

# Numerical Weather Prediction

## Parametrization of diabatic processes

### Convection I: General circulation and concepts

Peter Bechtold

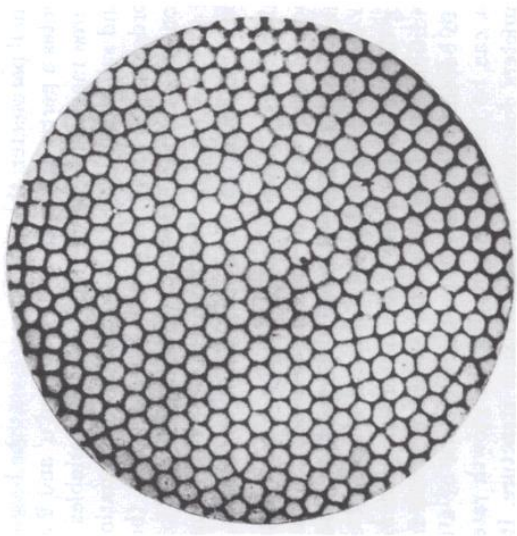


# Convection Parametrisation and Dynamics - Text Books

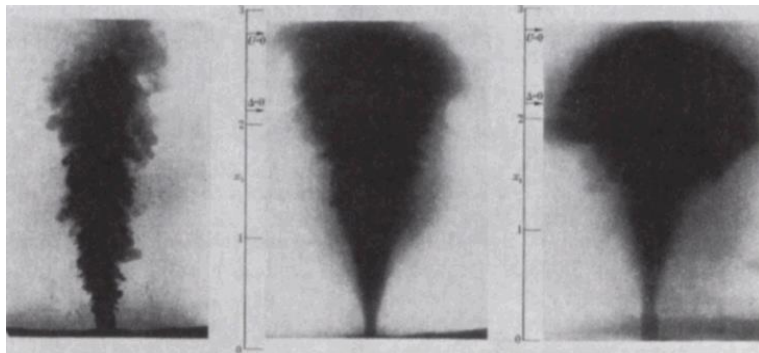
- Yano&Plant (Editors), 2015: Parameterization of atmospheric convection. *World scientific, Imperial College Press*
- Emanuel, 1994: Atmospheric convection, *OUP*
- Houze R., 1993: Cloud dynamics, *AP*
- Holton, 2004: An introduction to Dynamic Meteorology, *AP*
- Bluestein, 1993: Synoptic-Dynamic meteorology in midlatitudes, Vol II. *OUP*
- Peixoto and Ort, 1992: The physics of climate. *American Institute of Physics*
- Emanuel and Raymond, 1993: The representation of cumulus convection in numerical models. *AMS Meteor. Monogr.*
- Smith, 1997: The physics and parametrization of moist atmospheric convection. *Kluwer*
- Dufour et v. Mieghem: Thermodynamique de l'Atmosphère, 1975: *Institut Royal météorologique de Belgique*
- Anbaum, 2010: Thermal Physics of the atmosphere. *J Wiley Publishers*

*AP=Academic Press; OUP=Oxford University Press*

# Convection=heat the bottom&cool the top



Rayleigh-Benard cellular convection



Classic plume experiment



Pre-frontal deep convection July 2010 near Baden-Baden Germany



# Outline

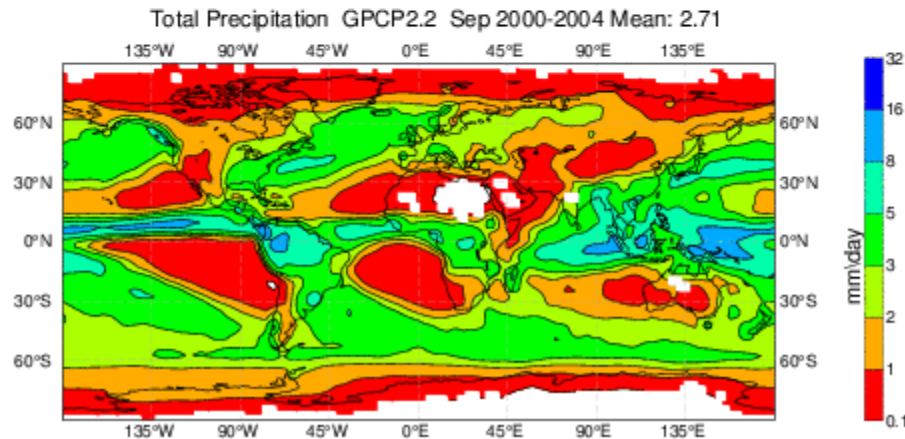
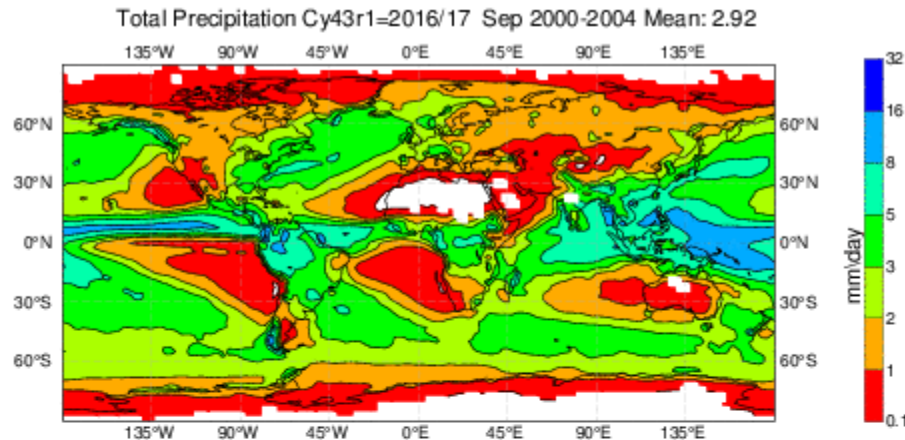
## General:

- Convection and tropical circulations
- Tropical waves
- Middle latitude Convection

## Useful concepts and tools:

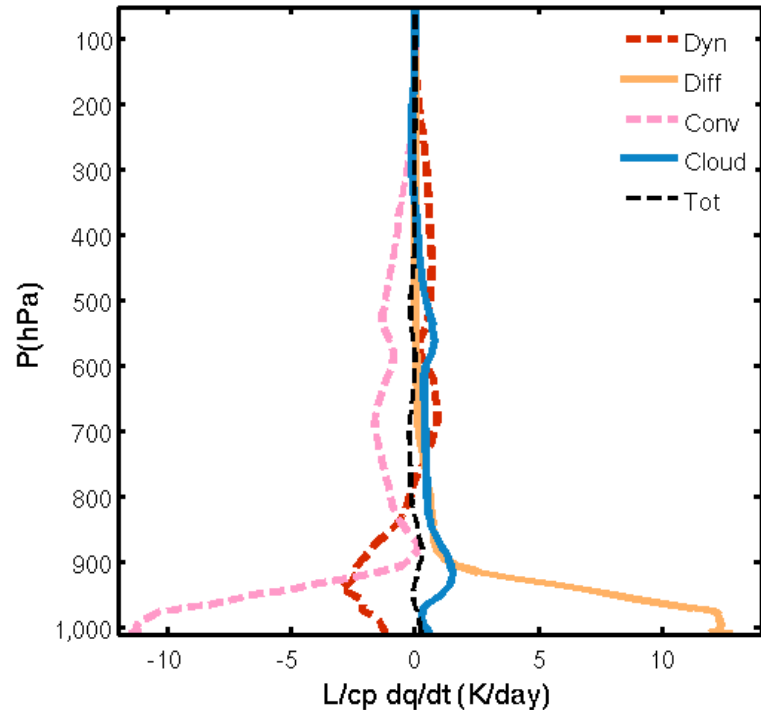
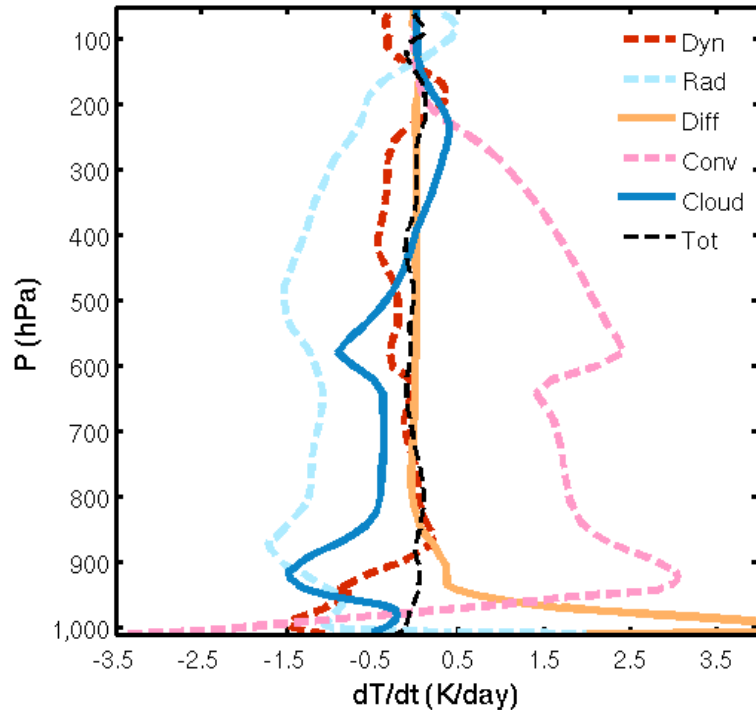
- Buoyancy
- Convective Available Potential Energy
- Soundings and thermodynamic diagrams
- Convective quasi-equilibrium
- Large-scale observational budgets

# It's raining again... 2000-2003 annual precipitation rate from IFS Cy43r1 (2016) GPCP2.2 dataset



about 2.7-2.8 mm/day is falling globally, but most i.e. 5-7 mm/day in the Tropics

# Model Tendencies – Tropical Equilibria

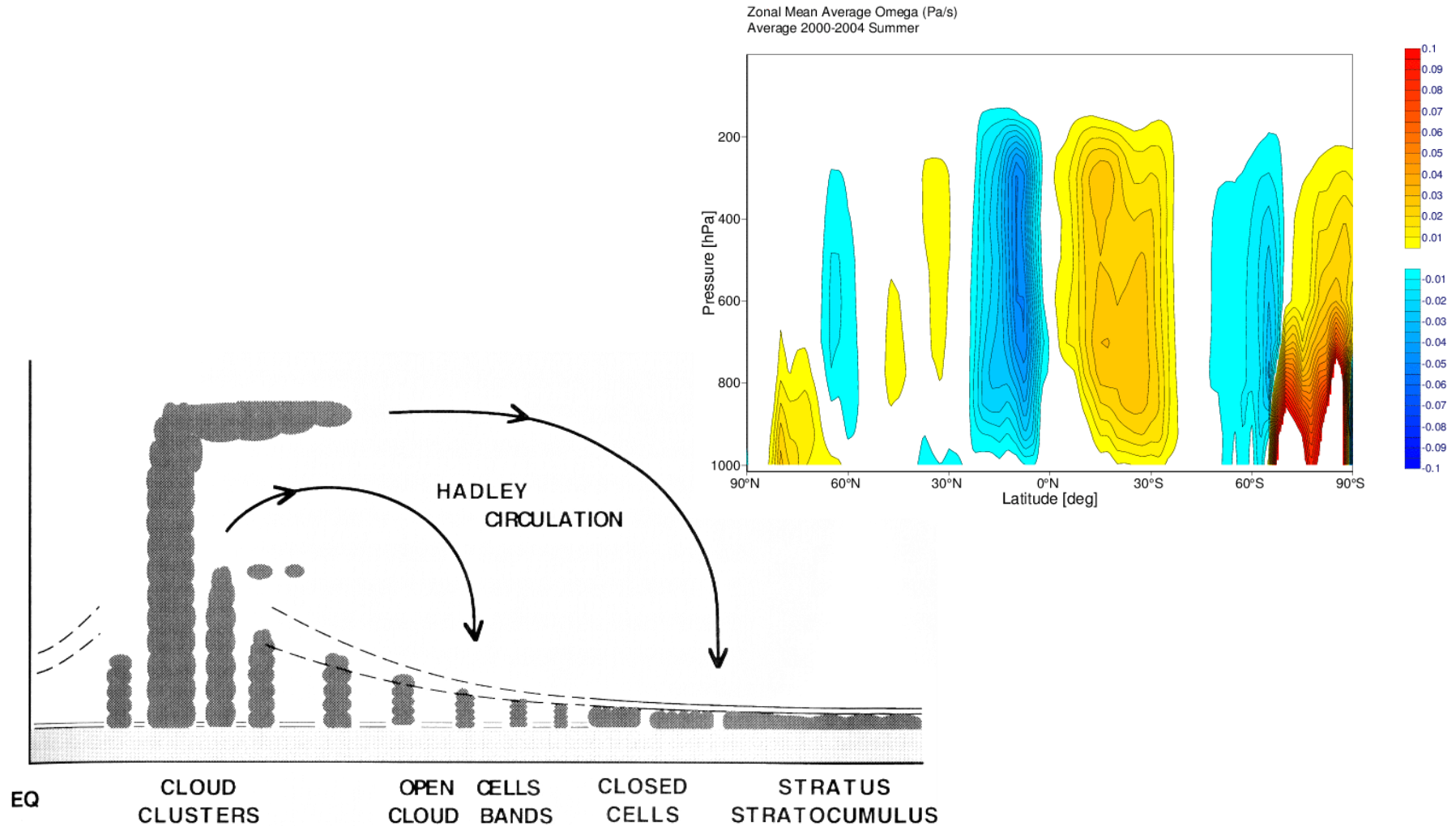


Above the boundary layer, for Temperature there is on average radiative-convective equilibrium; and convective-dynamic equilibrium over the large-scale disturbance, whereas for moisture there is roughly an equilibrium between dynamical transport (moistening) and convective drying. - *Global Budgets are very similar*

