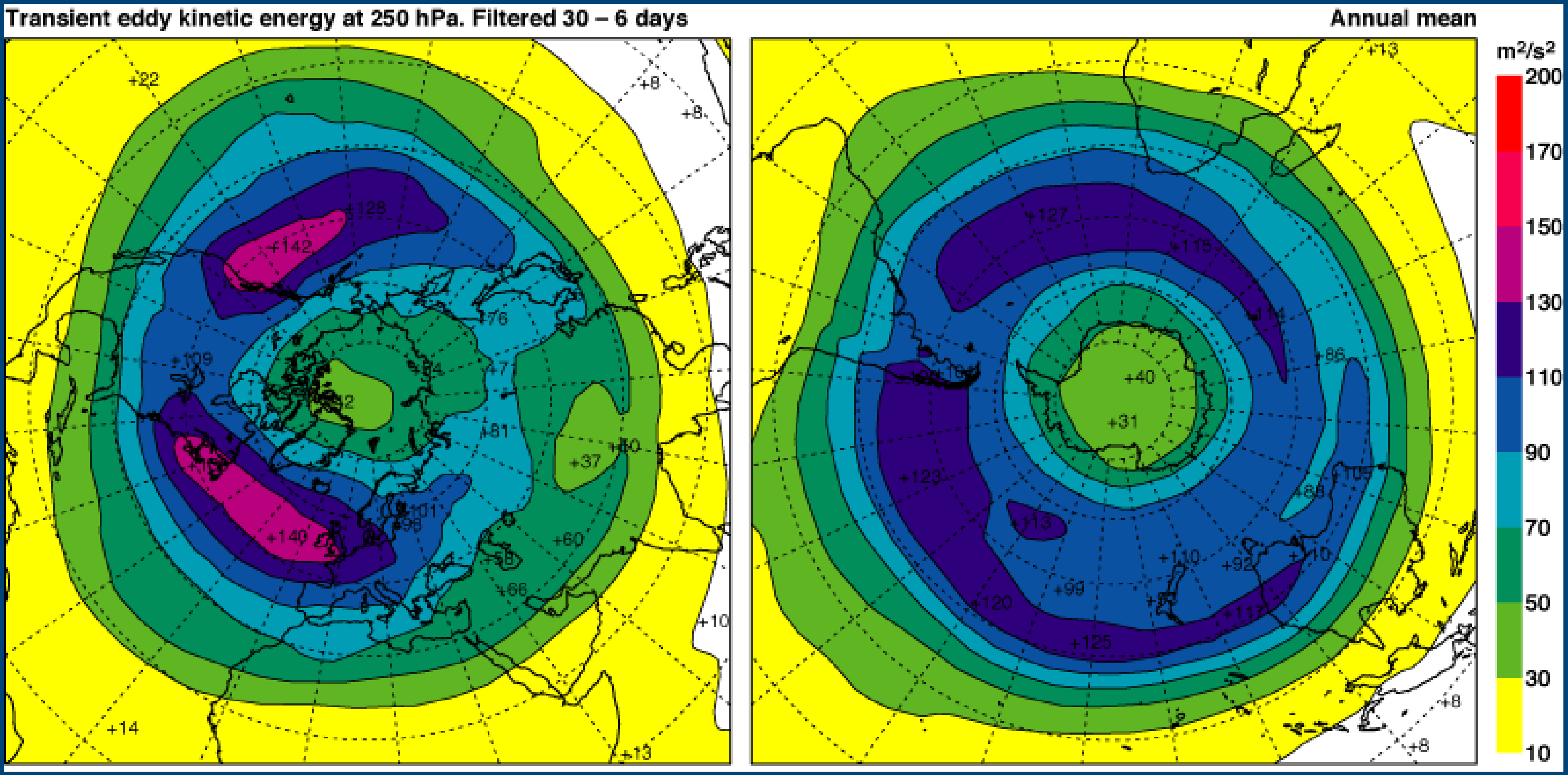
Annual mean 500 hPa geopotential height



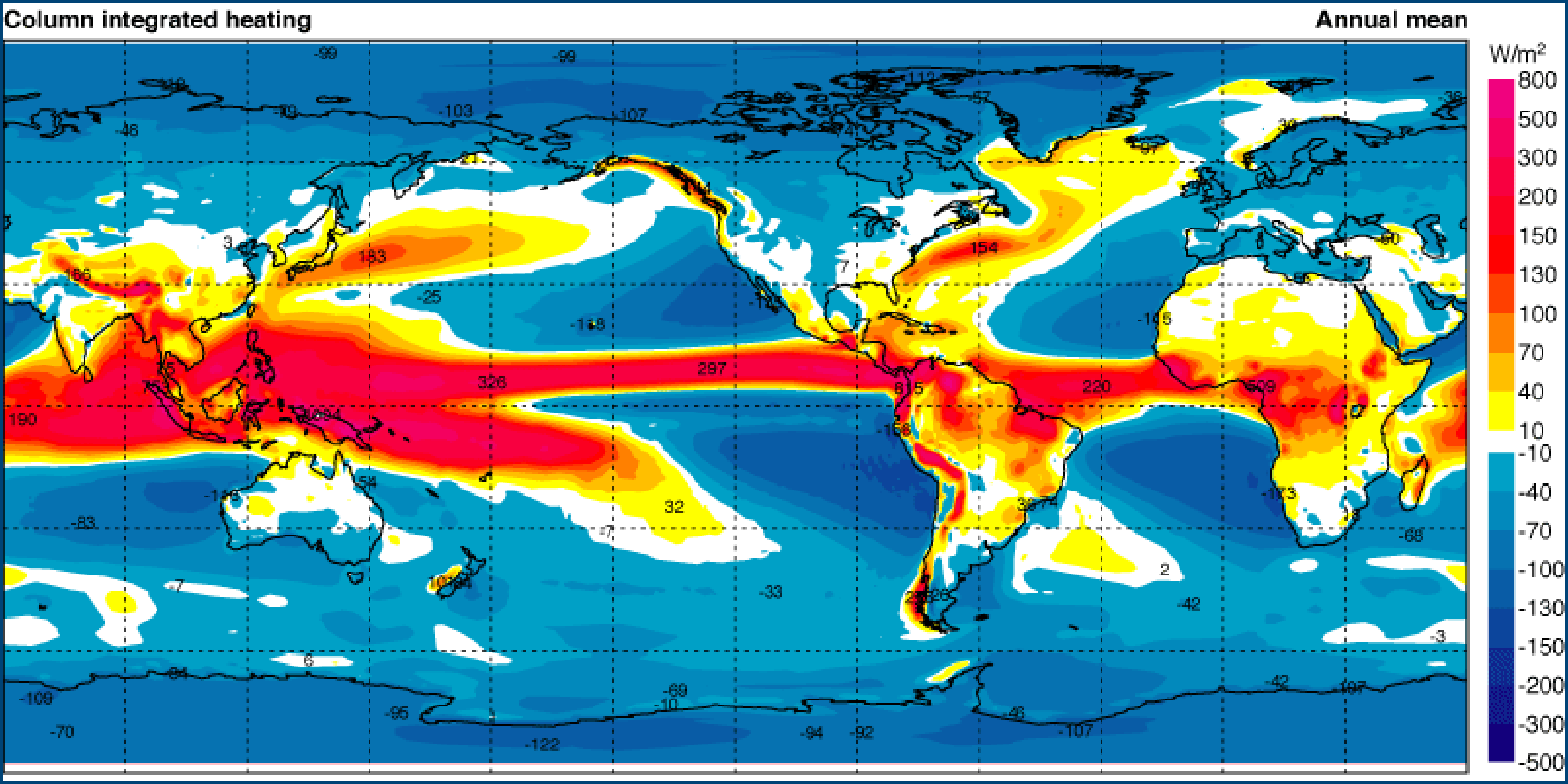
Stationary northward eddy flux of temperature



Low frequency eddy kinetic energy at 250 hPa (6-30 days)



Column integrated heating



- Sources of information:
  - ERA40 Atlas (available on ECMWF website)
  - Holton, J.R.: An introduction to dynamic meteorology. Academic Press, 4th edition, pp 535.
  - Held, I. and A. Hou, 1980: Nonlinear axially symmetric circulations in a nearly inviscid atmosphere. J. Atmos. Sci., 37, 515-533.