The driving force for atmospheric convection is the radiation

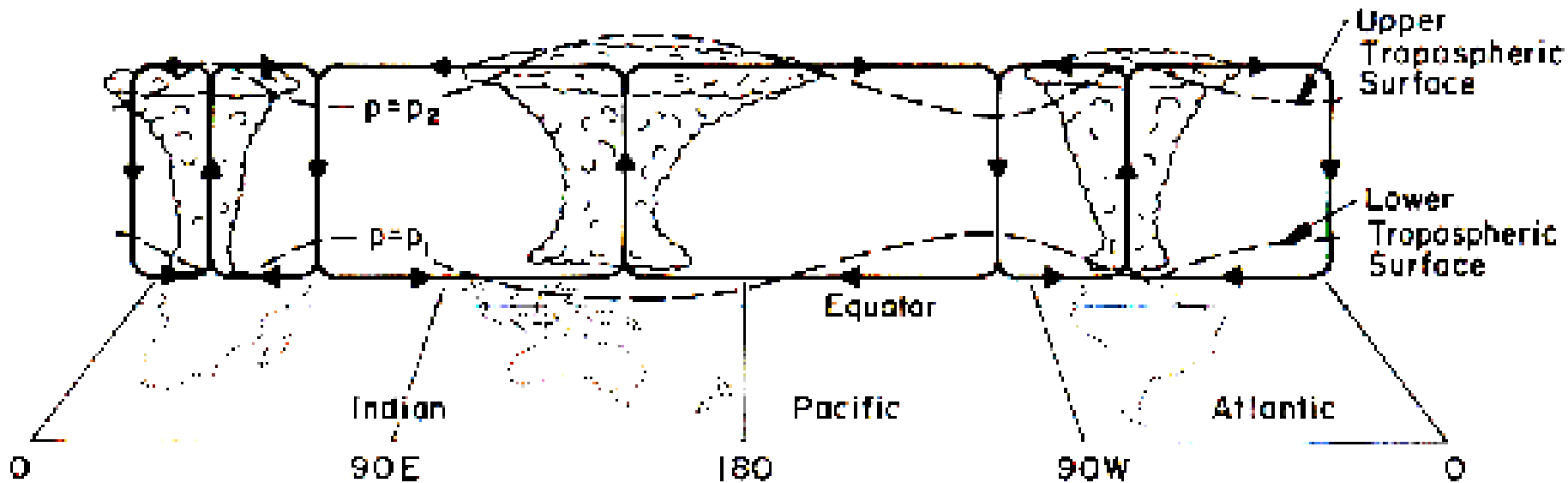
# Convection and tropical circulations (1)

## The ITCZ and Hadley meridional circulation



# Convection and tropical circulations (2)

## The Walker zonal Circulation and SST coupling

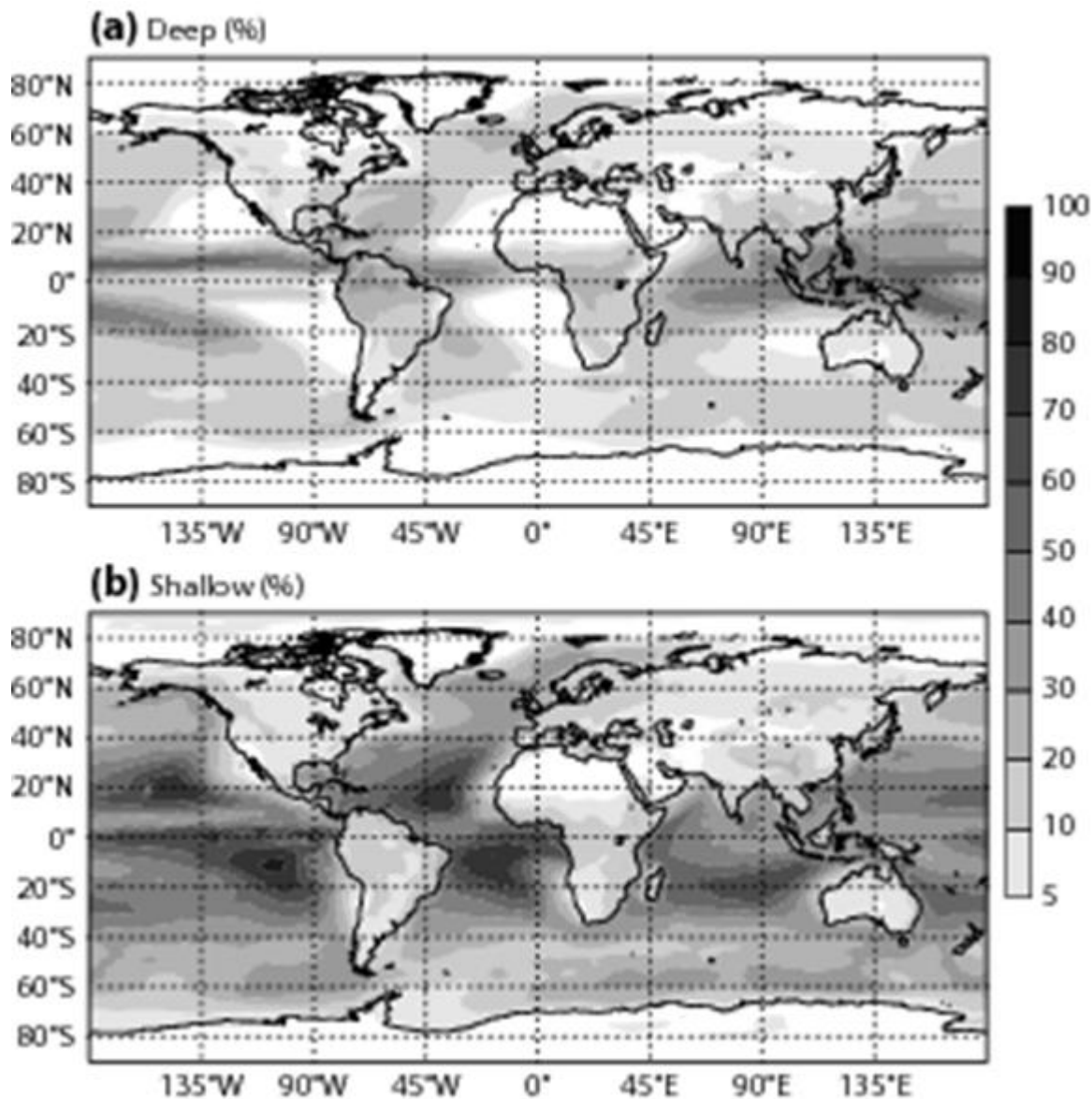


From Salby (1996)

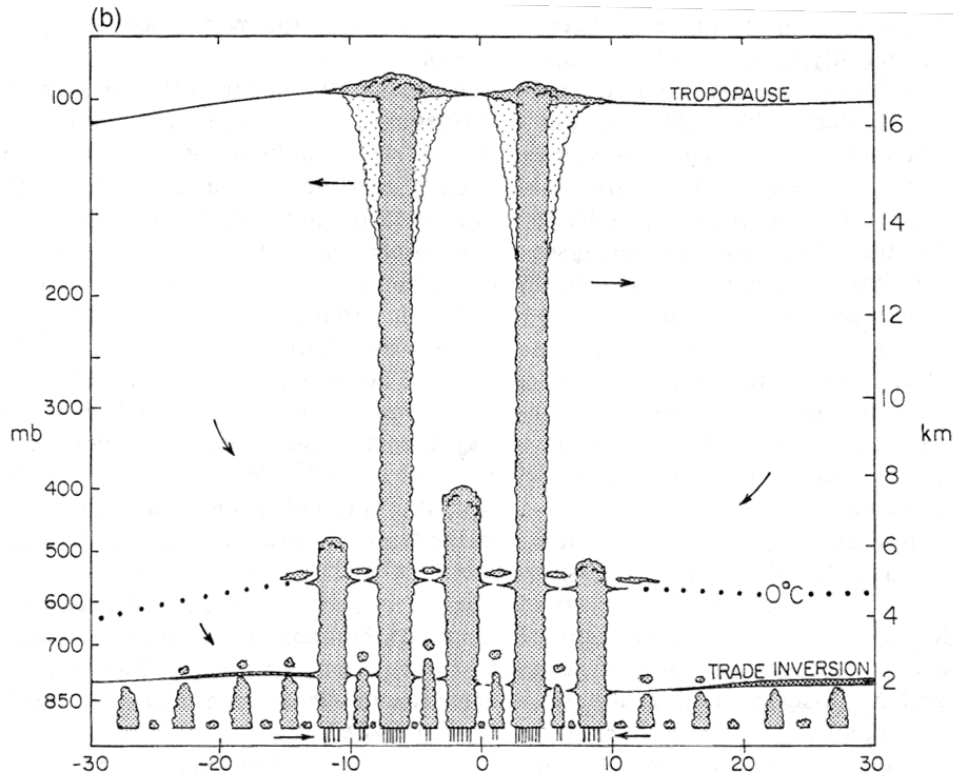


# Distribution of deep and shallow IFS cy40r1 (2014)

Deep type including congestus



# Vertical distribution of convective clouds



Johnson et al., 1999, JCL

Tri-modal distribution: Shallow cumulus, Congestus attaining the melting level, Deep penetrating convection

# Summary: the weather and thermal equilibria

- Suppose we have a series of nice clear sky anticyclonic days, then above the boundary-layer

$$\frac{d\theta}{dt} \approx 0 \Rightarrow w \frac{d\theta}{dz} = \left. \frac{d\theta}{dt} \right|_{rad} = -\frac{2K}{86400s} \Rightarrow w \sim -0.5 \text{ cm/s subsidence}$$

$\sim 0.5 \text{ K/100 m}$

- But what happens if we have a thunderstorm day with Pr=100 mm/day

$$\frac{c_p}{g} \int_{P_{top}=200hPa}^{P_{surf}=1000hPa} \frac{\partial T}{\partial t} dp = L_v \rho_{water} Pr(m/s)$$

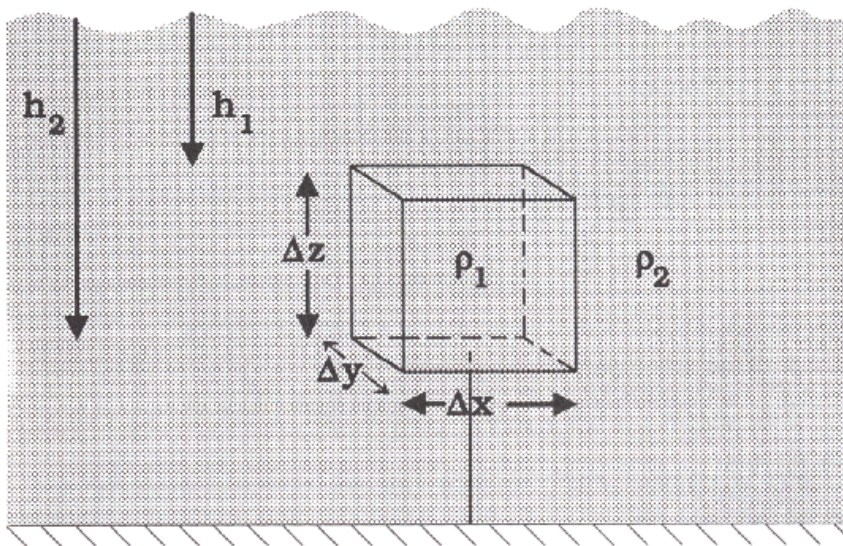
$$c_p = 1004 \text{ Jkg}^{-1} \text{ K}^{-1} \quad \rho_{water} = 1000 \text{ kgm}^{-3} \quad L_v = 2.5 \times 10^6 \text{ Jkg}^{-1}$$

$$g = 9.81 \text{ ms}^{-2} \quad Pr = 100 \text{ mm/day} = 1.16 \times 10^{-6} \text{ ms}^{-1}$$

100 mm/day precipitation heats the atmospheric column by 2893 W/m<sup>2</sup> or by 30 K/day on average. This heating must be compensated by uplifting of  $w \sim 10 \text{ cm/s}$  → heavy precip/convection requires large-scale perturbations.

# Buoyancy (1)- Archimedes said 'Eureka!'

Body in a fluid



Assume fluid to be in hydrostatic equilibrium  $\frac{dp_2}{dz} = -\rho_2 g$

$$\rho_2 = \text{const.} \longrightarrow p_2 = \rho_2 g h$$

Forces:

Top  $F_{top} = -\rho_2 g h_1 \Delta x \Delta y$

Bottom  $F_{bot} = \rho_2 g h_2 \Delta x \Delta y$

Gravity  $F_{grav} = -\rho_1 g \Delta x \Delta y \Delta z$

Net Force:  $F = F_{top} + F_{bot} + F_{grav} = \rho_2 g (h_2 - h_1) \Delta x \Delta y - \rho_1 g \Delta x \Delta y \Delta z = g (\rho_2 - \rho_1) \Delta x \Delta y \Delta z$

Acceleration:  $A = \frac{F}{M_{body}} = \frac{F}{\rho_1 \Delta x \Delta y \Delta z} = g \frac{(\rho_2 - \rho_1)}{\rho_1}$

Emanuel, 1994



## Buoyancy (2)

Vertical momentum equation:

$$\frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g$$

$$p = \bar{p} + p' \quad \rho = \bar{\rho} + \rho' \quad \frac{\partial \bar{p}}{\partial z} = -\bar{\rho}g$$

$$\frac{dw}{dt} = -\frac{1}{\bar{\rho} + \rho'} \frac{\partial(\bar{p} + p')}{\partial z} - g$$

$$\frac{1}{\bar{\rho} + \rho'} = \frac{1}{\bar{\rho}} \left( \frac{1}{1 + \rho'/\bar{\rho}} \right) = \frac{1}{\bar{\rho}} \left[ 1 - \frac{\rho'}{\bar{\rho}} + \left( \frac{\rho'}{\bar{\rho}} \right)^2 + \dots \right]$$

$$\rho' \ll \bar{\rho} \quad \longrightarrow \quad \text{Neglect second order terms}$$

## Buoyancy (3)

$$\frac{dw}{dt} = -\frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial z} - \frac{1}{\bar{\rho}} \frac{\partial p'}{\partial z} - g + \frac{\rho'}{\bar{\rho}} \frac{1}{\bar{\rho}} \frac{\partial \bar{p}}{\partial z} + \frac{\rho'}{\bar{\rho}} \frac{1}{\bar{\rho}} \frac{\partial p'}{\partial z}$$

$\parallel$   $\parallel$   
 $g$   $-g$

$$\frac{dw}{dt} = -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial z} - \frac{\rho'}{\bar{\rho}} g$$

$\parallel$

B - buoyancy acceleration

# Buoyancy (4) T and P and Contributions

Buoyancy

$$B = -\frac{\rho'}{\bar{\rho}} g$$

Dry air:

$$\rho = \frac{p}{RT} \rightarrow \rho' = \frac{p'}{RT} - \frac{\bar{p}T'}{RT^2} \rightarrow \frac{\rho'}{\bar{\rho}} = \frac{p'}{\bar{p}} - \frac{T'}{\bar{T}}$$

$$\frac{p'}{\bar{p}} \ll \frac{T'}{\bar{T}} \text{ and } B \approx g \frac{T'}{\bar{T}}$$



$$\frac{dw}{dt} \approx g \frac{T'}{\bar{T}} - \frac{1}{\bar{\rho}} \frac{\partial p'}{\partial z}$$

$T' > 0$  (warm parcel)  $\Rightarrow$  upward acceleration

## Buoyancy (5) moist atmosphere

effects of humidity and condensate need to be taken into account

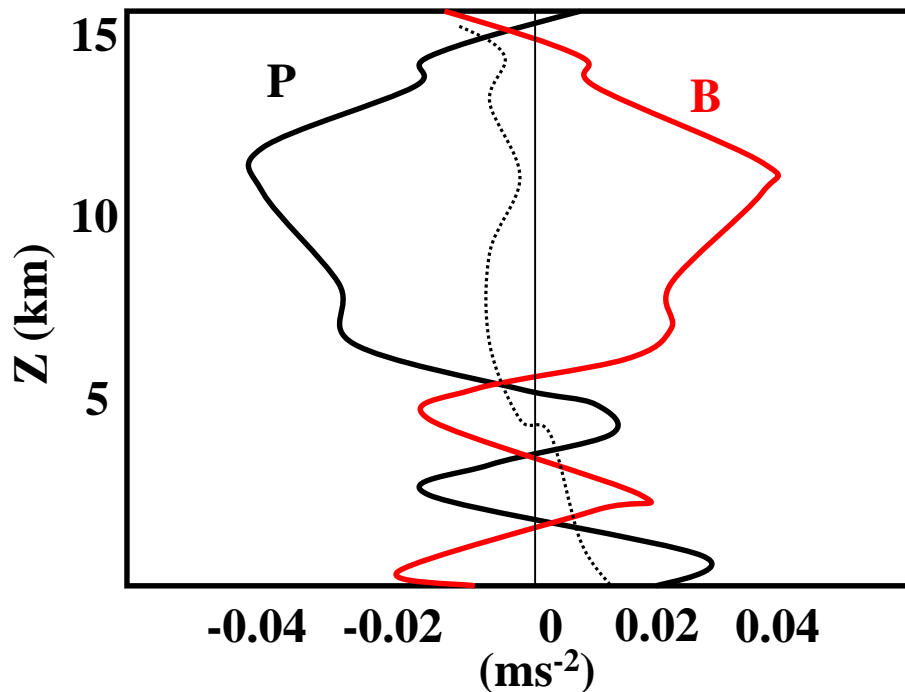
$$B = -g \frac{\rho'}{\bar{\rho}} \approx -g \left( \frac{T'}{\bar{T}} + 0.608q' - q_l \right)$$

In general **all 3 terms** are **important**. 1 K perturbation in T is equivalent to 5 g/kg perturbation in water vapor or 3 g/kg in condensate



# Non-hydrostat. Pressure gradient effects

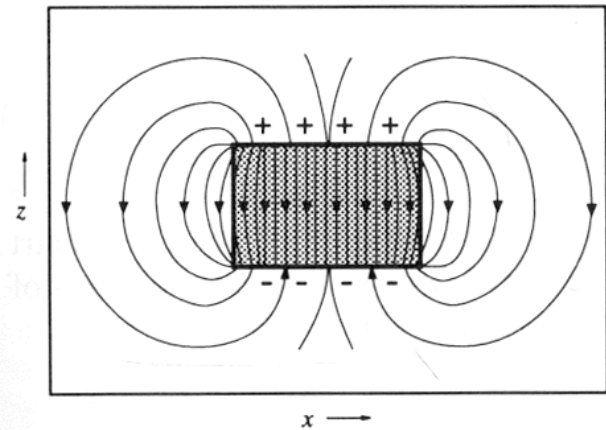
$$\frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p'}{\partial z} - \frac{\rho'}{\bar{\rho}} g$$



## CRM analysis of the terms

by Guichard and Gregory

Physics:



Vector field of the buoyancy pressure-gradient force for a uniformly buoyant parcel of finite dimensions in the x-z-plane. (Houze, 1993, Textbook)

# Convective Available Potential Energy (CAPE) and vertical kinetic energy

$$CAPE = \int \vec{F} \cdot d\vec{l} = \int_{base}^{top} B dz$$

$$CAPE \approx \int_{base}^{top} g \frac{T_{cld} - T_{env}}{T_{env}} dz$$

$$\frac{dw}{dt} = w \frac{dw}{dz} = \frac{1}{2} \frac{dw^2}{dz} \approx g \frac{T'}{\bar{T}}$$

$$w^2(z) = 2 \int_0^z g \frac{T'}{\bar{T}} dz = 2 \cdot CAPE$$

$$w = \sqrt{2 \cdot CAPE}$$

CAPE represents the amount of potential energy of a parcel lifted to its level of neutral buoyancy. This energy can potentially be released as kinetic energy in convection.

Example:

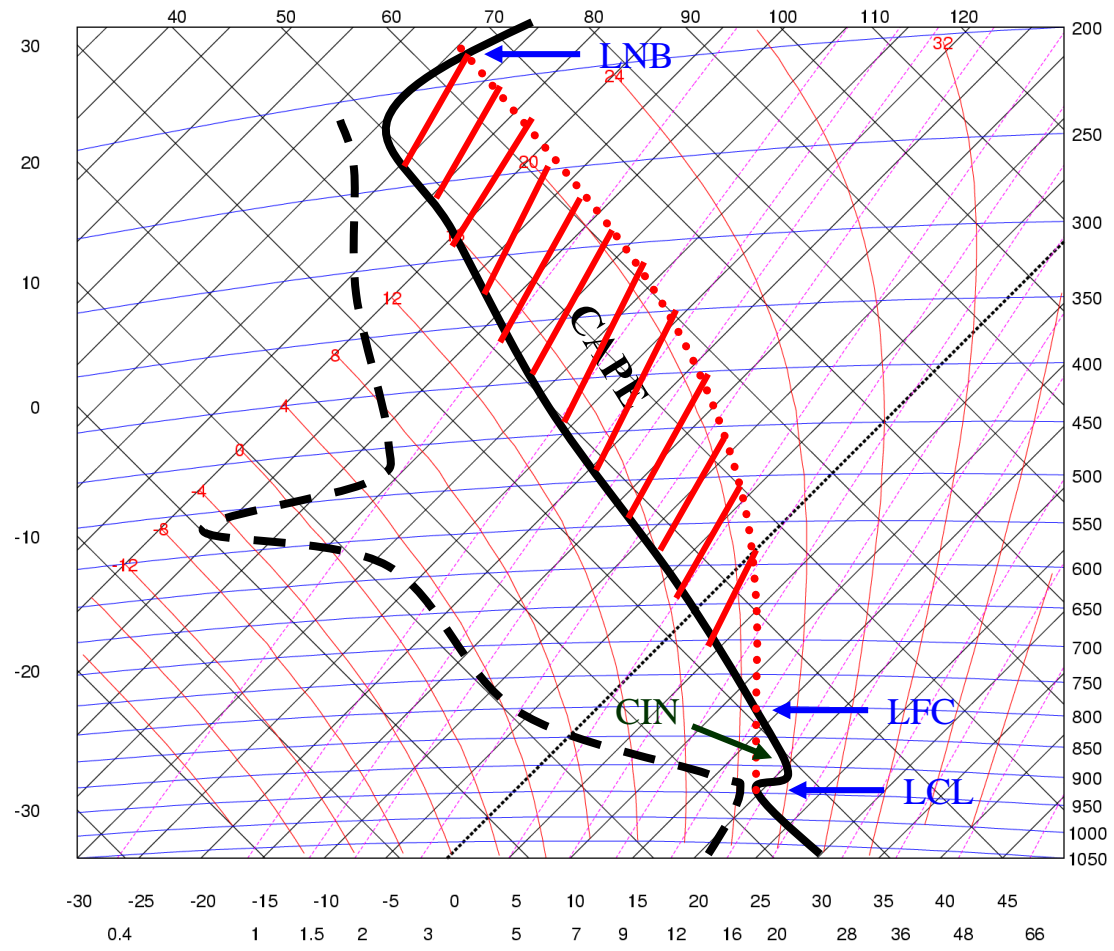
$T' = 5 \text{ K}$ ,  $T = 250 \text{ K}$ , cloud depth = 10 km

$$w \approx 60 \text{ m s}^{-1}$$

Much larger than observed - what's going on ?

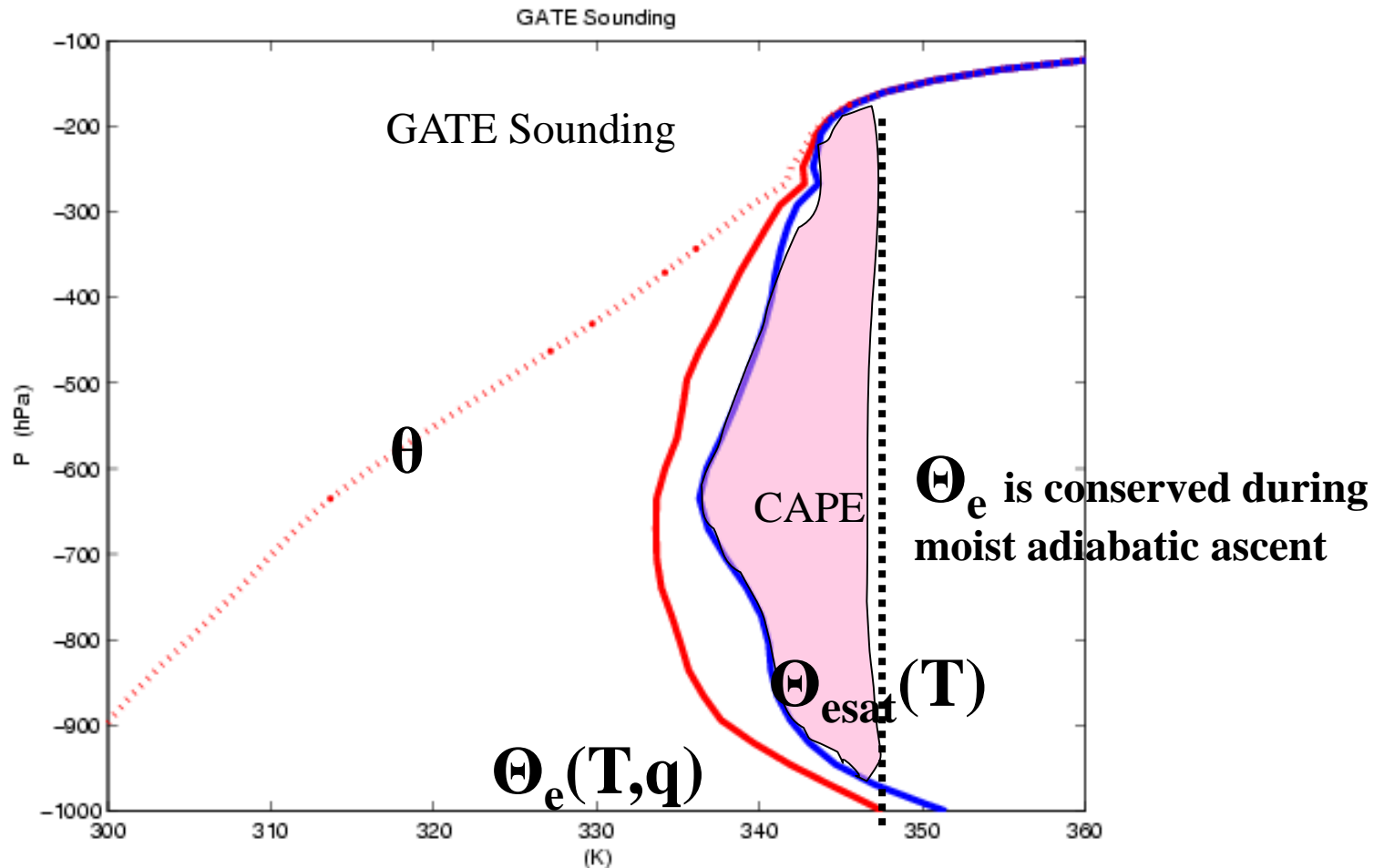
# Convection in thermodynamic diagrams (1)

## using Tephigram/Emagram



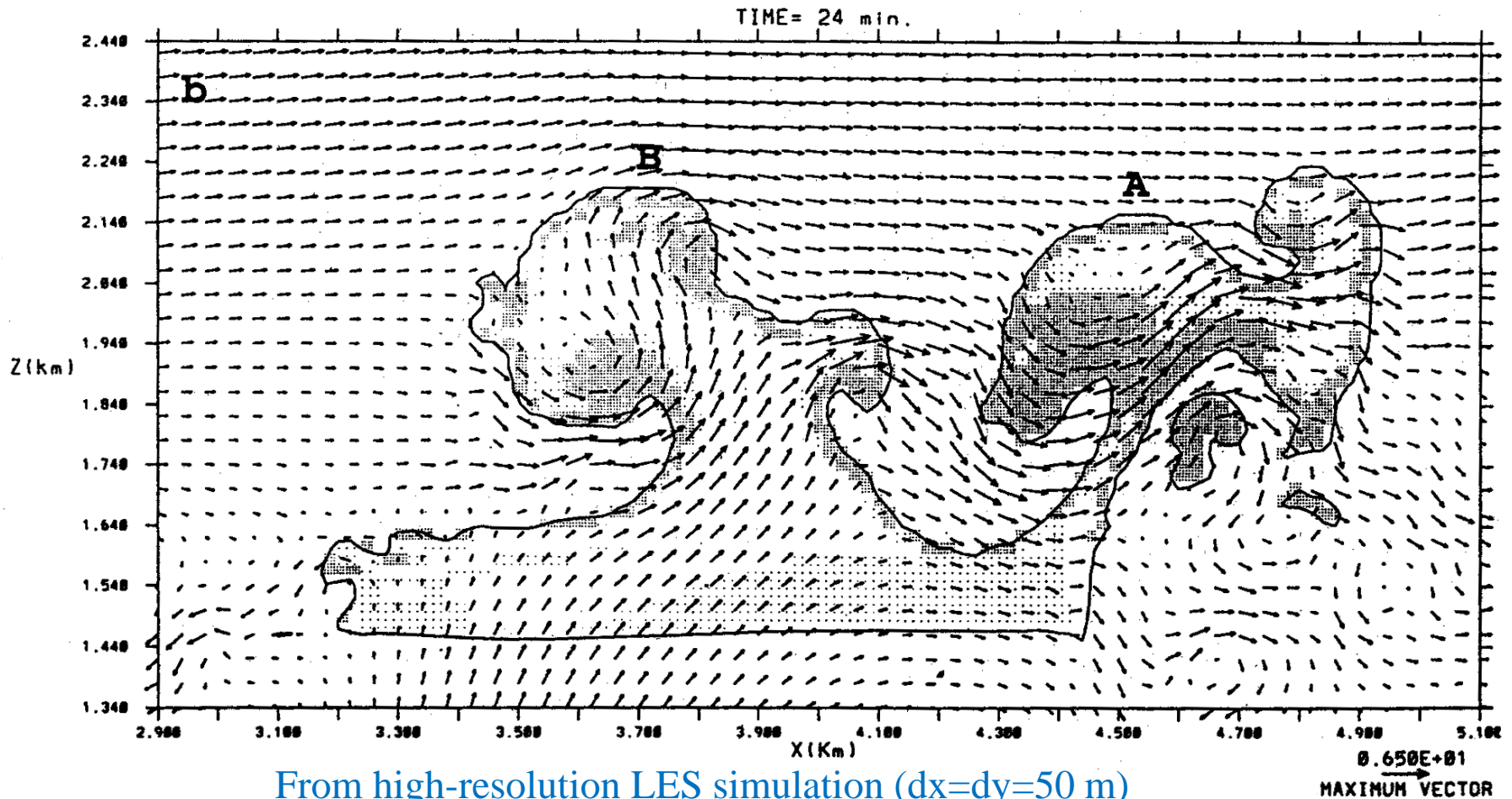
Idealised Profile

# Convection in thermodynamic diagrams (2) using equivalent Potential Temperatures



Note that no CAPE is available for parcels ascending above 900 hPa and that the tropical atmosphere is stable above 600 hPa ( $\theta_e$  increases) – downdrafts often originate at the minimum level of  $\theta_e$  in the mid-troposphere.

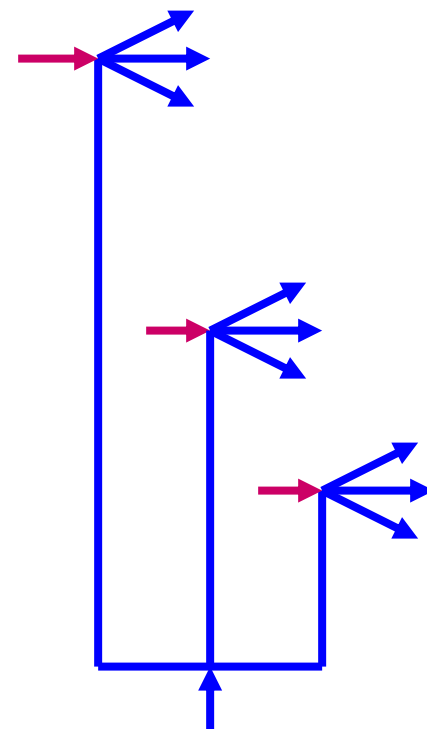
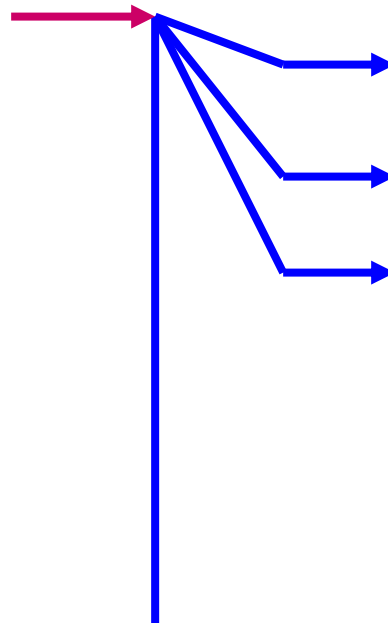
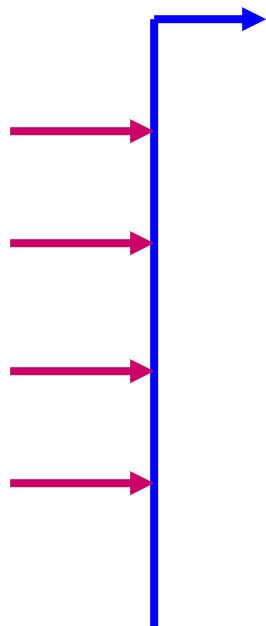
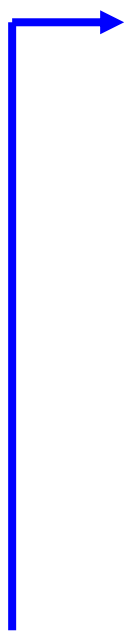
# Mixing and 3D flow subcloud and cloud-layer Circulations



From high-resolution LES simulation ( $dx=dy=50$  m)  
Vaillancourt, You, Grabowski, JAS 1997

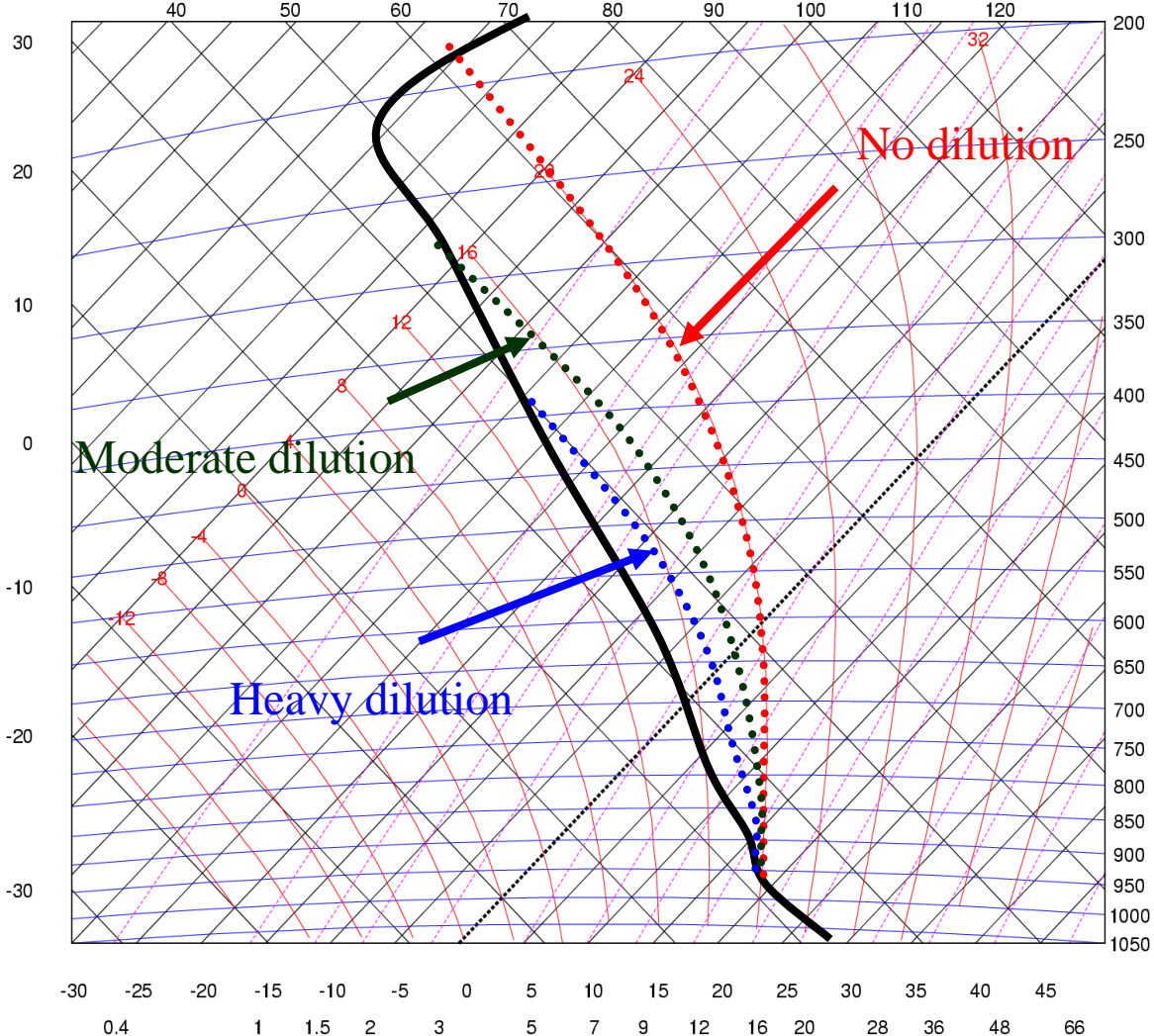
# Mixing models

undiluted entraining plume cloud top entrainment stochastic mixing



after Raymond, 1993

# Effect of mixing on parcel ascent



# Large-scale effects of convection (1)

## $Q_1$ and $Q_2$

Thermodynamic equation (dry static energy) :

$$\frac{\partial s}{\partial t} + \nabla \vec{v}_h s + \frac{\partial \omega s}{\partial p} = Q_R + L(c - e)$$

Why use  $s$  or  $\theta$ , not  $T$  ?

$$s = c_p T + gz$$

$$ds/dz = C_p dT/dz + g$$

If  $dT/dz = -g/c_p$  (dry adiabatic lapse rate), then  $ds = d\theta = 0$

Define averaging operator over area  $A$  such that:

$$\bar{\Phi} = \frac{1}{A} \int_A \Phi dA \quad \text{and} \quad \Phi = \bar{\Phi} + \Phi'$$

Apply to thermodynamic equation, neglect horizontal second order terms, use averaged continuity equation:

$$\underbrace{\frac{\partial \bar{s}}{\partial t} + \bar{\vec{v}}_h \nabla \bar{s} + \bar{\omega} \frac{\partial \bar{s}}{\partial p}}_{\text{“large-scale observable” terms}} = \bar{Q}_R + \underbrace{L(\bar{c} - \bar{e}) - \frac{\partial \bar{\omega}'s'}{\partial p}}_{\text{“sub-grid” terms}}$$

In convective regions these terms will be dominated by convection



# Large-scale effects of convection $Q_1$ , $Q_2$ and $Q_3$

Define:  $Q_1 \equiv Q_R + L(\bar{c} - \bar{e}) - \frac{\partial \overline{\omega' s'}}{\partial p}$  Apparent heat source

Analogous:  $Q_2 \equiv L(\bar{c} - \bar{e}) + L \frac{\partial \overline{\omega' q'}}{\partial p}$  Apparent moisture sink

$\vec{Q}_3 \equiv \frac{\partial \overline{\omega' \vec{v}'_h}}{\partial p}$  Apparent momentum source

This quantity can be derived from observations of the “large-scale” terms on the l.h.s. of the area-averaged equations and describe the influence of the “sub-grid” processes on the atmosphere.

Note that:

$Q_1 - Q_2 - Q_R \equiv -\frac{\partial \overline{\omega' h'}}{\partial p}$  with  $h = s + Lq$  Moist static energy

# Large-scale effects of convection (2)

## vertical integrals of $Q_1$ and $Q_2$

$$\int_{P_t}^{P_s} Q_1 \frac{dp}{g} \equiv \int_{P_t}^{P_s} Q_R \frac{dp}{g} + L \text{Pr} + \rho C_p (\overline{w'T'})_{P=P_s} = \int_{P_t}^{P_s} Q_R \frac{dp}{g} + L \text{Pr} + HS$$

Surface Precipitation  
flux

Surface sensible  
Heat flux

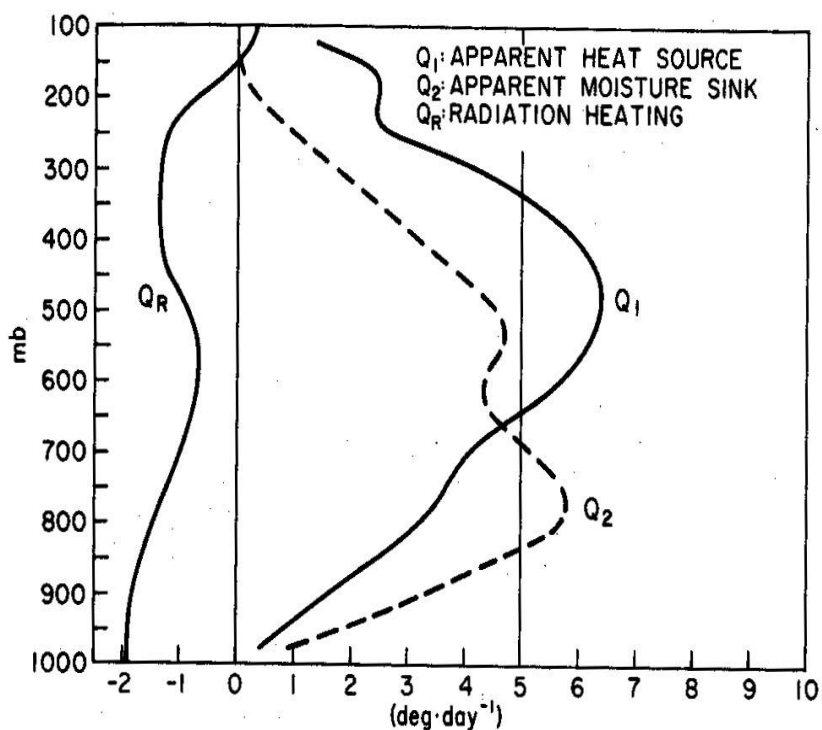
$$\int_{P_t}^{P_s} Q_2 \frac{dp}{g} \equiv L \text{Pr} - \rho L (\overline{w'q'})_{P=P_s} = L \text{Pr} - HL$$

Surface Precipitation

Surface latent  
Heat flux

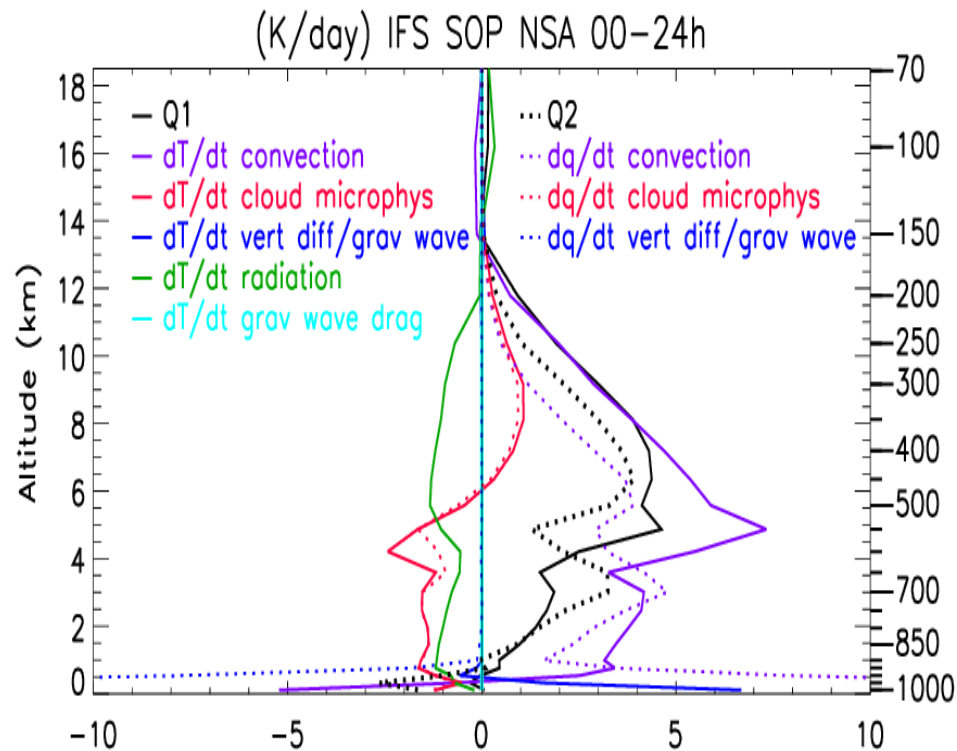
# Large-scale effects of convection (3)

## Budgets from Obs: Tropical Pacific



Yanai et al., 1973, JAS

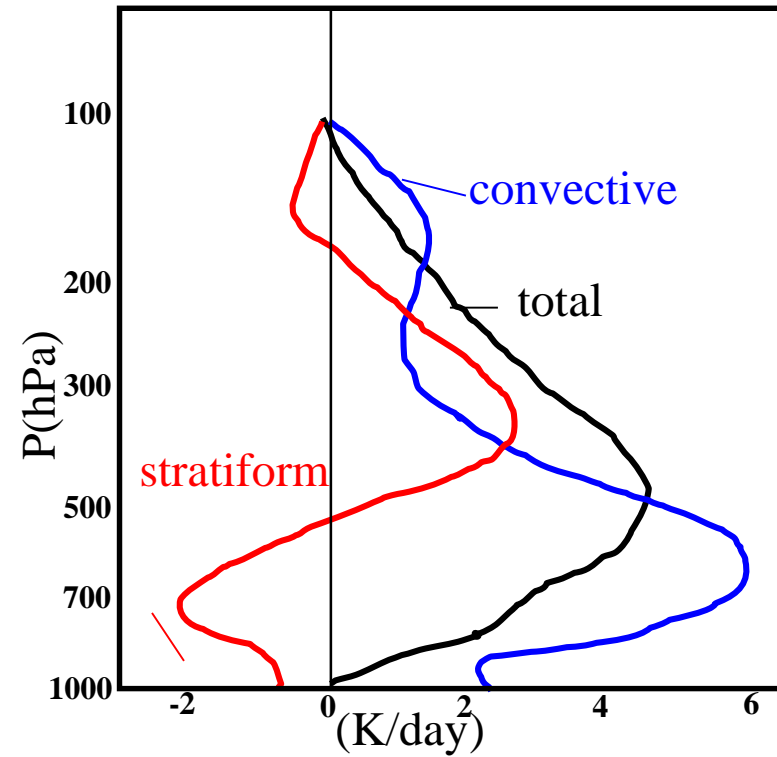
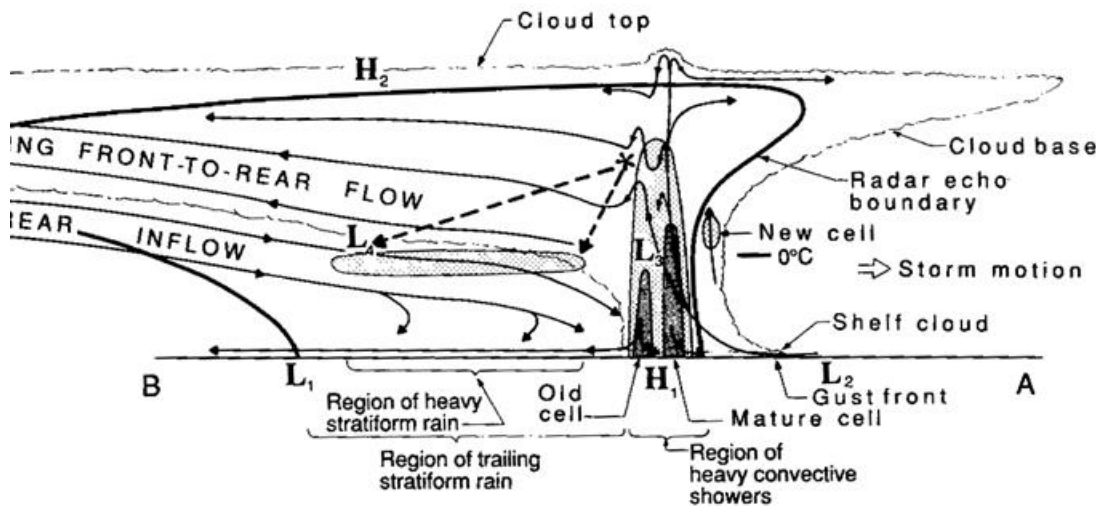
## Budgets Obs&IFS: Indian Ocean



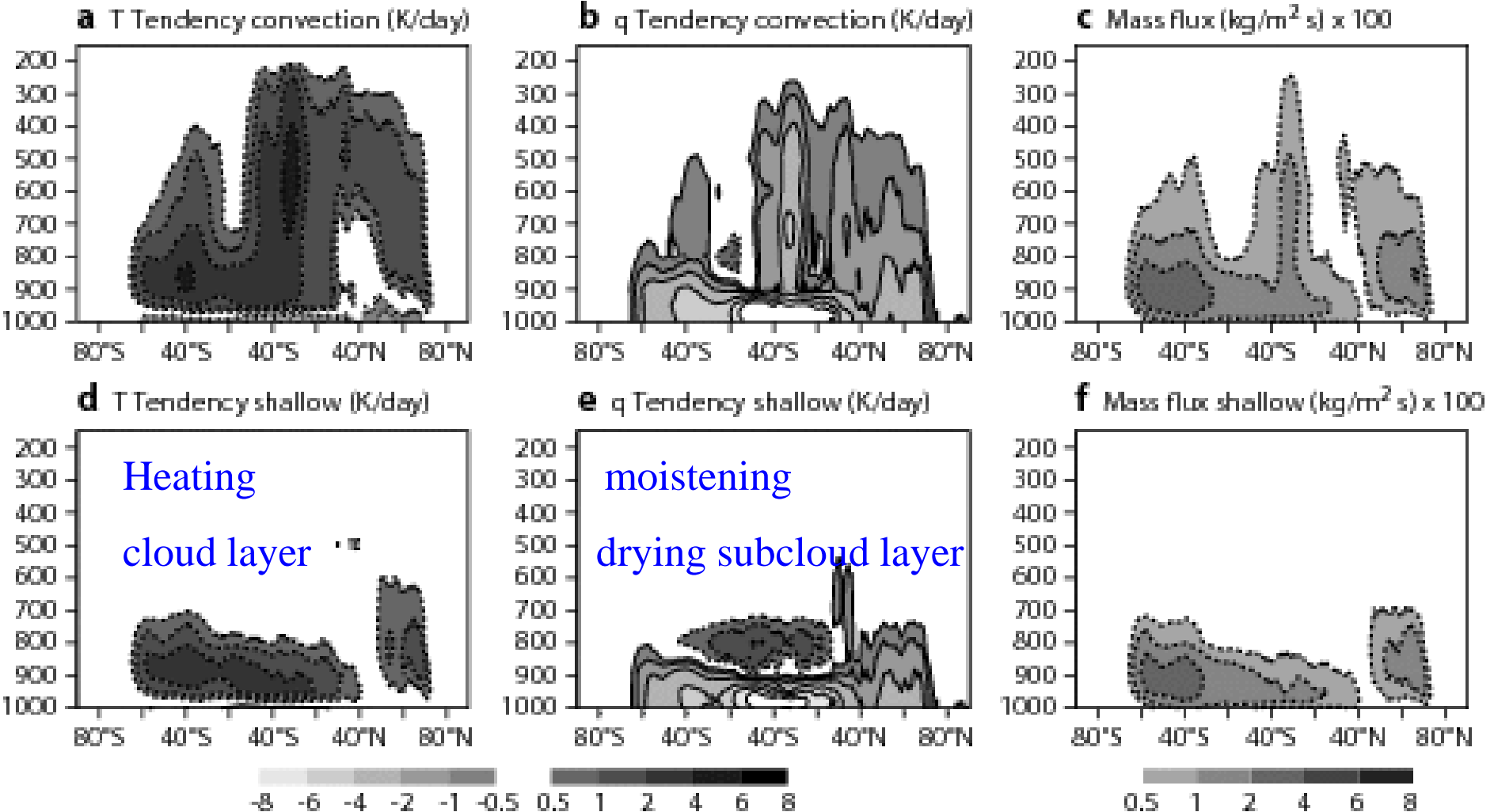
courtesy Ji-Eun Kim and Chidong Zhang

Note the typical tropical maximum of  $Q_1$  at 500 hPa,  $Q_2$  maximum is lower and typically around 700 -800 hPa

# Effects of mesoscale organization convective and stratiform heating modes



# Zonal mean convective tendencies (deep & shallow) July 2013 and mass flux in IFS



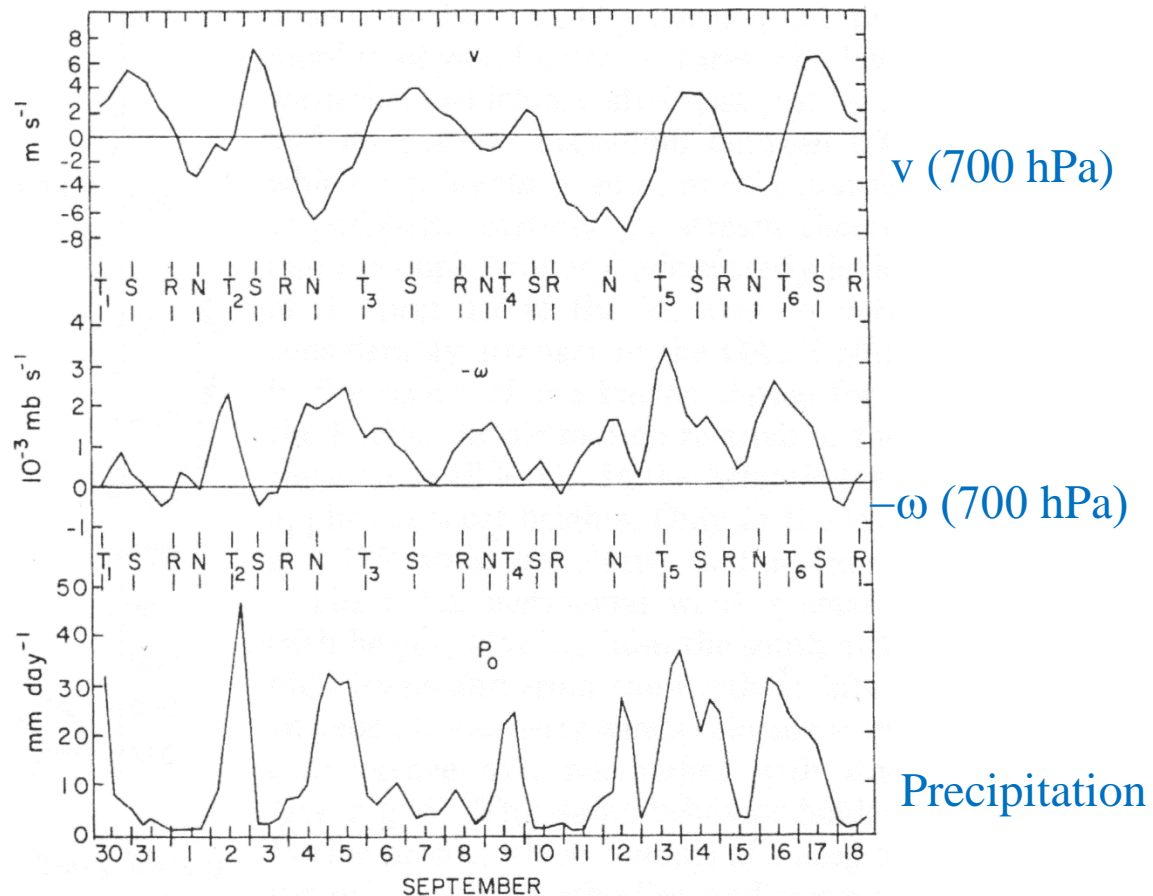
# Convective quasi-equilibrium

Arakawa and Schubert (1974) postulated that the level of activity of convection is such that their stabilizing effect balances the destabilization by large-scale processes.

Observational evidence:

GARP Atlantic Tropical  
Experiment (1974)

Thompson et al., JAS, 1979



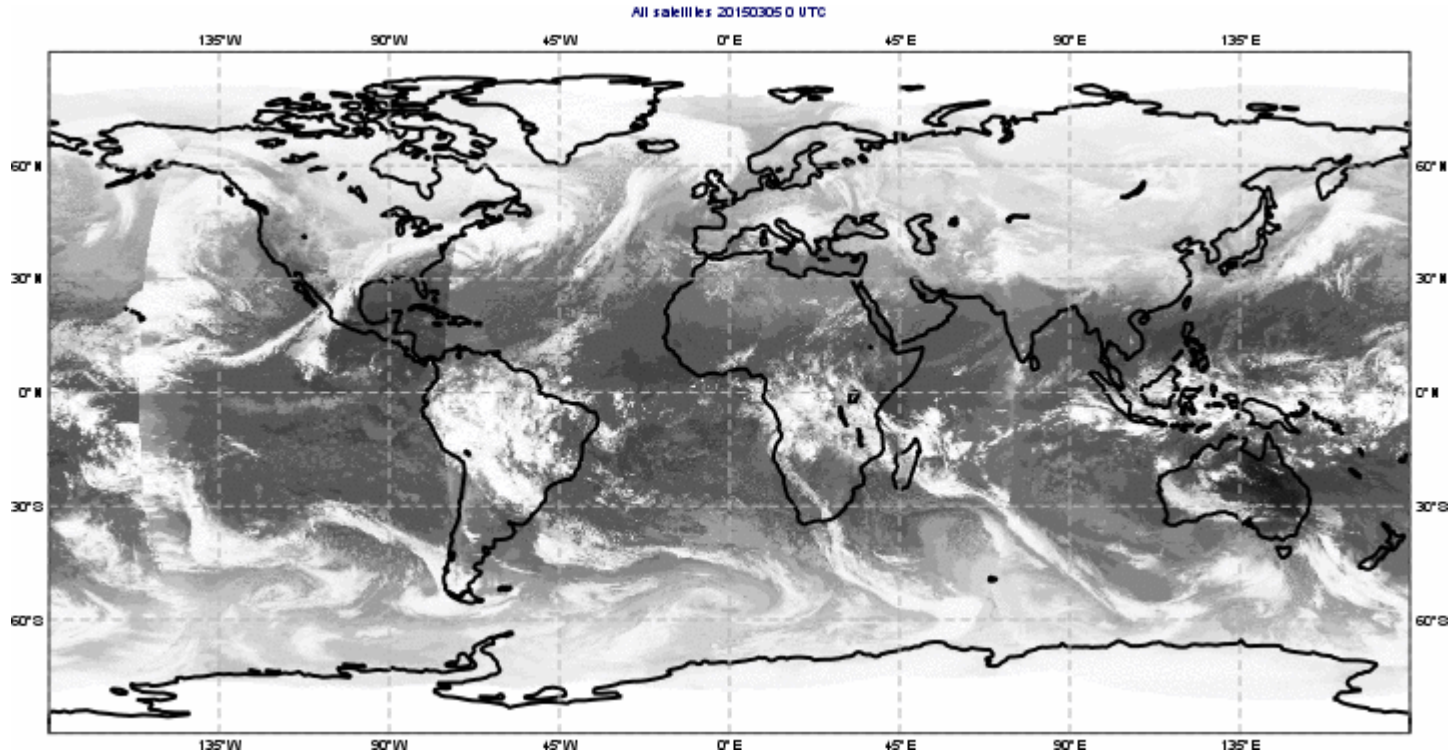
# Summary

- Convection affects the atmosphere through **condensation / evaporation and eddy transports**
- To first order convection stabilizes the environment and on large horizontal scales convection is in **quasi-equilibrium** with the large-scale forcing
- **Q1, Q2 and Q3** are quantities that reflect the time and space average effect of convection (“unresolved scale”) and stratiform heating/drying (“resolved scale”)
- An important parameter for the strength of convection is **CAPE**
- **Shallow convection** is present over very large (oceanic) areas, it determines the redistribution of the surface fluxes and the transport of vapor and momentum from the subtropics to the ITCZ
- The effect of convection (local heat source) is fundamentally different in the middle latitudes and the Tropics. In the Tropics **the Rossby radius of deformation  $R=N H/f$**  ( $N$ =Brunt Väisälä Freq,  $f$ =Coriolis parameter,  $H$ =tropopause height) is infinite, and therefore the effects are not locally bounded, but spread globally via gravity waves – **“throwing a stone in a lake”**



# Convectively coupled waves:

## Rossby, Kelvin, MJO and African easterly Waves



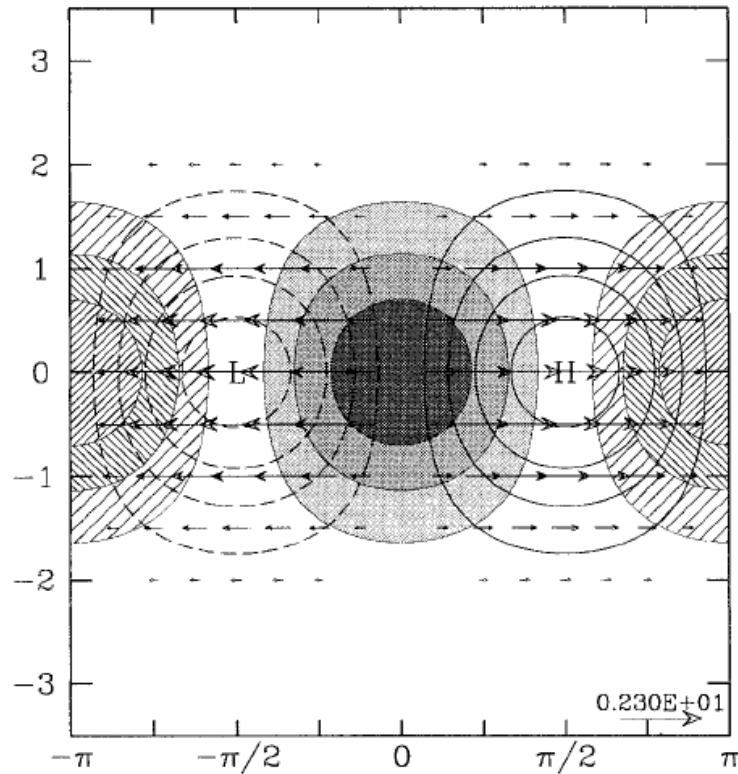
Analytical: solve shallow water equations (see Lecture Note)

$$u = u_0 f(y) e^{i(kx - \omega t)}; \quad f(y) = e^{-y^2/2}$$

$$v = \hat{v}(y) f(y) e^{i(kx - \omega t)}; \quad \hat{v}(y) = \textit{Hermite Polynomials}$$

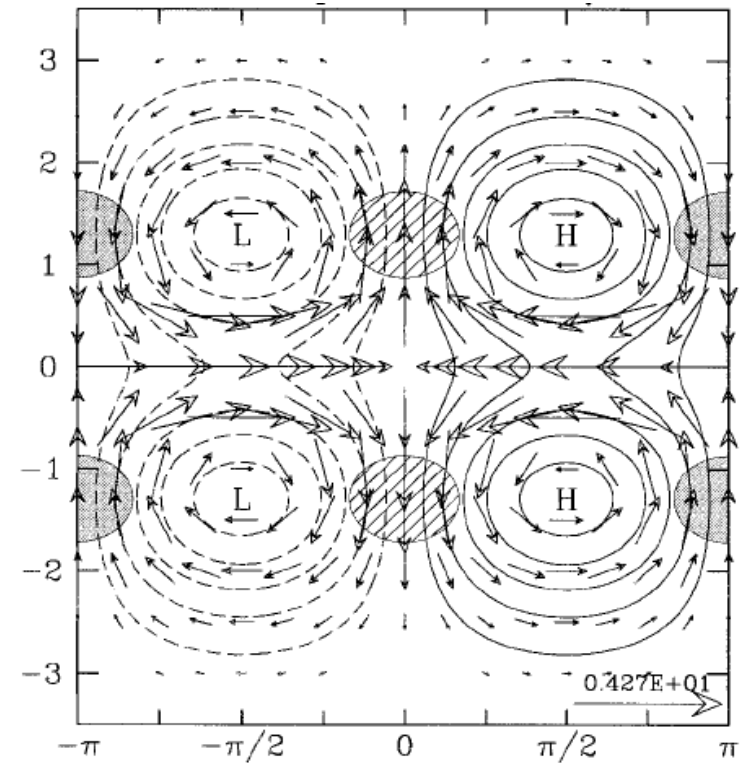


# The Kelvin wave



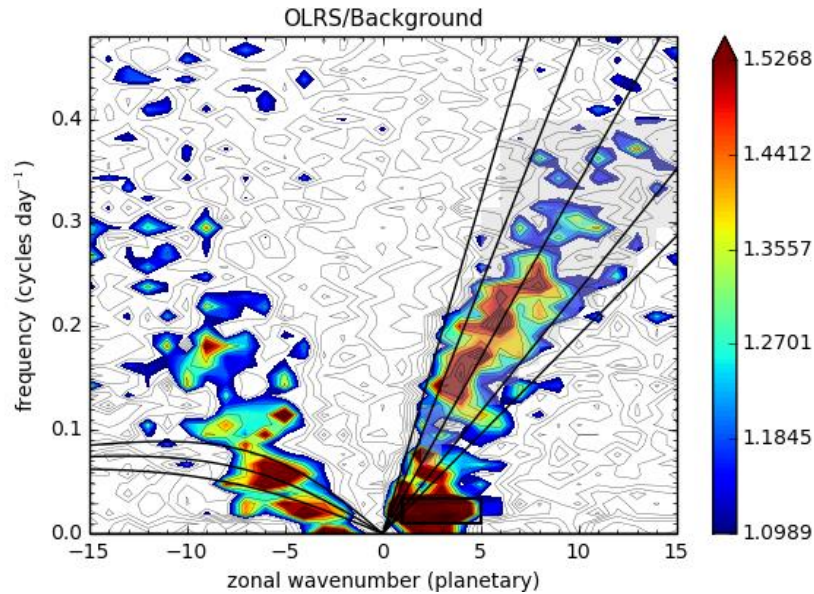
$V=0$ , eastward moving  $\sim 18$  m/s  
sym. around equator  
OLR anomaly shaded, winds max at equator

# The $n=1$ Rossby wave

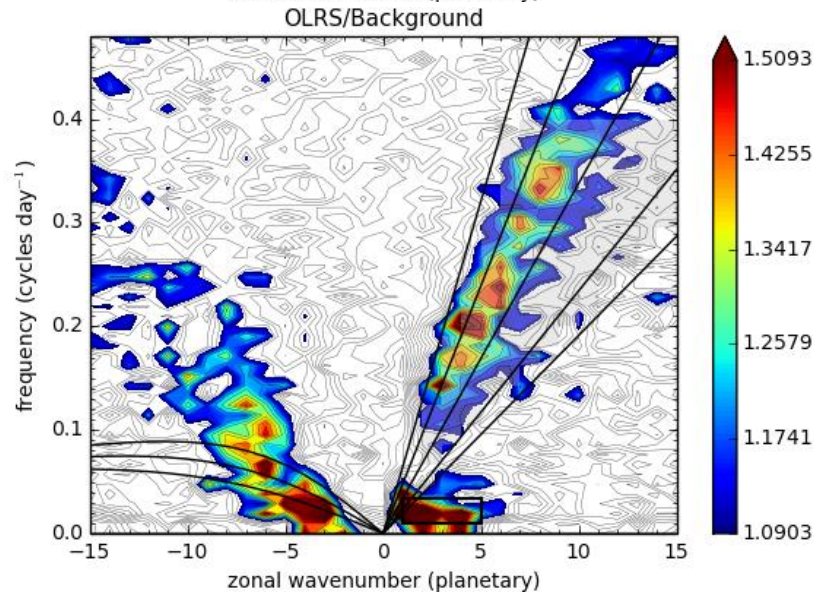


westward moving  $\sim 5$  m/s  
sym. around equator

# Wavenumber frequency Diagrams of OLR



**ECMWF Analysis**

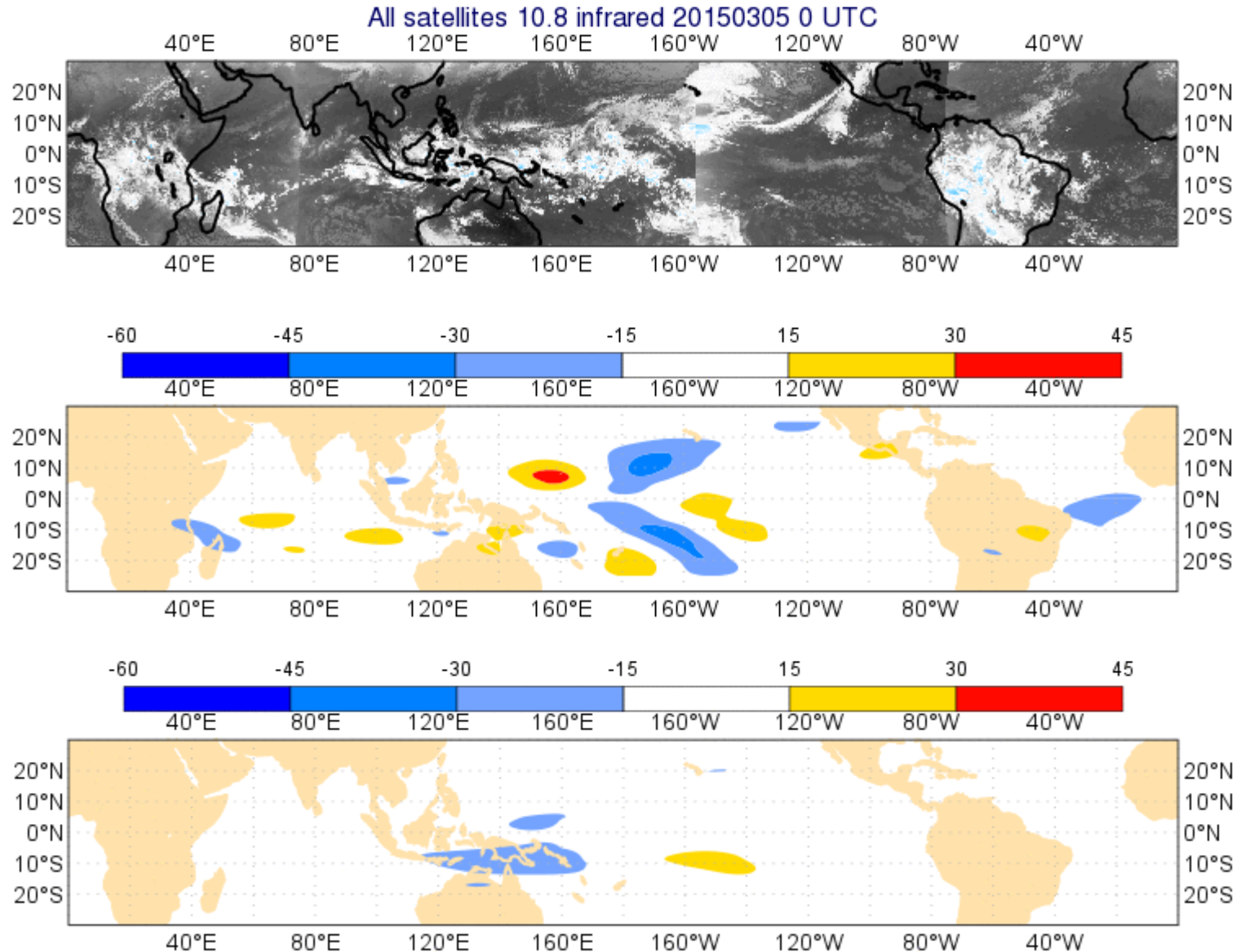


**Cy40r1 6y (2014)**

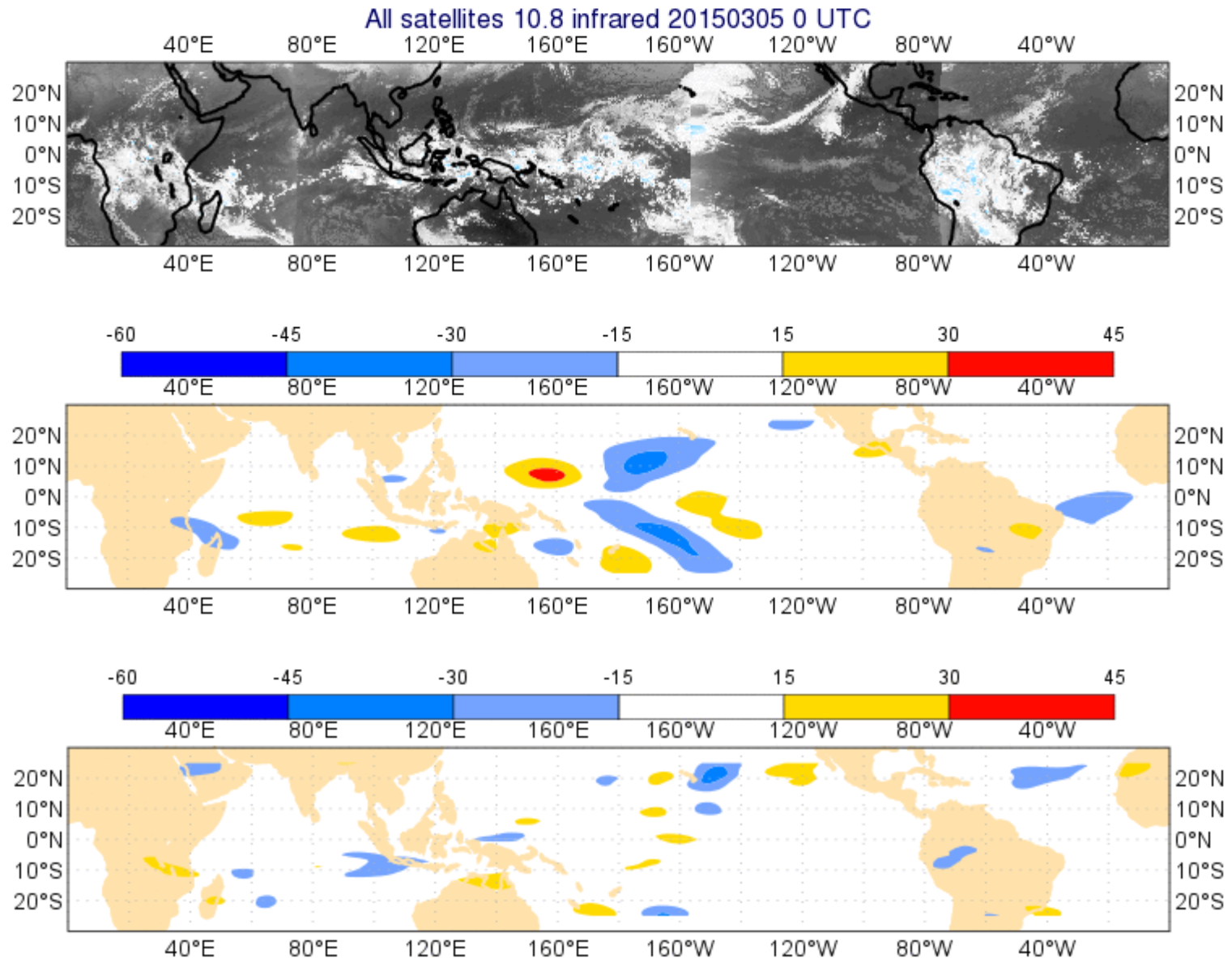
software courtesy  
Michael Herman (New  
Mexico Institute)

(all spectra have been  
divided by their own=  
smoothed background)

# Rossby & MJO 5.3.2015-18.3 2015



# Rossby & Kelvin 5.3.2015-16.3 2015

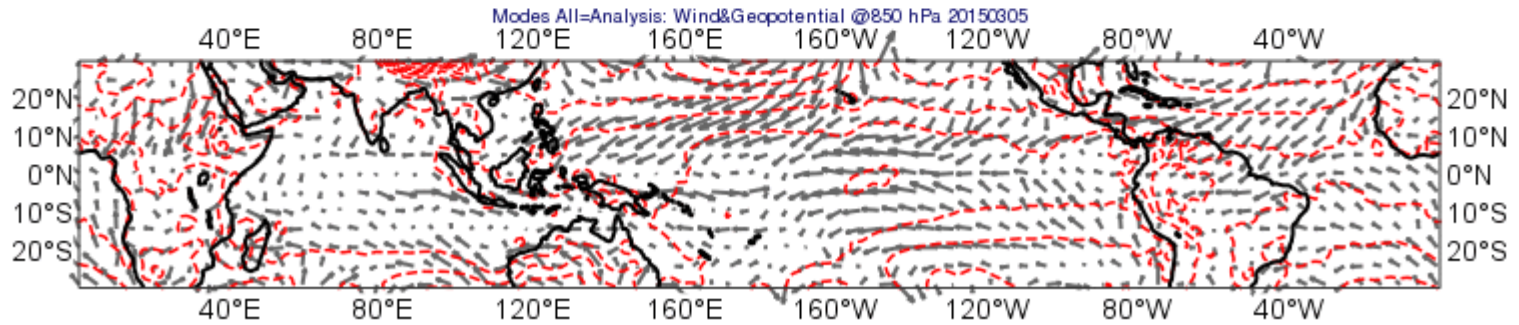




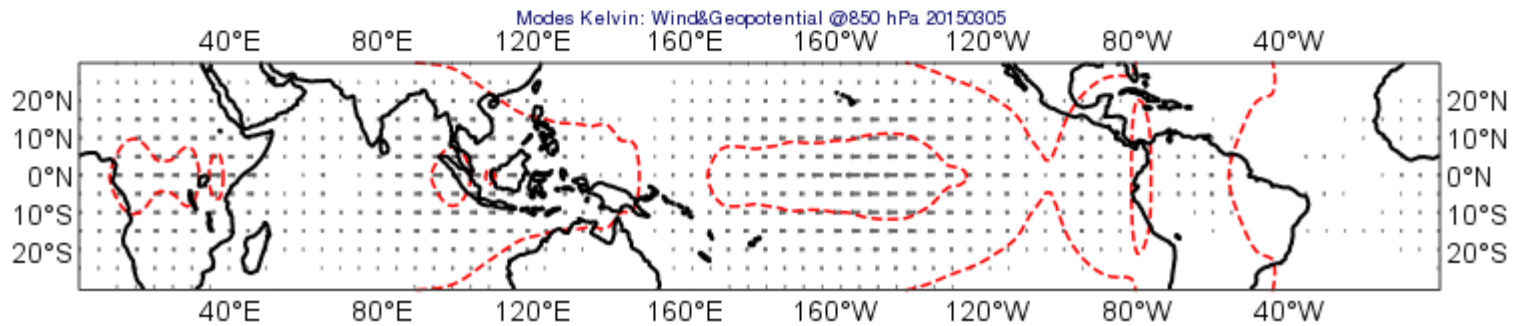
# Normal mode projection and filtering

Žagar et al. (Geosc. Mod. Dev. 2015)

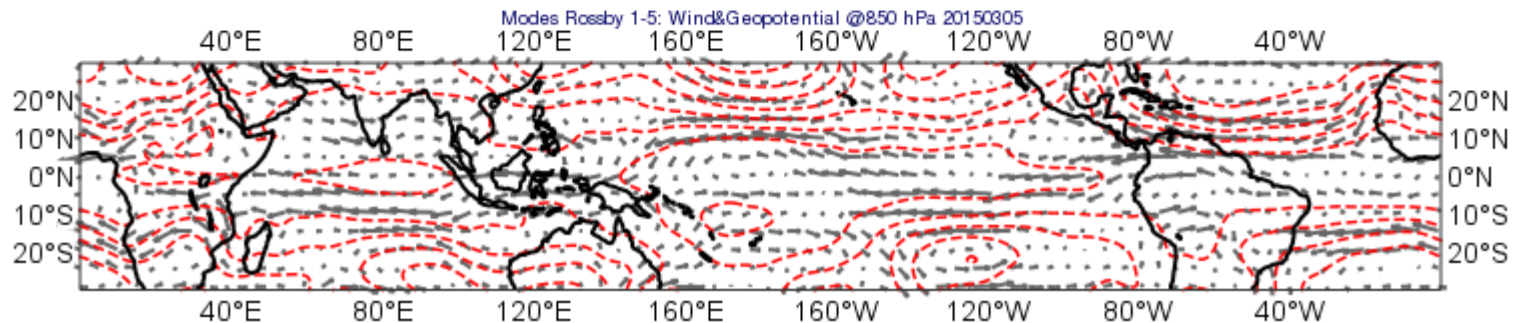
All  
=Analysis



Kelvin



Rot 1-5

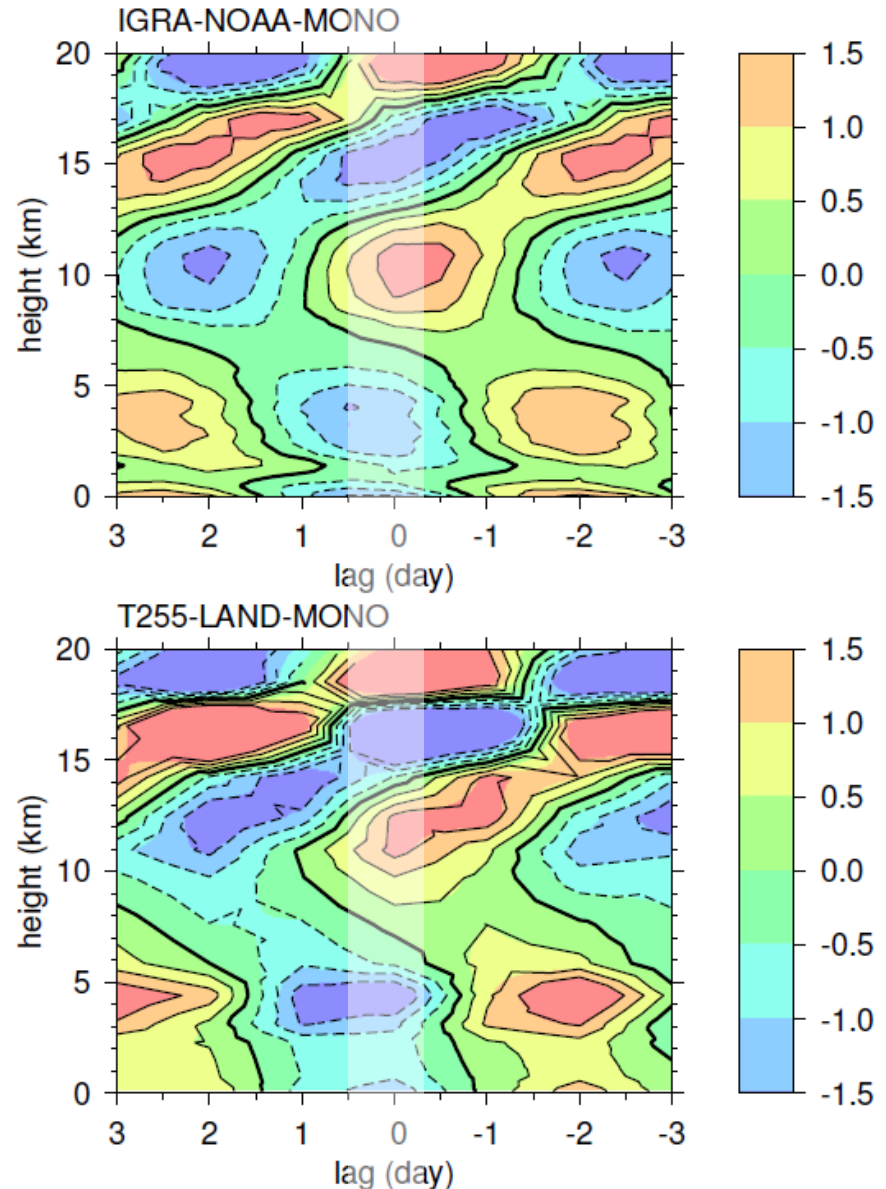


# Kelvin waves: vertical T-anomalies

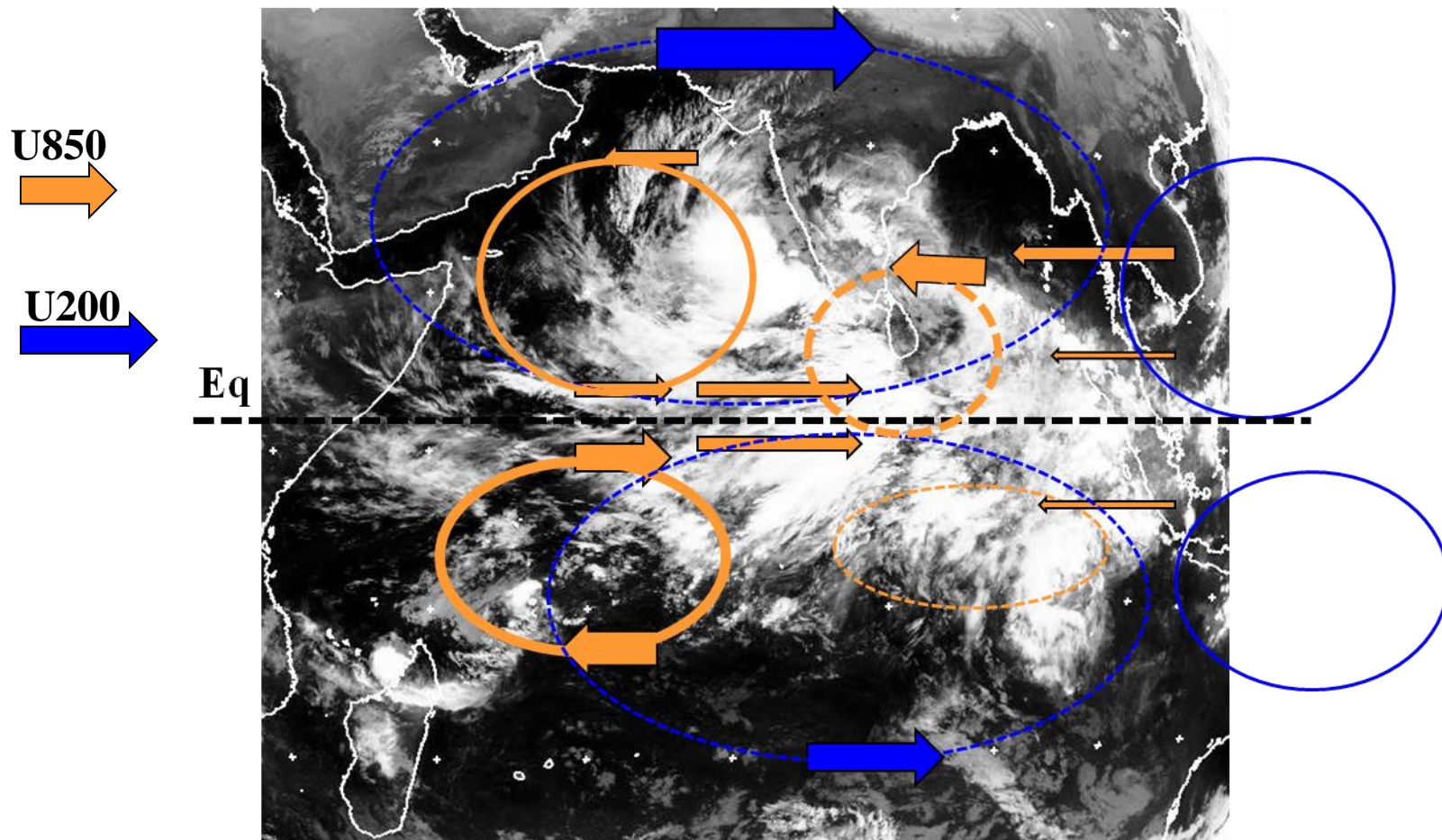
At  $z \sim 10$  km, warm anomaly and convective heating are in phase, leading to :

- the conversion of potential in kinetic energy =  $\alpha\omega$
- The generation of potential energy =  $NQ$

see also [G. Shutts \(2006, Dyn. Atmos. Oc.\)](#)



# The MJO over Indian Ocean



27 November 2011: Meteosat 7 + ECMWF Analysis



# African Easterly waves

Hovmoeller diagrams as an easy way to plot waves (propagation + amplitude)

